Direct Processes in Effective Hamiltonians to Mimic Microwave Communications in Noisy Environments

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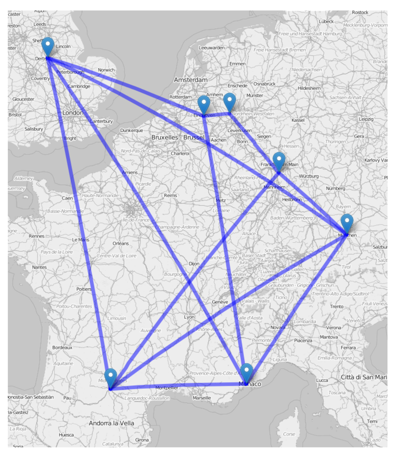
Introduction / Project

Project Nemf21



- Partners across Europe
- Industry and Academia
- Aim: Improvement of *Chip-2-Chip* and *On-Chip* communications

Current State

- Increasing number of transistors on ICs
- Connected by wires → heat
- Miniaturization → cross talk between wires

Work Done In Nice

Microwave Experiments
- *Reverberating* resonators
- Microwaves over *PCB* boards

Theoretical Numerical
- Model communication processes in presence of noisy environments
- Analyze stability of transmission

Problem Description and Theoretical Approach

Create model system $\boldsymbol{H}$ for communication process

$\Downarrow$ ←add process here ?

Obtain *Scattering Matrix* $\boldsymbol{S}$ for communication process

$\Downarrow$ ←add process here ?

Calculate transmission $\boldsymbol{T}$ between dedicated antennas

$\Downarrow$ ←add noise

Calculate transmission distribution $\boldsymbol{P(T)}$

Describe stability of $\boldsymbol{P(T)}$ under *noise* given by *RMT* Hamiltonian

Effective Hamiltonian and the Scattering Matrix

- Full Hamiltonian $(M+N \times M+N)$

$$H_{\text{total}} = \begin{pmatrix} H_{\text{channel}} & W^T \\ W & H_{\text{system}} \end{pmatrix}$$

- Effective Hamiltonian $(N \times N)$

$$H_{\text{eff}} = H_{\text{system}} - \tfrac{\mathrm{i}}{2} W W^T$$

- Scattering Matrix $(M \times M)$

$$S(E) = 1 - \mathrm{i} W \frac{1}{E - H_{\text{eff}}} W^T$$

- Transmission between specific antennas
$$T = |S_{12}(E=0)|^2$$
- Model noise via *RMT* on H_{system}
- Measure for direct processes: $\langle S_{12} \rangle_{\text{ensemble}}$ (Zero in standard *RMT*)
- Distribution of transmissions
$$P(T) = \left\langle \delta\left(T - |S_{12}|^2\right) \right\rangle_{\text{ensemble}}$$
- determine $\langle T \rangle$, $\mathrm{Var}(T)$, ...

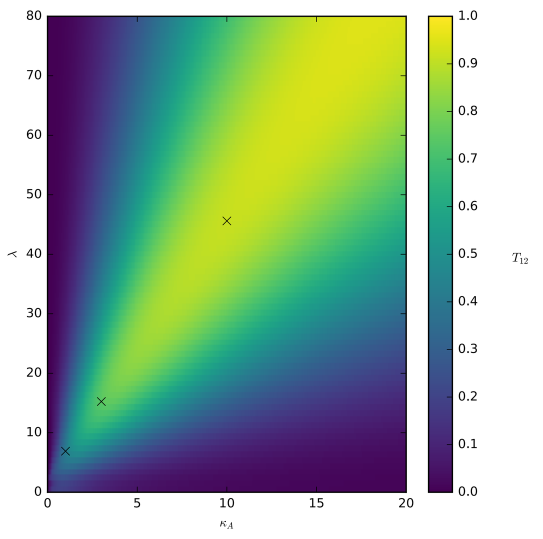
Direct Communication Processes

Characterized by: Non-zero $\langle S_{12} \rangle_{\text{ensemble}}$

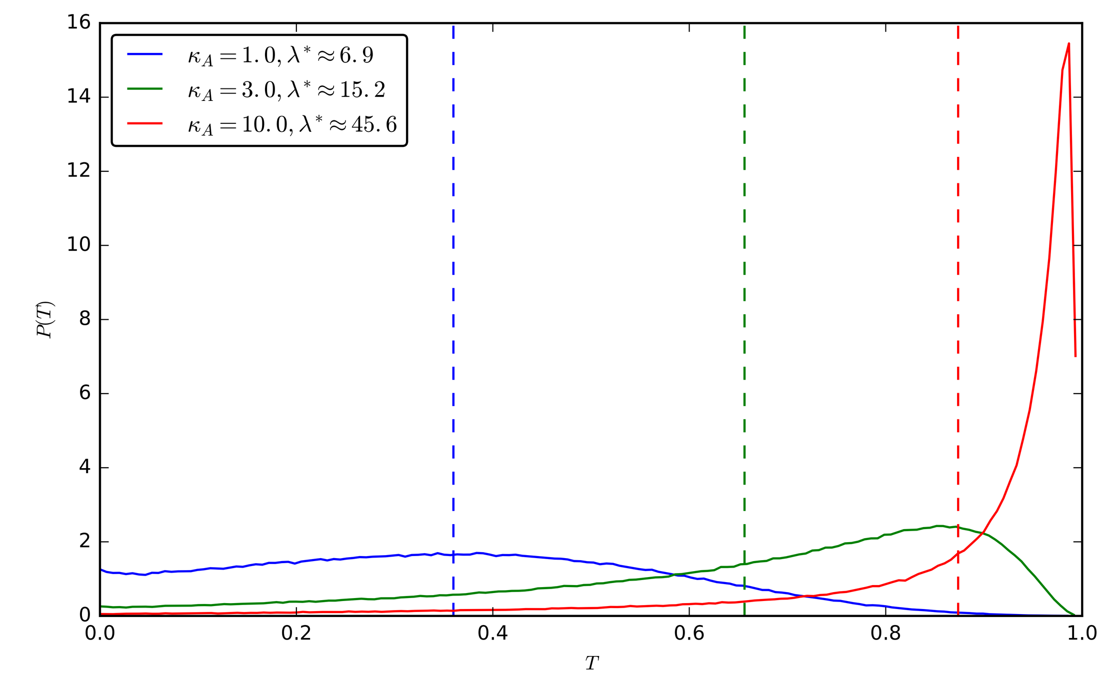
1. Change $\boldsymbol{S}$ directly
   - Scattering phases
     Advantage: Access to broad knowledge
     Drawback: Less direct, Link to micro-wave experiments?
2. Change *Anti-Hermitian* part of the H_{eff}
   Introduce correlation on levels/channels
3. Change *Hermitian* part of H_{eff}
   $$H_{\text{eff}} = H_{\text{GOE}} + H_{\text{direct}} + \tfrac{\mathrm{i}}{2} W W^T$$
   Advantage: Analyze spectrum of H_{eff} and the S-matrix (autocorrelation, $P(T)$)
   - Use $H_{\text{direct}} = \frac{\sqrt{N}}{\pi} \lambda \left(|1\rangle\langle 2| + |2\rangle\langle 1|\right)$
   - Together with constant W (*doorway states*)
   $$W^T = \sqrt{N} \sqrt{\frac{2\kappa_A \Delta}{\pi}} \begin{pmatrix} 1 & 0 & \\ & 1 & 0 \cdots \\ & & 1\ 0 \end{pmatrix}$$

Direct Process - Optimal Parameters

- Parameter dependence
  $T_{12}(\kappa_A, \lambda)$
  $\langle S_{12}(\kappa_A, \lambda) \rangle$
- *Optimal* choice $\lambda^*(\kappa_A)$
  defined by
  $\langle T_{12} \rangle \longrightarrow \max$
  $\lambda^*(\kappa_A) \approx \sqrt{N}\kappa_A$

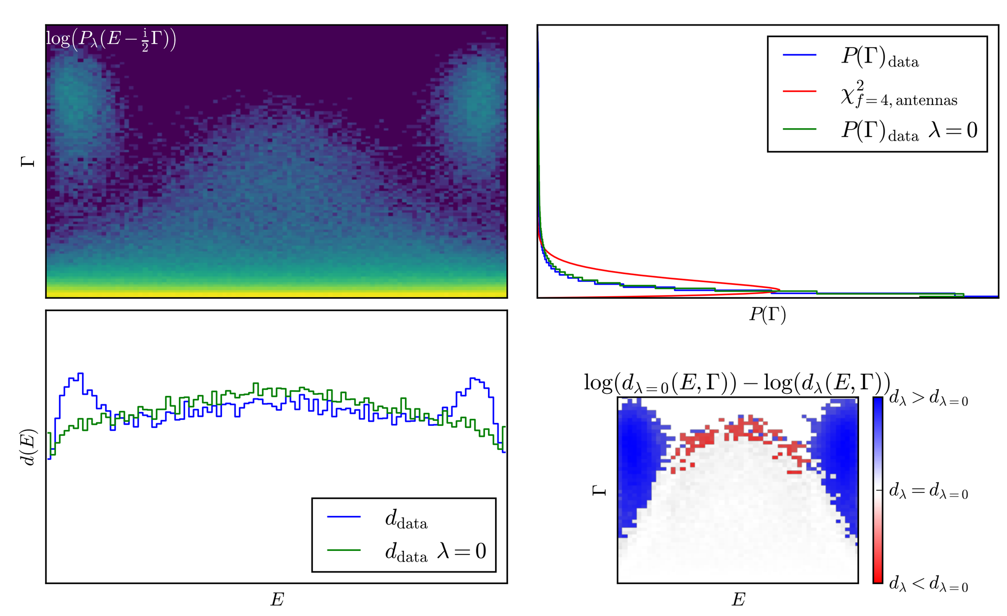


- Transmission probability $P(T)$



Spectrum of H_{eff} in Presence of a Direct Process

- Analyze *direct process* through spectrum of H_{eff}
- Example at optimal coupling $\kappa_A = 1$



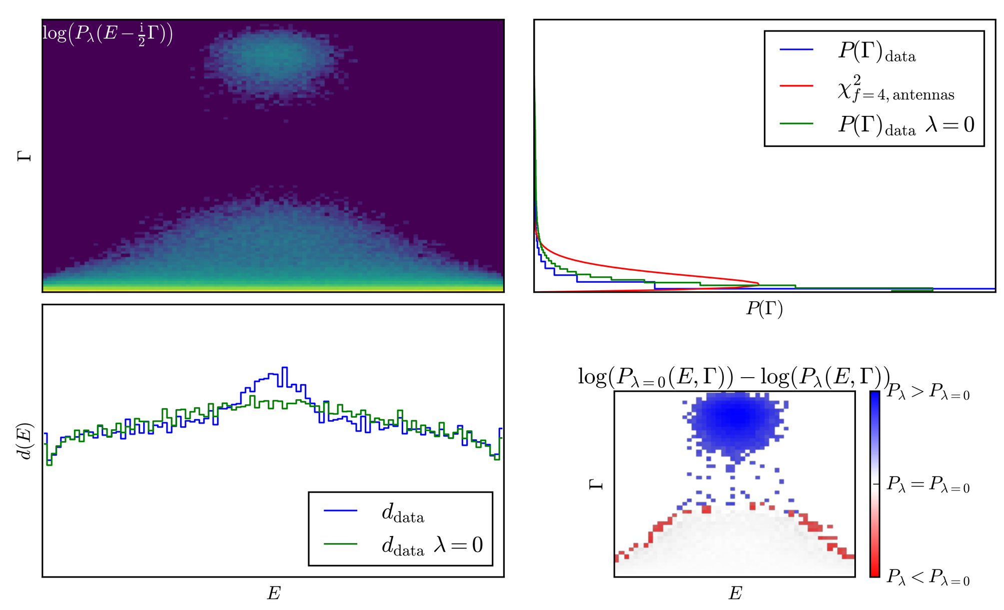
- λ introduces another energy scale
- shifts eigenvalues from center to edges

Theoretical Description

- Optimal parameter $\lambda^*(\kappa_A)$ suggest:
  $H_{\text{eff}} = \varepsilon H_{\text{GOE}} + H_{\text{direct}} + \tfrac{\mathrm{i}}{2} W W^T, \varepsilon \ll 1$
- 2×2 model for averages in S_{12}
  $$\langle f(S_{12}) \rangle = \iiint \mathrm{d}h_x P(h_x) \mathrm{d}h_a P(h_a) \mathrm{d}h_b P(h_b)$$
  $$f\left(-\mathrm{i} N \sigma^2 \frac{\left(1 - \frac{h_x^2}{\left(2\frac{\sqrt{N}\lambda}{\pi}\right)^2}\right)\left(\frac{\sqrt{N}\lambda}{\pi} + h_b\right)}{\left(E - \mathrm{i}\frac{N\Delta\kappa_A}{\pi} - h_a\right)^2 - \left(\frac{\sqrt{N}\lambda}{\pi} + h_b\right)^2} \right)$$
- Similar for non-perturbative 2×2 model
- *Method of Steepest Descent*
  $$P(T) = \left\langle \delta\left(T - |S_{12}|^2\right) \right\rangle = \delta\left(T - T_A\left(N\Delta^2 \tfrac{\kappa_A^2}{\lambda^2}\right)\right)$$
  with T_A as below.
- Right scaling for optimum $\lambda^*(\kappa_A) = \sqrt{N}\kappa_A$
- But no statement about $\mathrm{Var}(T)$ possible

Direct Communication Processes Alternatives

- Add to S
  How? By adding scattering phases
  Why? Usually done the other way around
  $S_{ab}^{\text{RMT}} = \left(U S^{\text{direct}} U^T\right)_{ab}$
- Model process in coupling matrices W
  Make channels linearly dependent
  $$W \cdot W^T = \frac{2\kappa_A}{\pi} \begin{pmatrix} 1 & \cos\left(\frac{\pi(1-\theta_{12})}{2}\right) & 0 & \\ \cos\left(\frac{\pi(1-\theta_{12})}{2}\right) & 1 & 0 & \\ & & 1\ 0 & \\ 0 & 0 & 0\ 0 & \\ & & & \ddots \end{pmatrix}$$
- Extreme Case $\theta_{12} = 1$:
  One over-coupled channel, one ghost channel
- Spectrum for maximally correlated W:



Open Questions

- Other models for communication processes
- Optimal modification of H_{eff}- Variational approach?
- Energy dependence of $S(E)$ for $\lambda^*(\kappa_A)$
- Connection to experiments
  - Estimate κ_A from reflection
    $T_A(\kappa_A) = 1 - |\langle S_{ii} \rangle|^2 = \frac{4\kappa_A}{|1+\kappa_A|^2}$
  - Decay of correlation functions?
- Extension to vector quantities $\vec{E}, \vec{B}$
- Modeling communication properties of H_{eff} by *POE* $\longrightarrow$ *GOE*