Brunel University London

Department of
Economics and Finance

Economics and Finance Working Paper Series

Guglielmo Maria Caporale, Luis Gil-Alana
and C. James Orlando

Linkages between the US and European Stock Markets:
A Fractional Cointegration Approach

February 2015

LINKAGES BETWEEN THE US

AND EUROPEAN STOCK MARKETS:

A FRACTIONAL COINTEGRATION APPROACH

Guglielmo Maria Caporale, Brunel University London, UK

Luis A. Gil-Alana, University of Navarra, ICS, Pamplona, Spain

C. James Orlando, University of Navarra, Pamplona, Spain

January 2015

Abstract

This paper analyses the long-memory properties of US and European stock indices, as well as their linkages, using fractional integration and fractional cointegration techniques. The empirical evidence suggests the presence of unit roots in both the S&P 500 Index and the Euro Stoxx 50 Index, and also that cointegration only holds over the subsample ending in March 2009, i.e. when the global financial crisis was still severe; subsequently, the US and European stock markets diverged and followed different recovery paths, possibly as a result of various factors such as diverging growth and monetary policy.

Corresponding author: Professor Guglielmo Maria Caporale, Department of Economics and Finance, Brunel University, London, UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. Email: Guglielmo-Maria.Caporale@brunel.ac.uk

1

1. Introduction

Globalisation has led to international financial markets becoming increasingly interconnected, with equities displaying a high degree of co-movement across countries. This paper analyses linkages between US and European stock markets. Specifically, it applies fractional integration and cointegration techniques with the aim of testing co-movement between the S&P 500 Index and the Euro Stoxx 50 Index over the period from 1986 to 2013. Interestingly, we find that following the Great Recession of 2008 and early 2009, the pattern of co-movement changed, namely, after the trough in both US and European stock markets in the first quarter of 2009, the recovery paths were very different. It is well known that Europe and the US have experienced diverging growth and monetary policy in recent years. The global financial crisis that had originated in the US then led to a serious debt crisis in the Eurozone, and to the ECB eventually adopting its own version of Quantitative Easing (QE) in the form of the so-called long-term refinancing operation (LTRO) in December 2011. The initial monetary policy response had been much more expansionary in the US, the Fed immediately espousing QE; tight fiscal policy was another factor leading to much weaker growth in Europe than in the US, which also meant lower Treasury yields. Other features of European stock markets, such as their being more cyclical and including less technology shares, may also have contributed to their underperformance.

The structure of this paper is as follows. Section 2 contains a brief discussion of the literature on long memory in stock markets and cross-market linkages. Section 3 outlines the empirical methods used for the analysis. Section 4 describes the data and the main empirical results, while Section 5 offers some concluding remarks.

2.   Literature review

There is an extensive literature testing whether stock prices are unpredictable and follow a random walk as implied by the Efficient Market Hypothesis or are instead mean-reverting. Two well-known studies by Fama and French (1988) and Poterba and Summers (1988) both found that US stock prices exhibit mean reversion. Techniques such as variance-ratio tests, regression coefficient and univariate unit root tests were used in other papers, for instance those by Fama (1995) and Choudhry (1997), also providing evidence of mean reversion. By contrast, Alvarez-Ramirez et al. (2008) concluded that both the S&P 500 and Dow Jones Industrial Average indices followed a random walk after 1972.

However, it is now well known that the unit root tests traditionally carried out (e.g., those by Dickey and Fuller (1979, 1981), Phillips and Perron (1988), and Ng and Perron (2001)) have very low power. This has led researchers to using other approaches to analyse long-run mean reversion, also known as 'long memory'. The literature on long memory in stock returns has produced mixed evidence. Greene and Fielitz (1977) found evidence of persistence in daily US stock returns using R/S methods. Similar conclusions were reached by Crato (1994), Cheung and Lai (1995), Barkoulas and Baum (1996), Barkoulas, Baum, and Travlos (2000), Sadique and Silvapulle (2001), Henry (2002), Tolvi (2003) and Gil-Alana (2006), for monthly, weekly, and daily stock market returns respectively. Several other studies, however, could not find any evidence of long memory. They include Aydogan and Booth (1988), Lo (1991), who used the modified R/S method and spectral regression methods, and Hiemstra and Jones (1997).

A number of papers have focused in particular on the Standard and Poor's (S&P) 500 Index. Granger and Ding (1995a,b) used power transformation or absolute value of

the returns as a proxy for volatility, and estimated a long-memory process to examine persistence in volatility, establishing some stylized facts regarding the temporal and distributional properties of these series. However, in a following study, Granger and Ding (1996) found that the parameters of the long memory model varied considerably across subsamples. The issue of fractional integration with structural breaks in stock markets has been examined by Mikosch and Starica (2000) and Granger and Hyung (2004) among others. Stochastic volatility models using fractional integration have been estimated by Crato and de Lima (1994), Bollerslev and Mikkelsen (1996), Ding and Granger (1996), Breidt, Crato and de Lima (1997, 1998), Arteche (2004), Baillie, Han, Myers and Song (2007), etc.

Another strand of the literature focuses not only on individual time series, but also on the co-movement between international stock markets. It dates back to Panto et al. (1976), who used correlations to test for stock market interdependence. Subsequent studies relied on the cointegration framework developed by Engle and Granger (1987) and Johansen (1991, 1996) to examine long-run linkages. For instance, Taylor and Tonks (1989) showed that markets in the US, Germany, Netherlands and Japan exhibited cointegration over the period October 1979 - June 1986. Jeon and Von-Furstenberg (1990) used the VAR approach and found an increase in cross-border cointegration since 1987. For post-crash periods and times of heightened volatility, Lee and Kim (1994) showed that the US and Japanese markets had tighter linkages. Copeland and Copeland (1998) and Jeong (1999) found a leadership role for the US relative to smaller markets. Wong et al. (2005) used fractional cointegration and reported linkages between India and the US, the UK and Japan. Syllignakis and Kouretas (2010) studied instead the integration of European and US stock markets, finding strong long-run linkages between US and German stock prices. Bastos and

4

Caiado (2010) found evidence of cointegration for a wider sample of forty-six developed and emerging countries. The present study contributes to this literature by using fractional cointegration techniques to test for long-run linkages between the US and European financial markets and highlighting a change in their relationship.

3.   Empirical methodology

The empirical analysis is based on the concepts of fractional integration and cointegration. For our purposes, we define an I(0) process as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Therefore, a time series $\{x_t, t = 1, 2, \dots \}$ is said to be I(d) if it can be represented as

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \dots, \tag{1}$$

with $x_t = 0$ for $t \leq 0$, where L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is $I(0)$. By allowing d to be fractional, we introduce a much higher degree of flexibility in the dynamic specification of the series in comparison to the classical approaches based on integer differentiation, i.e., d = 0 and d = 1.

Given the parameterisation in (1), different models can be obtained depending on the value of d. Thus, if d = 0, $x_t = u_t$, x_t is said to be "*short memory*", and the observations may be weakly autocorrelated, i.e. with the autocorrelation coefficients decaying at an exponential rate; if d > 0, x_t is said to be "*long memory*", so named because of the strong association between observations far apart in time. If d belongs to the interval (0, 0.5) x_t is still covariance stationary, while d $\geq$ 0.5 implies nonstationarity. Finally, if d < 1, the series is mean reverting, implying that the effect of the shocks disappears in the long run, in contrast to what happens if d $\geq$ 1, when the effects of shocks persist forever.

We estimate d using a parametric Whittle function in the frequency domain (Fox and Taqqu, 1986; Dahlhaus, 1989) along with a Lagrange Multiplier (LM) test developed by Robinson (1994a) that has the advantage that it remains valid even in the presence of nonstationarity. Some semi-parametric methods (Robinson, 1995a,b) will also be used for the analysis.

For the multivariate case, we apply fractional cointegration methods. First we test for homogeneity in the orders of integration of the two series using a procedure developed by Robinson and Yajima (2002); then, since the two parent series appear to be I(1), we run a standard OLS regression of one variable against the other, and examine the order of integration of the estimated errors. A Hausman test of the null hypothesis of no cointegration against the alternative of fractional cointegration (Marinucci and Robinson, 2001) is also carried out.

4.   Data and empirical results

The series used for the analysis are the S&P 500 Index and the Euro Stoxx 50 Index (downloaded from Yahoo! Finance), representing two of the most liquid markets in the world. The frequency is monthly and the sample period goes from December 31, 1986, to December 31, 2013.

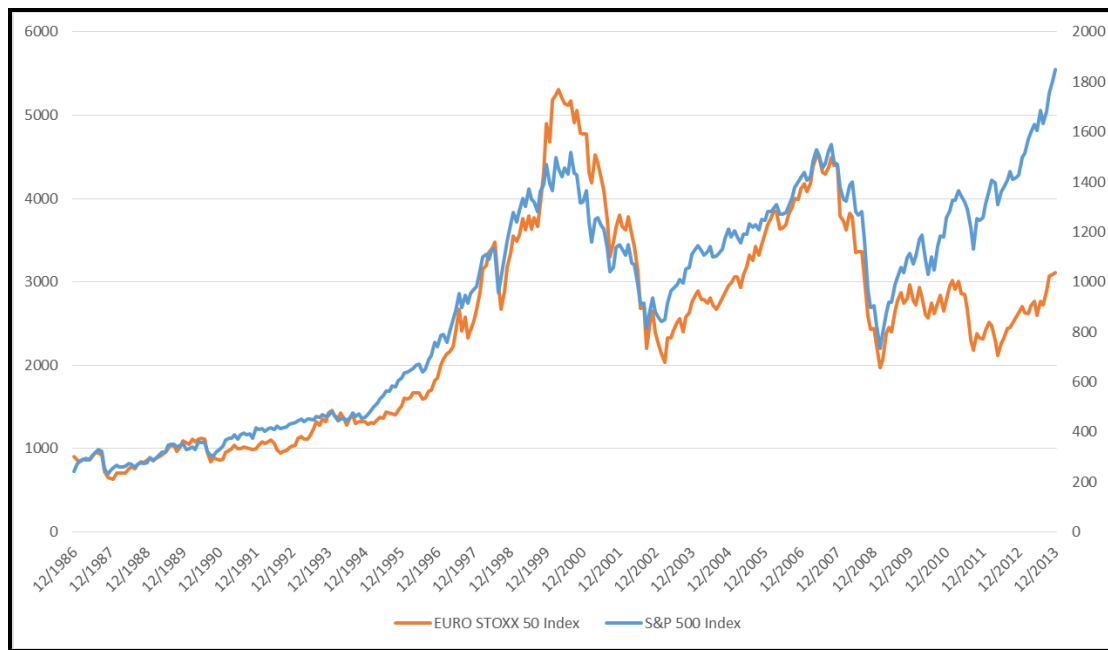 **Figure 1: Euro STOXX 50 and S&P 500**

Figure 1 displays the two series. They exhibit very similar behaviour from the beginning of the sample until 2009, with two peaks occurring in 2000 and 2007, followed by a sharp decline in 2001 and 2008. Since the start of the recovery from the global financial crisis in 2009, a much faster recovery is observed in the S&P 500 than in Euro Stoxx 50.

As a first step we estimate the fractional differencing parameter in the following model,

$$y_t \ = \ \beta_0 \ + \ \beta_1 t \ + \ x_t, \quad (1 - L)^d x_t = u_t, \qquad t \ = \ 1, 2, \ldots \tag{2}$$

where y_t is the observed series, β_0 and β_1 are the coefficients corresponding to an intercept and a linear time trend, and x_t is assumed to be I(d), where d can take any real value. Therefore the error term, u_t, is I(0), and is assumed in turn to be a white noise, a non-seasonal and seasonal (monthly) AR(1) process and to follow the exponential spectral model of Bloomfield (1973), which is a non-parametric approach that produces autocorrelations decaying exponentially as in the AR case.

7

Table 1 shows the estimates of the fractional differencing parameter for the original series, while Table 2 focuses on the log-transformed data. In both cases, we display the estimates of d, along with their corresponding 95% confidence intervals, in the three cases of no regressors ($\beta_0 = \beta_1 = 0$ a priori in (2)), an intercept (β_0 unknown and $\beta_1 = 0$ a priori) and an intercept with a linear trend (β_0 and β_1 unknown).

For the original series (see Table 1), if u_t is assumed to be a white noise, the estimates of d are slightly above 1 and the unit root null hypothesis is rejected in favour of d > 1 in both series. The results are very similar with seasonal AR disturbances. However, if u_t is assumed to be autocorrelated (either following a non-seasonal AR(1) process or the more general model of Bloomfield), although d is estimated to be above 1, the unit root null hypothesis cannot be rejected.

[Insert Tables 1 and 2 about here]

The same conclusions hold for the log-transformed series (see Table 2). Although the estimated value of d is above 1 in the majority of cases, in the most realistic case of autocorrelated disturbances the I(1) hypothesis cannot be rejected. When using Bloomfield's (1973) specification for the disturbances, the estimated value of d is 0.98 for the log S&P 500 Index, and slightly higher, 1.01, for the log-Euro Stoxx 50 Index. In both cases, an intercept seems to be sufficient to describe the deterministic components.

[Insert Table 3 about here]

Table 3 displays the estimates of d obtained using a "local" Whittle semi-parametric approach (Robinson, 1995) for a selected range of bandwidth parameters m $= (T)^{0.5} \pm 3$; the unit root hypothesis cannot be rejected in any case for either series.[1] These results are consistent with those of other papers also providing evidence of unit

---

[1] The estimates were obtained using first-differenced data, then adding 1 to obtain the proper estimates of d. Alternative semiparametric methods also based on the Whittle function (Velasco and Robinson, 2000; Abadir et al., 2007) produced essentially the same results.

roots in stock indices in most developed economies (Huber, 1997; Liu et al., 1997; Ozdemir, 2008; Narayan, 2005, 2006; Narayan and Smyth, 2004, 2005; Qian et al., 2008; etc.).

Various studies in the literature have documented non-linear dynamics in stock prices. For instance, Hsieh (1991) explored 'Chaos Dynamics' in stock prices not following a normal distribution; Abhyankar et al. (1995) provided evidence of non-linearity in the London Financial Times Stock Exchange (FTSE) index that cannot be fully explained by a GARCH model; Kosfeld and Robé (2001) showed various types of non-linearities in German bank stocks. Therefore we also carried out some non-linearity tests; specifically, we apply a recent procedure of Cuestas and Gil-Alana (2012) that allows the examination of the degree of integration of the series in the presence of non-linear deterministic terms. The estimated model is

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t, \qquad (1 - L)^d x_t = u_t, \qquad (3)$$

where $P_{i,T}(t)$ are the Chebyshev time polynomials, defined by:

$$P_{0,T}(t) = 1,$$

$$P_{i,T}(t) = \sqrt{2}\cos\left(i\pi(t-0.5)/T\right), \qquad t = 1, 2, ..., T; \quad i = 1, 2, ... \qquad (4)$$

Here, m indicates the order of the Chebyshev polynomial: if $m = 0$ the model contains an intercept, if $m = 1$ it also includes a linear trend, and if $m > 1$ it becomes non-linear, and the higher m, the less linear the approximated deterministic component becomes.[2]

[Insert Table 4 about here]

Table 4 displays the d-coefficient estimates and their 95% confidence bands for different degrees of linear (m = 1) and non-linear (m = 2, 3) behaviour in the logged-transformed series. It can be seen that the unit root model cannot be rejected in any

---

[2] See Hamming (1973) and Smyth (1998) for a detailed description of these polynomials.

case; the estimated coefficients for the linear and non-linear trends (not reported) were found to be statistically insignificant in all cases, which implies a rejection of the hypothesis of non-linear trends in the two series.[3]

Next, we investigate the issue of time variation in the fractional differencing parameter d by carrying out recursive analysis, starting with the first 120 observations (the first 10 years of the sample), and then adding one at a time. In particular, we focus on the log-transformed series and the specification with Bloomfield disturbances, with an intercept but not a linear trend, which is the model chosen on the basis of various diagnostic tests on the residuals.[4]

[Insert Figure 2 about here]

The two series appear to behave in a very similar way, although the estimates of d are slightly higher for the Euro Stoxx 50 Index. Those for the S&P 500 are all below 1, but the unit root null cannot be rejected. The estimated value of d increases when extending the sample recursively up to the 141st observation (the month following the 1998 Russian financial crisis); then it remains stable before jumping after the 191st observation (the start of the recovery in stock markets after the early 2000s recession), and is stable again till reaching 265 observations (right before the start of the recovery in global financial markets), when a new shift occurs.[5]

A similar behaviour of d is found in the case of the Euro Stoxx 50 Index, namely an upward trend for the first 191 observations (despite a downward shift after 143 observations), and then a jump after 266 observations. The unit root null hypothesis, i.e., the I(1) case, cannot be rejected for any subsample, which confirms the results from

---

[3] Very similar results were obtained with the unlogged data, and allowing for autocorrelated errors.

[4] In addition to t-tests for the deterministic terms, LR tests and various likelihood information criteria were used for model selection.

[5] The sample containing the first 141 observations ends in August 1998, the one with 191 ends in October 2002, and finally, the sample containing 265 observations ends in December 2008.

the full sample analysis; since both series appear to be I(1) throughout the sample it is legitimate to test for cointegration.

A necessary condition for cointegration is that the two parent series have the same degree of integration. In our case, the confidence intervals reported in Table 1 and 2 clearly suggest that the unit root (I(1)) hypothesis cannot be rejected for either series. However, we also perform a test of the homogeneity of the orders of integration in the bivariate systems (i.e., H_o: $d_x = d_y$), where d_x and d_y are the orders of integration of the two individual series, by using an adaptation of the Robinson and Yajima (2002) statistic $\hat{T}_{xy}$ to log-periodogram estimation. This is calculated as:

$$\hat{T}_{xy} = \frac{m^{1/2}\left(\hat{d}_x - \hat{d}_y\right)}{\left(\frac{1}{2}\left(1 - \hat{G}_{xy} / (\hat{G}_{xx}\hat{G}_{yy})\right)\right)^{1/2} + h(n)} \tag{5}$$

where $h(n) > 0$ and $\hat{G}_{xy}$ is the $(xy)^{\text{th}}$ element of

$$\hat{G} = \frac{1}{m}\sum_{j=1}^{m}\text{Re}\left[\hat{\Lambda}(\lambda_j)^{-1}I(\lambda_j)\hat{\Lambda}(\lambda_j)^{-1*}\right], \quad \hat{\Lambda}(\lambda_j) = diag\left\{e^{i\pi\hat{d}_x/2}\lambda^{-\hat{d}_x}, e^{i\pi\hat{d}_y/2}\lambda^{-\hat{d}_y}\right\},$$

with a standard normal limit distribution (see Gil-Alana and Hualde (2009) for evidence on the finite sample performance of this procedure). As expected, the results strongly support the hypothesis that the two orders of integration are the same, with a unit root being present in both cases.

Next, we examine the cointegrating relationship by estimating the following regression,

$$y_{1t} = \beta_0 + \beta_1 y_{2t} + x_t, \qquad (1 - L)^d x_t = u_t, \qquad t = 1, 2, ... \tag{6}$$

where y_{1t} is the logged S&P 500 Index and y_{2t} the logged Euro Stoxx 50 Index. We consider the two cases of uncorrelated (white noise) and correlated (Bloomfield) errors.

The fact that the two individual series are I(1) validates the use of standard OLS methods under the standard setting of cointegration (Phillips and Durlauf, 1986). In a fractional setting, things are more complicated and the properties depend on the specific orders of integration of the parent series and that of the cointegrating regression (Gil-Alana and Hualde, 2009).[6]

[Insert Table 5 about here]

Table 5 displays the estimated value of d in the cointegrating regression along with the other parameters in the cointegrating relationship. The estimated value of d in the residuals from the above regressions is 0.97 with white noise errors and 0.98 with autocorrelated disturbances, and the unit root null cannot be rejected in either case. This constitutes strong evidence against the hypothesis of cointegration, since the cointegrating residuals display a similar order of integration to the original series.

[Insert Figure 3 about here]

Next, we carry out recursive cointegration analysis, again starting with a sample of 121 observations. The results for d are displayed in Figure 3. It can be seen that the estimated value of d is below 1 (implying fractional cointegration and mean-reverting errors) in all the subsamples before reaching 268 observations, when the confidence intervals start including the unit root case, thus rejecting the hypothesis of cointegration. This point in the sample corresponds to March 2009, namely the trough of the financial crisis and the moment when global markets began to exit it. Our analysis indicates that at that stage the pattern of co-movement that had existed for the previous 22 years between the US and European stock markets began to break down, and different recovery paths were followed. As mentioned before, different policy responses, namely the very prompt adoption of QE by the Fed in contrast to fiscal tightening and very

_____

[6] Alternative methods for the estimation of β_0 and β_1 in (6) were also employed including a Narrow Band Least Squared (NBLS) estimator as proposed in Robinson (1994b) and a Fully Modified NBLS as in Nielsen and Frederiksen (2011).

limited monetary easing in Europe in the presence of a serious debt crisis, have led to different growth experiences on the two sides of the Atlantic, the European economies lagging behind and their stock markets underperforming, also as a result of some of their features, i.e. being more cyclical and less exposed to technology.

Finally, we perform the Hausman test for no cointegration of Marinucci and Robinson (2001) comparing the estimate $\hat{d}_x$ of d_x with the more efficient bivariate one of Robinson (1995), which uses the information that $d_x = d_y = d_*$. Marinucci and Robinson (2001) show that

$$H_{im} = 8m\left(\hat{d}_* - \hat{d}_i\right)^2 \quad \rightarrow_d \chi_1^2 \qquad as \quad \frac{1}{m} + \frac{m}{T} \rightarrow 0, \qquad (7)$$

with $i = x, y$, and where $m < [T/2]$ is again a bandwidth parameter, analogous to that introduced earlier; $\hat{d}_i$ are univariate estimates of the parent series, and $\hat{d}_*$ is a restricted estimate obtained in the bivariate context under the assumption that $d_x = d_y$. In particular,

$$\hat{d}_* = -\frac{\sum\limits_{j=1}^{s} 1_2'\hat{\Omega}^{-1}Y_j v_j}{21_2'\hat{\Omega}^{-1}1_2 \sum\limits_{j=1}^{s} v_j^2}, \qquad (8)$$

where 1_2 indicates a (2x1) vector of 1s, and with $Y_j = [\log I_{xx}(\lambda_j), \log I_{yy}(\lambda_j)]^T$, and $v_j = \log j - \frac{1}{s}\sum\limits_{j=1}^{s}\log j.$ The limiting distribution above is presented heuristically, but Marinucci and Robinson (2001) argue that it seems sufficiently convincing for the test to warrant serious consideration.

[Insert Table 6 about here]

Table 6 displays the results for the Hausman test of no cointegration of Marinucci and Robinson (2001). The null of no cointegration cannot be rejected for the full sample, the estimated order of integration for the cointegrating error being about 1.01, which is very close to the values obtained for the individual series. By contrast, the null is rejected in favour of fractional cointegration for the subsample ending in December 2008, although the estimated value of d in the cointegrating error is close to 1, which implies highly persistent deviations from the long-run equilibrium relationship.

5.   Conclusions

This paper analyses the long-memory properties of US and European stock indices, as well as their linkages, using fractional integration and fractional cointegration techniques respectively. The empirical evidence suggests the presence of unit roots in both the S&P 500 Index and the Euro Stoxx 50 Index. This result is robust to using a variety of parametric and semi-parametric methods. Given the fact that the two series exhibit the same order of integration, we also examine the possibility of a long-run equilibrium relationship linking them. The results indicate that cointegration does not hold over the full sample; however, there is evidence of fractional cointegration over the subsample ending in March 2009, indicating that the effects of shocks affecting the long-run relationship vanish at a very slow rate.

It appears that the recovery paths followed by US and European stock markets after reaching their lowest price level (as a result of the Great Recession) have been very different. The Eurozone debt crisis combined with fiscal tightening and no significant monetary easing led to much weaker growth in Europe than in the US, where the Fed immediately embarked on an extensive QE programme. This has also affected European financial markets, with downward pressures on both bond yields and stock

prices. In the case of the latter, other factors such as less prominence of technology stocks have also resulted in underperforming stock indices.

References

Abadir, K.M., Distaso, W. and Giraitis, L. (2007). Nonstationarity-extended local Whittle estimation, *Journal of Econometrics* 141, 1353-1384.

Abhyankar, A., Copeland, L. S. and W. Wong, 1995, Nonlinear dynamics in real-time equity market indices: evidence from the United Kingdom, The Economic Journal, 864-880.

Alvarez-Ramirez, J., Alvarez, J., Rodriguez, E. and Fernandez-Anaya, G., 2008, Time-varying Hurst exponent for US stock markets, Physica A: Statistical Mechanics and Its Applications, 6159-6169.

Arteche, J., 2004, Gaussian semiparametric estimation in long memory in stochastic volatility and signal plus noise models, Journal of Econometrics 119, 131-154.

Aydogan, K. and G.G. Booth, 1988, Are there long cycles in common stock returns? Southern Economic Journal 55, 141-149.

Baillie, R.T., Y.W. Han, R.J. Myers and J. Song, 2007, Long memory models for daily and high frequency commodity future returns, Journal of Future Markets 27, 643-668.

Barkoulas, J.T. and C.F. Baum, 1996, Long term dependence in stock returns. Economics Letters, 53, 253-259.

Barkoulas, J.T., C.F. Baum and N. Travlos, 2000, Long memory in the Greek stock market. Applied Financial Economics, 10, 177-184.

Bastos, J. A. and Caiado, J., 2010, The structure of international stock market returns, CEMAPRE Working Paper, No. 1002.

Bloomfield, P. (1973). An exponential model in the spectrum of a scalar time series, *Biometrika* 60, 217-226.

Bollerslev, T. and H.O. Mikkelsen, 1996, Modeling and pricing long memory in stock market volatility, Journal of Econometrics 73, 151-184.

Breidt, F., N. Crato and P. de Lima, 1997, Modeling persistent volatility of asset returns, Computational Inteligence for Financial Engineering 23, 266-272.

Breidt, F., N. Crato and P. de Lima, 1998, The detection and estimation of long memory in stochastic volatility, Journal of Econometrics 83, 325-348.

Cheung, Y.- W. and K.S. Lai, 1995, A search for long memory in international stock market returns, Journal of International Money and Finance, 14, 597-615.

Choudhry, T, 1997, Stochastic trends in stock prices: Evidence from Latin American markets, Journal of Macroeconomics, 19, 285-304.

Copeland, M. and Copeland, T., 1998, Lads, lags, and trading in global markets, Financial Analysts Journal, 54, 70-80.

Crato, N., 1994, Some international evidence regarding the stochastic behaviour of stock returns. Applied Financial Economics, 4, 33-39.

Crato, N. and P.J.F. de Lima, 1994, Long range dependence in the conditional variance of stock returns, Economics Letters 45, 281-285.

Cuestas, J.C. and Gil-Alana, L.A. 2012. A Non-Linear Approach with Long Range Dependence Based on Chebyshev Polynomials, *Working Papers* 2012-013, The University of Sheffield, Department of Economics.

Dahlhaus, R. (1989). "Efficient parameter estimation for self-similar process", *Annals of Statistics* 17, 1749-1766.

Dickey, D.A. and W.A. Fuller, 1979."Distributions of the estimators for autoregressive time series with a unit root."*Journal of the American Statistical Association* 74, 427-431.

Dickey, D.A. and W.A. Fuller, 1981, Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root ," Econometrica , 49, 1057-1072.,

Ding, Z. and C.W.J. Granger, 1996, Modeling volatility persistence of speculative returns: A new approach, Journal of Econometrics 73, 185-215.

Engle, R.F. and C.W.J. Granger, 1987, Cointegration and error-correction: Representation, estimation and testing, Econometrics 35, May, 1433159.

Euro Stoxx 50 Index, 2014, July 2. ^STOXX50E stock quote, Yahoo! Finance, Retrieved from http://finance.yahoo.com/q?s=%5ESTOXX50E.

Fama, E., 1995, Random Walks in Stock Market Prices, Financial Analysts Journal,75-80.

Fama, E. and K. French, 1988, Permanent And Temporary Components Of Stock Prices, Journal of Political Economy, 246-246.

Fox, R. and M.S. Taqqu, (1986) "Large sample properties of parameter estimates for strongly dependent stationary Gaussian time series" *Annals of Statistics* 14, 517–532.

Gil-Alana, L.A., 2006, Fractional integration in daily stock market returns, Review of Financial Economics 15, 28-48.

Gil-Alana, L.A. and Hualde J. (2009). Fractional integration and cointegration. An overview with an empirical application. *The Palgrave Handbook of Applied Econometrics* 2, 434-472.

Goodhart, C.A.E., 1988, The international transmission of asset price volatility, in: Financial market volatility (Federal Reserve Bank of Kansas City, Kansas City). 79-121.

Granger, C.W.J. and Z. Ding, 1995a, Some properties of absolute returns. An alternative measure of risk. Annales d'Economie et de Statistique, 40, 67-91.

Granger, C.W.J. and Z. Ding, 1995b, Stylized facts on the temporal and distributional properties of daily data from speculative markets. UCSD Working Paper.

Granger, C.W.J. and Z. Ding, 1996, Varieties of long memory models. Journal of Econometrics, 73, 61-78.

Granger, C.W.J. and N. Hyung, 2004, Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns, Journal of Empirical Finance 11, 399-421.

Greene, M.T. and B.D. Fielitz, 1977, Long term dependence in common stock returns, Journal of Financial Economics 5, 339-349.

Hamming, R. W. 1973.*Numerical Methods for Scientists and Engineers*, Dover.

Henry, O.T., 2002, Long memory in stock returns. Some international evidence. Applied Financial Economics, 12, 725-729.

Hiemstra, C. and J.D. Jones, 1997, Another long at long memory in common stock returns. Journal of Empirical Finance, 4, 373-401.

Hsieh, D.A., 1991, Chaos and nonlinear dynamics: application to financial markets, The Journal of Finance, 46(5), 1839-1877.

Huber, P., 1997, Stock market returns in thin markets: evidence from the Vienna stock exchange. Applied Financial Economics 7, 493-498.

Jeon, B. and Von-Furstenberg, 1990, Growing International Comovement in Stock Price Indexes, Quarterly Review of Economics and Finance, Vol. 30, No. 30, pp. 17-30.

Jeong, J., 1999, Cross-border transmission of stock price volatility: evidence from the overlapping trading hours, Global Finance Journal, 10, 53-70.

Johansen, S., 1991, Estimation and hypothesis testing of cointegration vectors in Gaussian Vector AutoRegressive models, Econometrica 59, 1551-1580.

Johansen, S. 1996, Likelihood-based inference in cointegrated vector autoregressive models, Oxford, Oxford University Press.

King, M. and S. Wadhwani, 1990, Transmission of volatility between stock markets, London School of Economics, Review of Financial Studies 3, 1, 5-33.

Kosfeld, R. and S. Robé, 2001, Testing for nonlinearities in German bank stock returns, Empirical Economics, 26(3), 581-597.

Lee, S.B., and K.J. Kim, 1994, Does the October 1987 crash strengthen the co-movement in stock price indexes, Quarterly Review of Economics and Business, Vol. 3, pp. 89-102.

Liu, C.Y., S.H. Song and P. Romilley, 1997, Are Chinese stock markets efficient?, A cointegration and causality analysis, Applied Economics Letters 4, 511-515.

Lo, A.W., 1991, Long-term memory in stock prices. Econometrica, 59, 1279-1313.

Marinucci, D. and P.M. Robinson (2001) Semiparametric fractional cointegration analysis, *Journal of Econometrics* 105, 225-247.

Merit. 1. and G. Merit. 1989. Potenttal gains from international portfolio diversification and inter-temporal stability and seasonality in internattonal stock market relationships, Journal of Banking and Finance 13, 6277640.

Mikosch, T. and C. Starica, 2000, Change of structure in financial time series, long range dependence and the GARCH model, Centre for Analytical Finance, University Of Aarhus, Working Paper Series No. 58.

Narayan, P. K (2005). Are the Australian and New Zealand stock prices nonlinear with a unit root? Applied Economics 37, 2161-2166.

Narayan, P. K (2006). The behavior of US stock prices: evidence from a threshold autoregressive model, Mathematics and Computers in Simulation 71, 103-108.

Narayan, P. K. and R. Smyth (2004). Is South Korea's stock market efficient? Applied Economics Letters 11, 707-710.

Narayan, P.K. and R. Smyth (2005). Are OECD stock prices characterized by a random walk? Evidence from sequential trend break and panel data models, Applied Financial Economics 15, 547-556.

Nielsen, M.O. and P.S. Frederiksen, (2011) Fully Modified Narrow-Band Least Squares Estimation of Weak Fractional Cointegration, The Econometrics Journal 14, 1, 77-120.

Ng, S. and P. Perron, 2001, Lag length selection and the construction of unit root tests with good size and power, Econometrica, 69, 1519–1554.

Opong, K.K., Mulholland, G., Fox, A.F., and Farahmand, K., 1999, The behaviour of some UK equity indices: An application of Hurst and BDS tests, Journal of Empirical Finance, 6(3), 267-282.

Ozdemir, Z.A., 2008, Efficient market hypothesis: evidence from a small open economy, Applied Economics 40, 633-641.

Panto, D.B., V.P. Lessig, and M. Joy, 1976, Comovement of International Equity Markets: A Taxonomic Approach, Journal of Financial and Quantitative Analysis 11, 415-432.

Phillips, P.C.B. and S.N. Durlauf, 1986, Multiple time series regressions with integrated processes, Review of Economic Studies 53, 473-495.

Phillips, P.C., and Perron P. 1988. "Testing for Unit Roots in Time Series Regression." Biometrika, 75, 335-346.

Poterba, J. and L. Summers, 1988, Mean reversion in stock prices, Journal of Financial Economics, 27-59.

Qian, Xi-Yuan. S, Fu-Tie. and Z. Wei-Xing (2008). Nonlinear behavior of the Chinese SSEC index with a unit root. Evidence from threshold unit root tests, Physica A 387, 503-510.

Robinson, P.M. (1994a), "Efficient tests of nonstationary hypotheses", Journal of the American Statistical Association 89, 1420-1437.

Robinson, P. M. (1994b), 'Semiparametric analysis of long-memory time series', Annals of Statistics 22, 515-539.

Robinson, P.M., 1995, Log-periodogram regression of time series with long range dependence, *Annals of Statistics* 23, 1048-1072.

Robinson, P.M. and Y. Yajima (2002), Determination of cointegrating rank in fractional systems, *Journal of Econometrics* 106, 217-241.

S&P 500 Index, 2014, July 2, ^GSPC stock quote, Yahoo! Finance, Retrieved from http://finance.yahoo.com/q?s=%5EGSPC.

Sadique, S. and P. Silvapulle, 2001, Long-term memory in stock market returns. International evidence. International Journal of Finance and Economics, 6, 59-67.

Smyth, G.K. 1998.*Polynomial Aproximation*, John Wiley & Sons, Ltd, Chichester.

Syllignakis M. and Kouretas, G., 2010. German, US and Central and Eastern European Stock Market Integration, Open Economies Review, Springer, vol. 21(4), pages 607-628.

Taylor, M. P. and Tonks, I., 1989, The Internationalisation of Stock Markets and the Abolition of U.K. Exchange Control, The Review of Economics and Statistics, Vol.71, pp. 332-336.

Tolvi, J., 2003, Long memory and outliers in stock market returns. Applied Financial Economics, 13, 495-502

Velasco, C. and Robinson, P.M. (2000). Whitle pseudo maximum likelihood estimation for nonstationary time series. *Journal of the American Statistical Association* **95**, 1229-1243.

Wong, W.K., Agarwal, A. and Du, J., 2005, Financial Integration for India Stock Market, aFractional Cointegration Approach, National University of Singapore Working Paper, No. 0501, pp. 1-29.

Table 1: Estimates of d for each series using the raw data

	No regressors	An intercept	A linear time trend
i) White noise disturbances			
U.S. stock market	1.09 (1.02, 1.17)	1.07 (1.00, 1.15)	1.07 (1.00, 1.15)
Euro stock market	1.10 (1.03, 1.18)	1.10 (1.04, 1.19)	1.10 (1.09, 1.19)
ii) AR(1) disturbances			
U.S. stock market	1.09 (0.98, 1.22)	1.05 (0.92, 1.17)	1.05 (0.93, 1.17)
Euro stock market	1.10 (0.96, 1.24)	1.09 (0.97, 1.23)co	1.09 (0.97, 1.23)
iii) Bloomfield disturbances			
U.S. stock market	1.08 (0.96, 1.21)	**1.04 (0.93, 1.19)**	1.04 (0.93, 1.19)
Euro stock market	1.11 (0.98, 1.25)	**1.09 (0.98, 1.23)**	1.09 (0.98, 1.23)
iv) monthly AR(1) disturbances			
U.S. stock market	1.08 (1.02, 1.17)	1.06 (0.99, 1.15)	1.06 (0.99, 1.15)
Euro stock market	1.09 (1.03, 1.18)	1.10 (1.03, 1.19)	1.10 (1.03, 1.19)

The values in parenthesis refer to the 95% band for the non-rejection values of d. In bold, the most significant model for each series according to the deterministic terms and the type of I(0) disturbances.

Table 2: Estimates of d for each series using the logged transformed data

	No regressors	An intercept	A linear time trend
i) White noise disturbances			
U.S. stock market	1.01 (0.94, 1.10)	1.06 (0.99, 1.16)	1.06 (0.99, 1.16)
Euro stock market	0.99 (0.92, 1.07)	1.09 (1.02, 1.19)	1.09 (1.02, 1.19)
ii) AR(1) disturbances			
U.S. stock market	1.39 (1.27, 1.55)	0.98 (0.85, 1.12)	0.98 (0.87, 1.11)
Euro stock market	1.37 (1.25, 1.52)	1.01 (0.89, 1.15)	1.01 (0.89, 1.15)
iii) Bloomfield disturbances			
U.S. stock market	0.99 (0.87, 1.14)	**0.98 (0.87, 1.11)**	0.97 (0.88, 1.11)
Euro stock market	0.98 (0.86, 1.12)	**1.01 (0.90, 1.14)**	1.01 (0.90, 1.14)
iv) monthly AR(1) disturbances			
U.S. stock market	1.01 (0.93, 1.10)	1.06 (0.98, 1.16)	1.06 (0.98, 1.16)
Euro stock market	0.99 (0.92, 1.07)	1.09 (1.02, 1.19)	1.09 (1.02, 1.19)

The values in parenthesis refer to the 95% band for the non-rejection values of d. In bold, the most significant model for each series according to the deterministic terms and the type of I(0) disturbances.

Table 3: Estimates of d based on the "local" Whittle semiparametric approach

Bandwidth nb.	Log SP&500	Log Euro Stock	Lower 95%	Upper 95% I(1)
15	1.08	1.09	0.78	1.21
16	1.13	1.12	0.79	1.20
17	1.12	1.12	0.80	1.19
18	1.03	1.06	0.80	1.19
19	1.04	1.07	0.81	1.18
20	1.05	1.10	0.81	1.18
21	1.06	1.14	0.82	1.17

The fourth and the fifth columns refer to the 95% lower and upper confidence bands for the I(1) hypothesis.

Table 4: Estimates of d based on a model with non-linear deterministic trends

	$m = 1$ (linear)	$m = 2$ (non-linear)	$m = 3$ (non-linear)
Log of U.S. stock	1.07 (0.99, 1.16)	1.06 (0.98, 1.16)	1.05 (0.97, 1.14)
Log of Euro stock	1.09 (1.00, 1.19)	1.08 (0.99, 1.17)	1.08 (0.99, 1.16)

The values in paraenthesis refer to the 95% band for the non-rejection values of d

Table 5: Estimates of d in the cointegrating regression

	d	Intercept	Slope
White noise errors	0.97 (0.92, 1.05)	1.373 (6.864)	0.601 (20.656)
Bloomfield errors	0.98 (0.86, 1.11)	1.131 (5.676)	0.641 (22.108)

The values in parenthesis in the second column refers to the 95% band for the non-rejection values of d. In the third and fourth columns t-values are reported.

Table 6: Testing the null of no cointegration against fractional cointegration

Log SP&500 / Log Euro Stock	H_x	H_y	$\hat{d}_*$
Whole sample (1986 – 2013)	0.0057	0.360	1.011
Sub-sample (1986 – 2008)	3.343	4.701	0.938

H_x and H_y refer respectively to the hypothesis in (7) for each one of the two series using the Hausman test of Marinucci and Robinson (2001). The values in the fourth column is the estimated value of d^*. $\chi_1^2(5\%) = 3.84$.

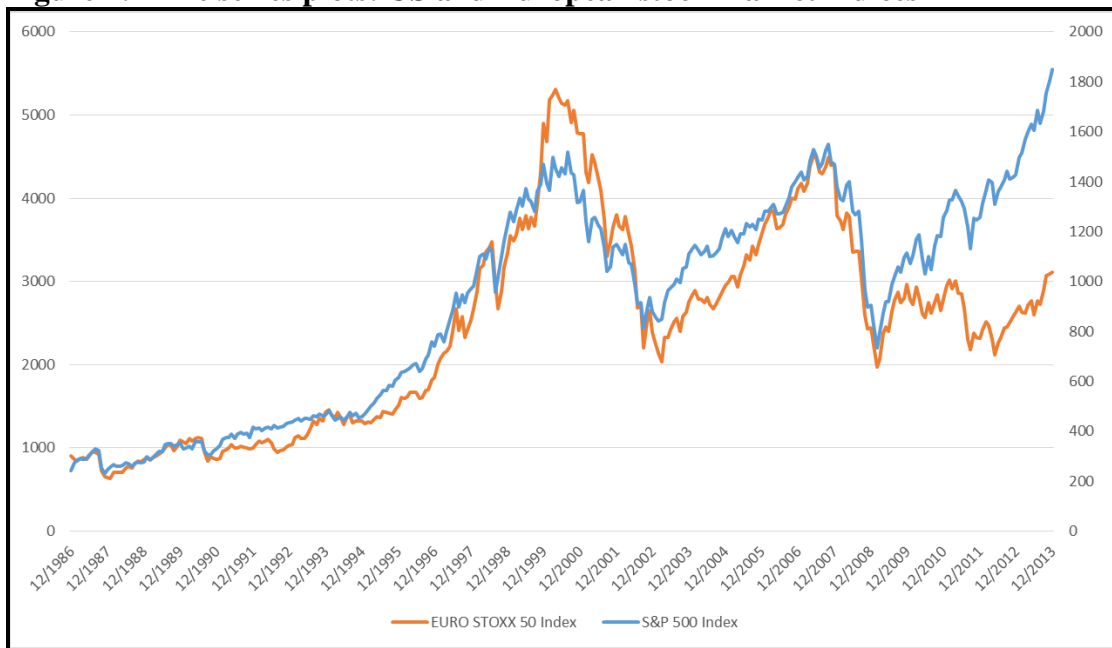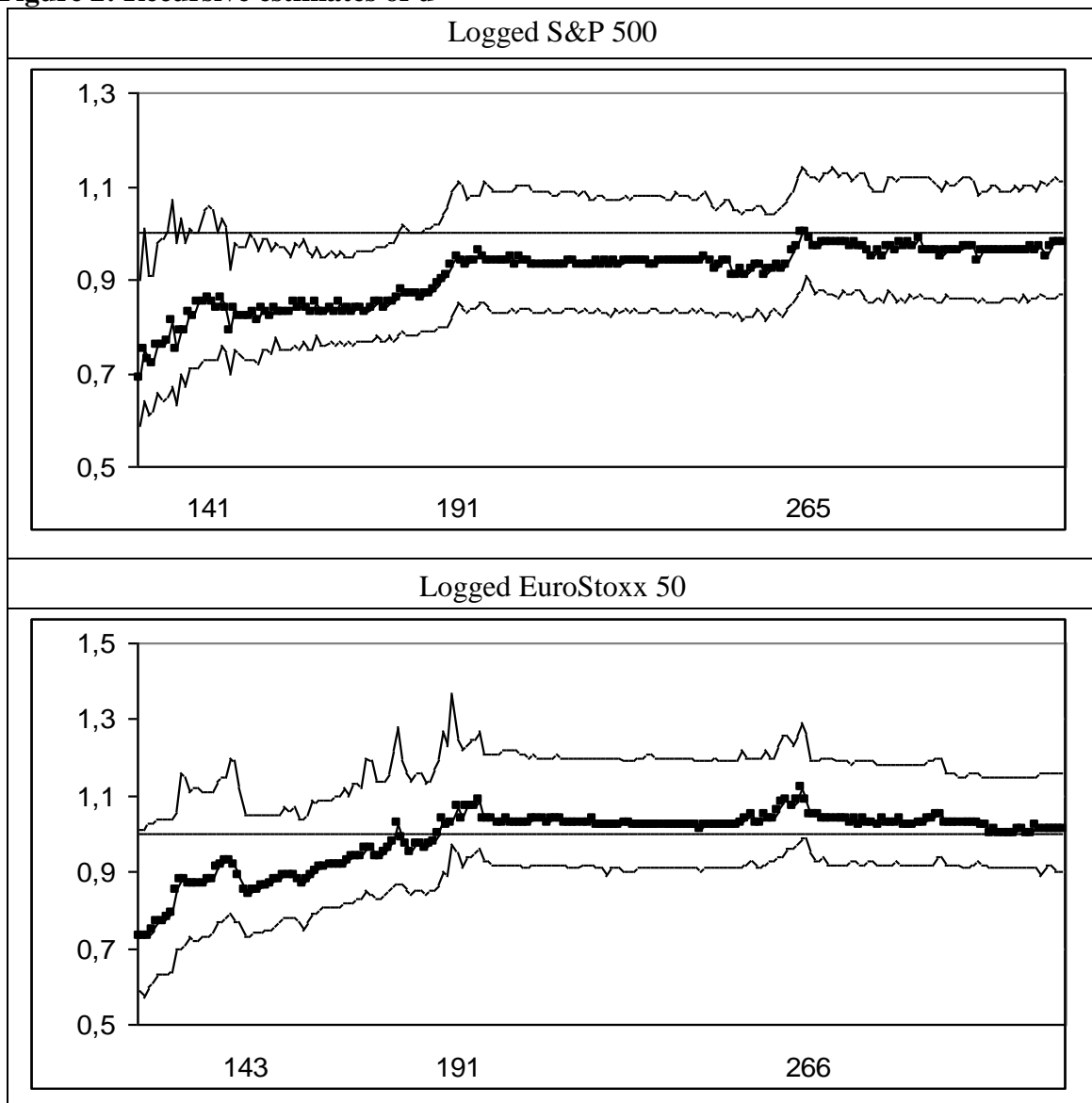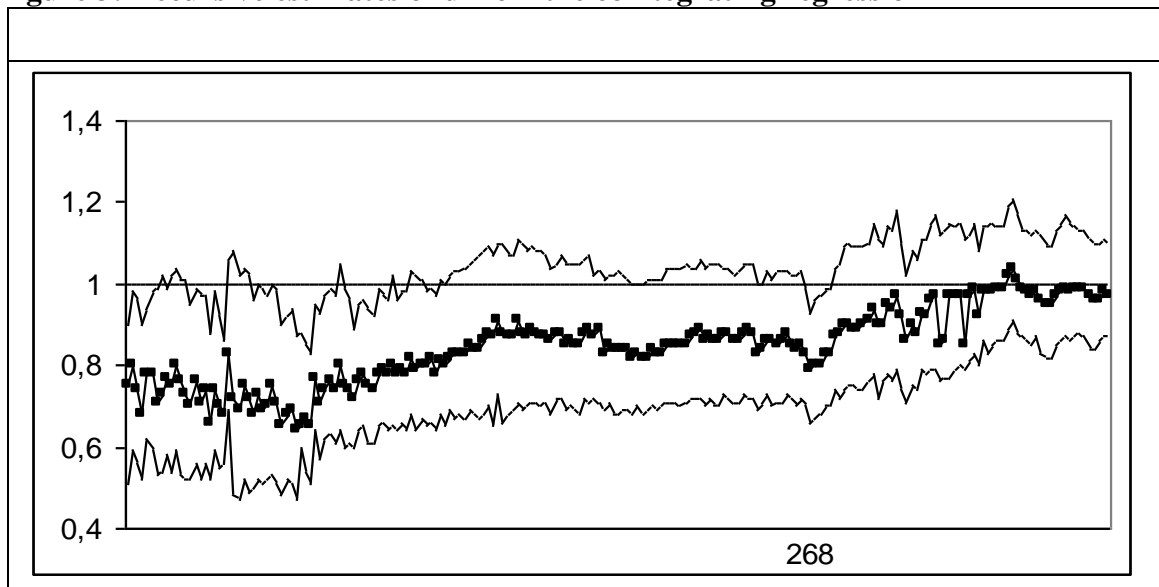**Figure 1: Time series plots: US and European stock market indices**

Figure 2: Recursive estimates of d

Logged S&P 500

Logged EuroStoxx 50

The thick line refers to the estimated values of d. The thin lines are the 95% confidence intervals.

Figure 3: Recursive estimates of d from the cointegrating regression

The thick line refers to the estimated values of d. The thin lines are the 95% confidence intervals.