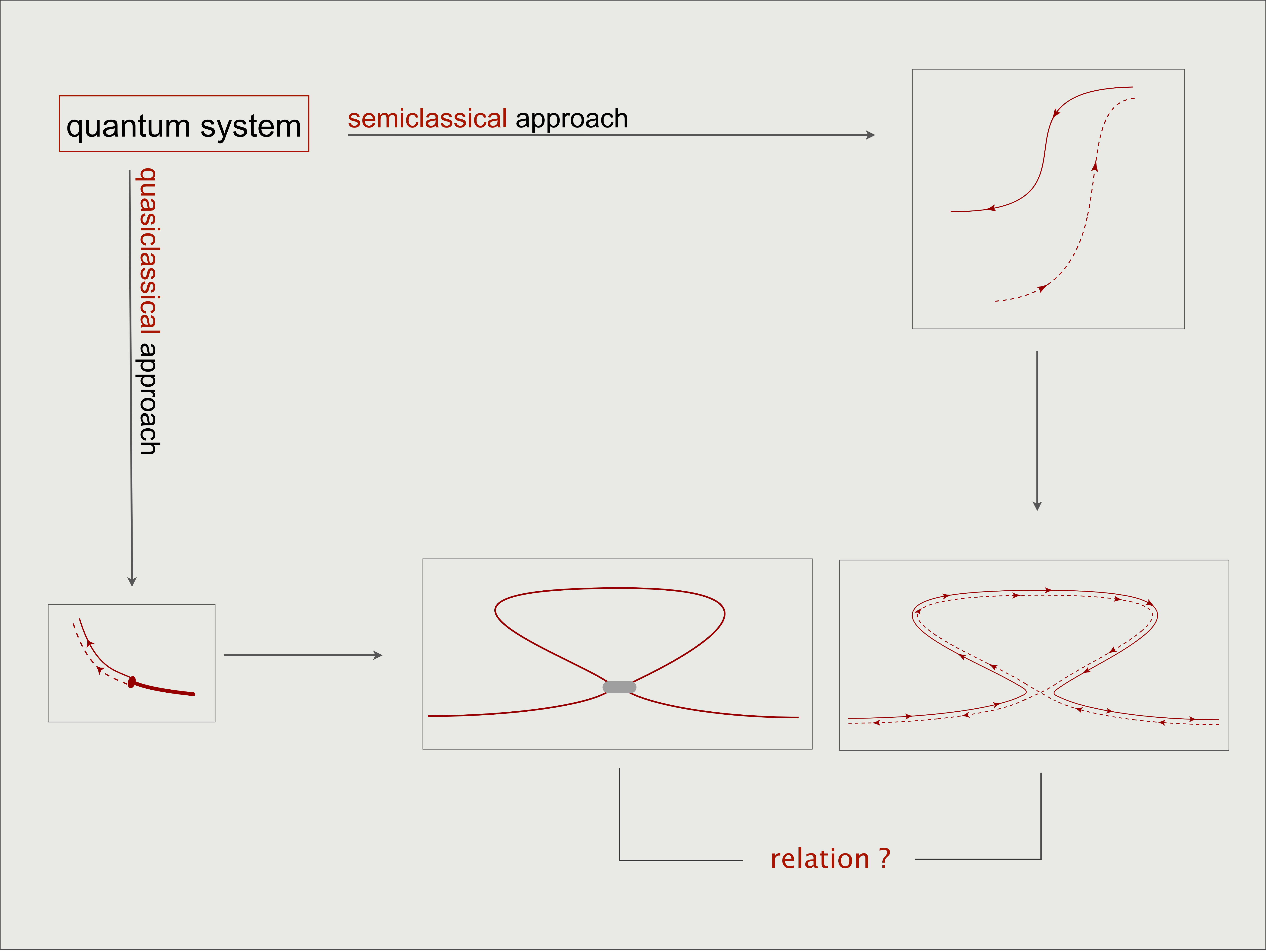
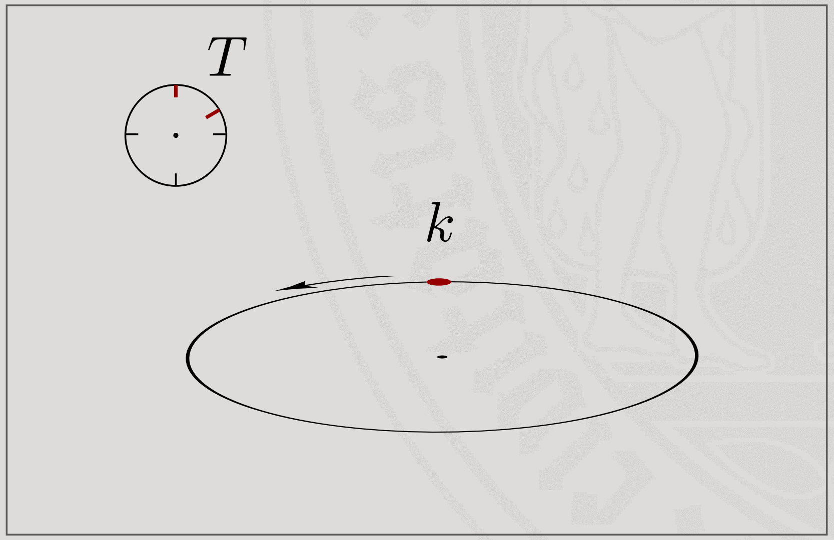

quantum system
semiclassical approach
quasiclassical approach
relation ?

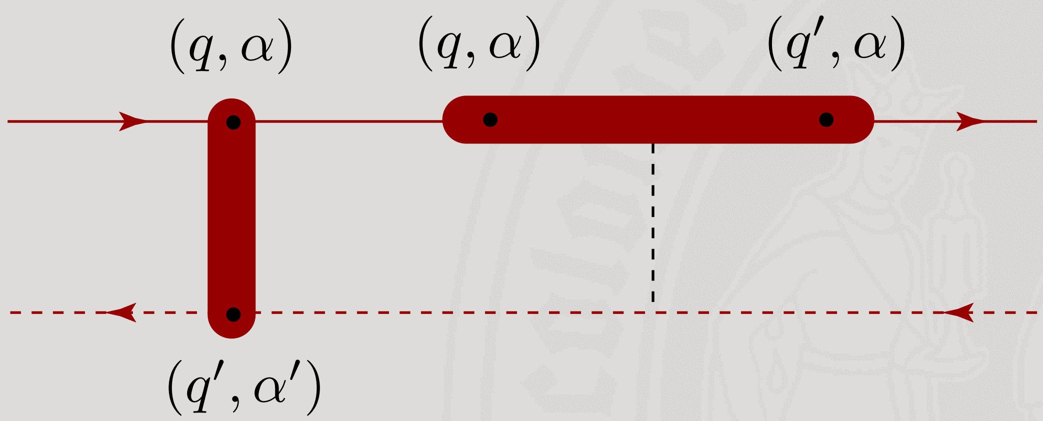
field theory approach to the standard map

Brunel, Dec. 18, 2009

Alexander Altland & Chushun Tian, Cologne University



- review: quasiclassical approach to nonlinear dynamics
- standard map
- field theory of the standard map

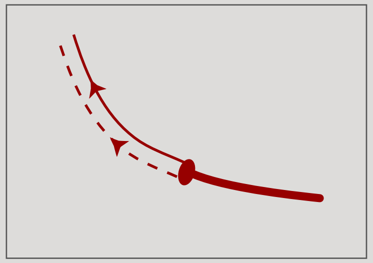


	semiclassics	quasiclassics
degrees of freedom	$\bar\psi_{q,\alpha}\psi_{q',\alpha} \to \mathcal{A}_\alpha(q,q')$	$\psi_{q,\alpha}\bar\psi_{q',\alpha'} \to Q_{\alpha,\alpha'}(x)$
formulation of theory	path integral	field integral
effective theory	path superposition (e.g. Gutzwiller double sum)	effective field theory in phase space
applications (traditional)	single particle	many particle

1968 Eilenberger equation

▷ Eilenberger equation (Eilenberger 68, Larkin & Ovchinikov 68)

first order nonlinear evolution equation in phase space. Describes evolution of quasiclassical Green function $\hat{G}_{\alpha,\alpha'}(\mathbf{x})$ along phase space trajectories. Sucessfully applied mostly to problems of superconductivity.



1968 Eilenberger equation

1995 ballistic sigma model

ballistic sigma model (Eilenberger 68; Altshuler et al. 95; Khmelnitskii & Muzykhantskii, 95)

Variational principle behind the Eilenberger equation. Allied to the nonlinear sigma model of disordered systems. Native version plagued by problems (no quantum interference, 'mode locking problem', 'zero mode' problem, repetition problem ...)

Hamilton function

$$S[T] = i \int_{\Gamma} d^{2f-1}x \,\mathrm{tr}\, \left(T * \Lambda\{H,T\} + \omega \Lambda T \Lambda * T^{-1} \right)$$

Moyal pr.

energy argument

integral over energy shell

1968 Eilenberger equation

1995 ballistic sigma model

1996 standard map

sigma model approach to standard map (Zirnbauer, a.a., 1996)

less mathematical difficulties. Successful description of diffusion, quantum interference and localization in the QKR.

In conclusion, the theory [1] in its present form may be expected to be valid only when a sort of band random matrix approach can be used *ad hoc*. However, it fails to take into account dynamical features which go beyond the random matrix theory description. A careful investigation of the conditions of validity of the suggested theory is needed.

Casati, Izrailev, Sokolov, 96

1968 — Eilenberger equation

1995 — ballistic sigma model

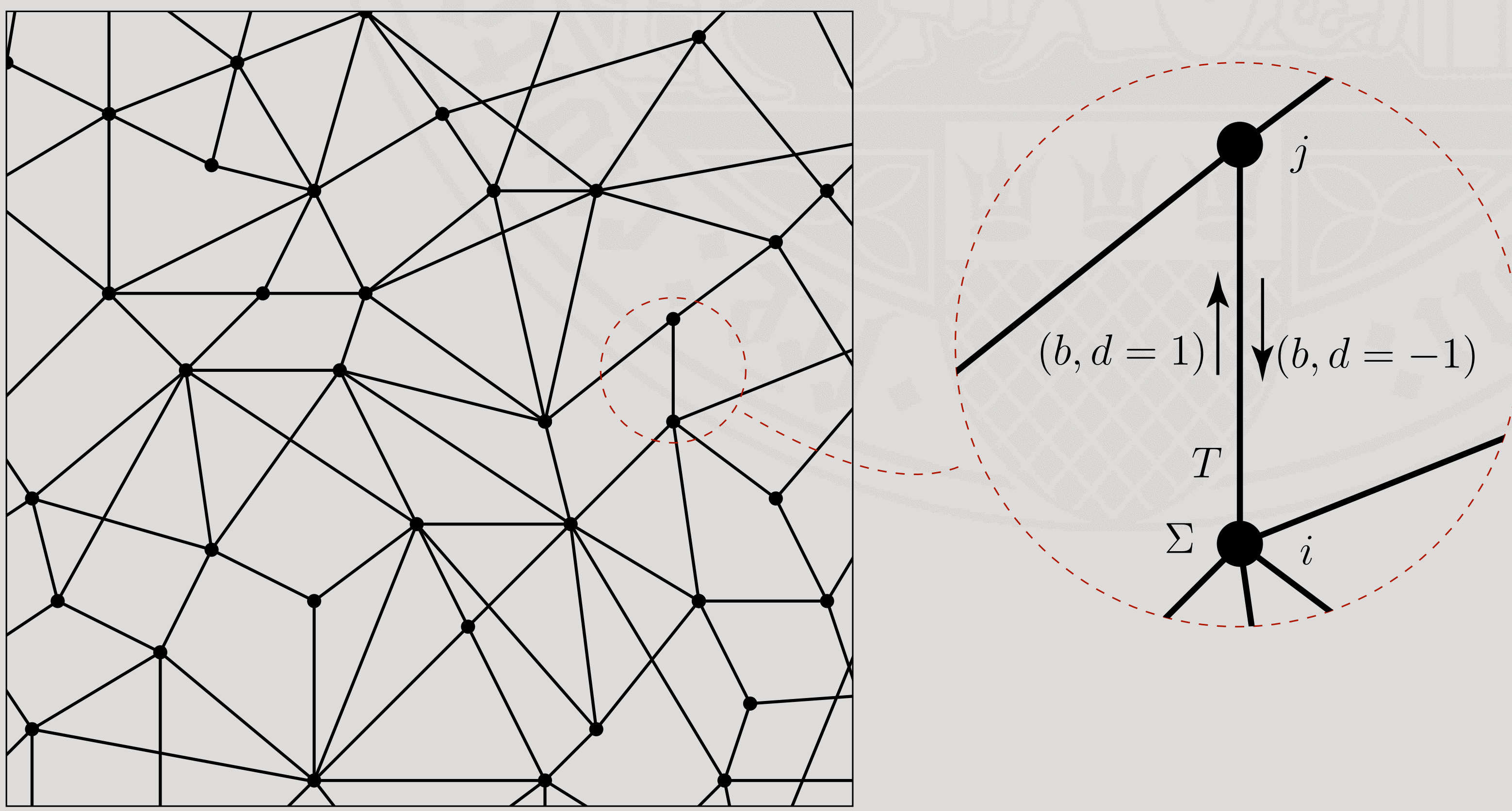
1996 — standard map

2005 — quantum graphs

universal correlations in quantum graphs (Gnutzmann, a.a. 2005)

spectral average in quantum graphs generates ergodic flow on system's phase space ➜ phase one of field theory construction in good shape.

For basic graphs (simple, connected) low energy theory stabilized ➜ universality



1968 — Eilenberger equation

1995 — ballistic sigma model

1996 — standard map

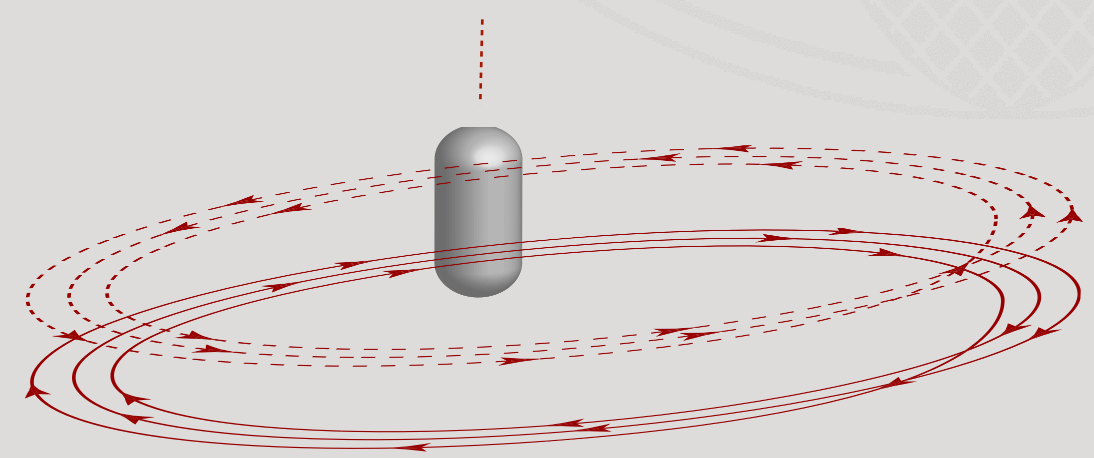
2005 — quantum graphs

2006 — sigma model pert. theory

universality from sigma model (Müller, Micklitz, a.a., 2006)

Connections to semiclassical analysis ('Sieber-Richter') established on perturbative level. Universal spectral correlations from field theory.

a particular problem: repetitions



solution for harmonic oscillator (Zirnbauer, priv. comm.)

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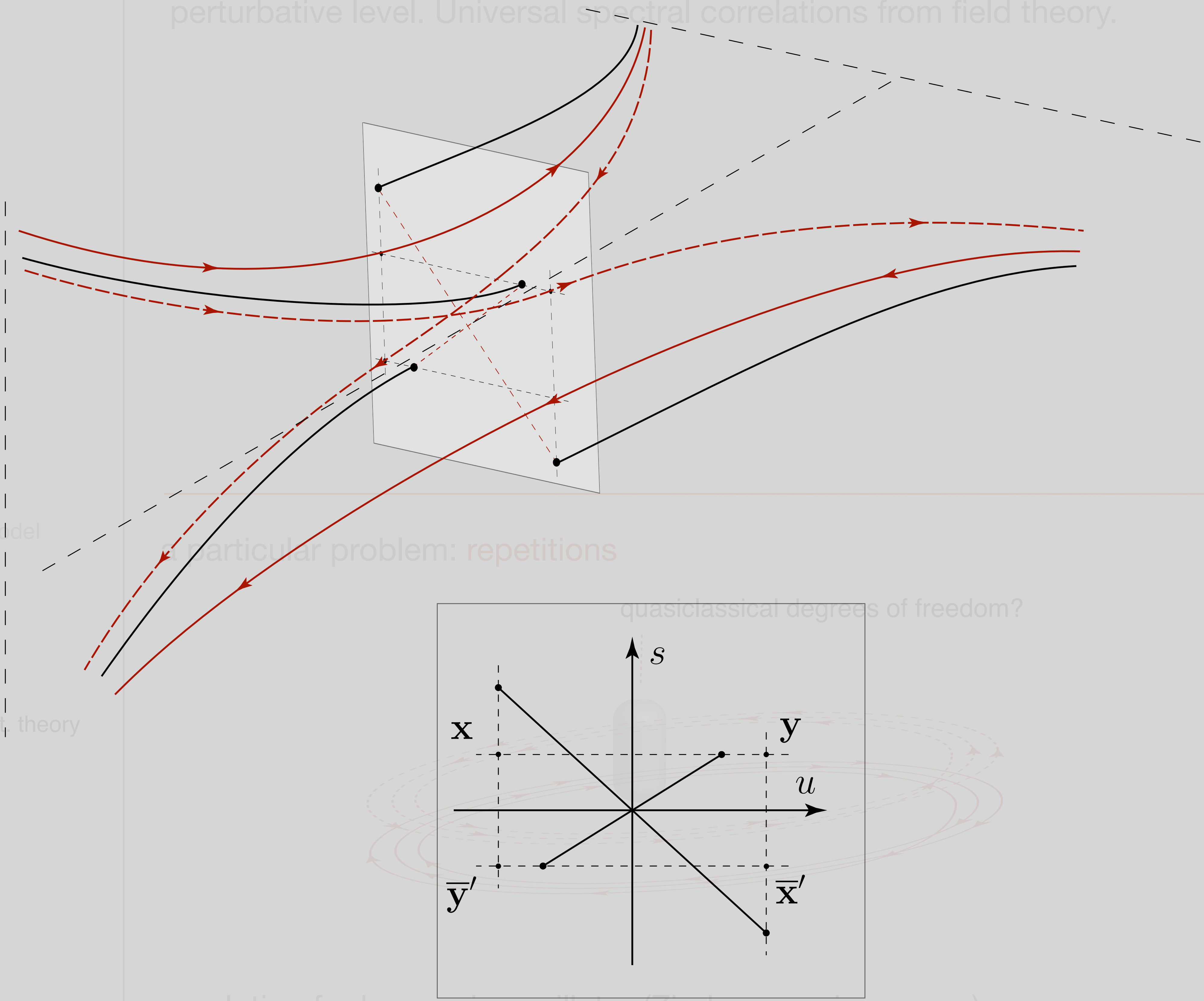
quasiclassical degrees of freedom?



solution for harmonic oscillator (Zirnbauer, priv. comm.)

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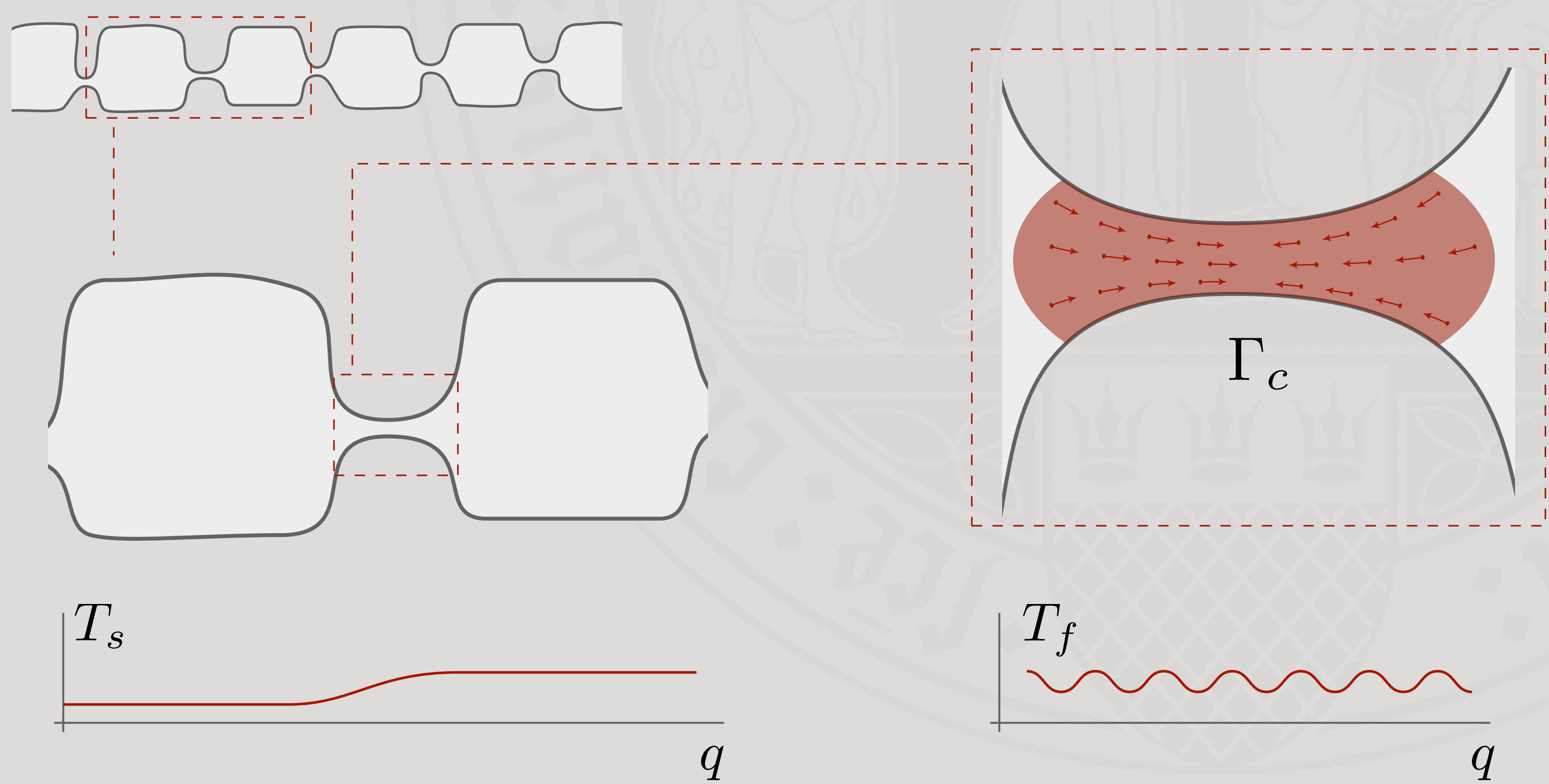
2008 — Anderson localization

Anderson localization in chaotic cavities (Brouwer, a.a., 2008)

Two approaches to localization in cavity arrays

- semiclassical analysis ➜ DMPK equation of localization
- quasiclassical analysis



1968 — Eilenberger equation

1995 — ballistic sigma model

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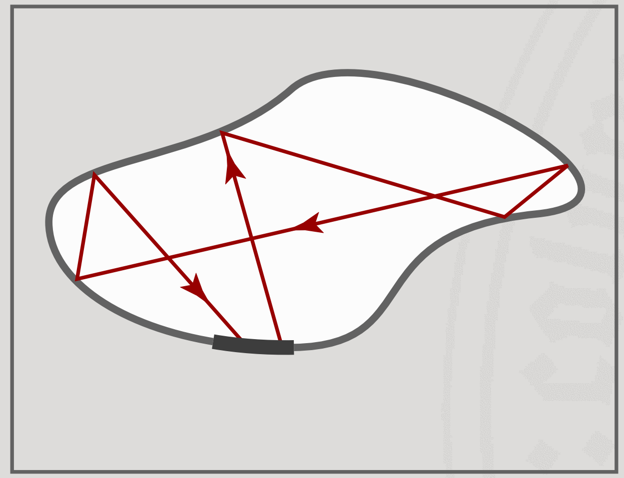
2008 — Anderson localization

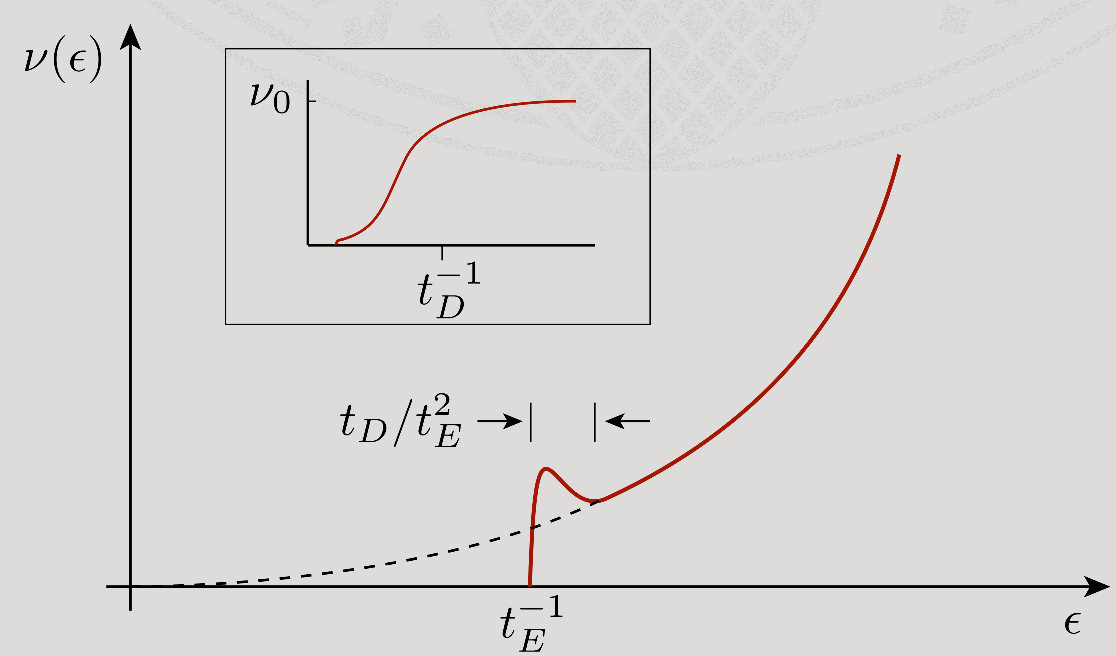
2009 — Andreev gap

spectral gap in chaotic Andreev billiards (Micklitz, a.a., 2009)



▷ (generalized) Eilenberger equation predicts formation of a spectral gap at the inverse Ehrenfest time.

▷ cf. semiclassical approach (Richter et al. 09)



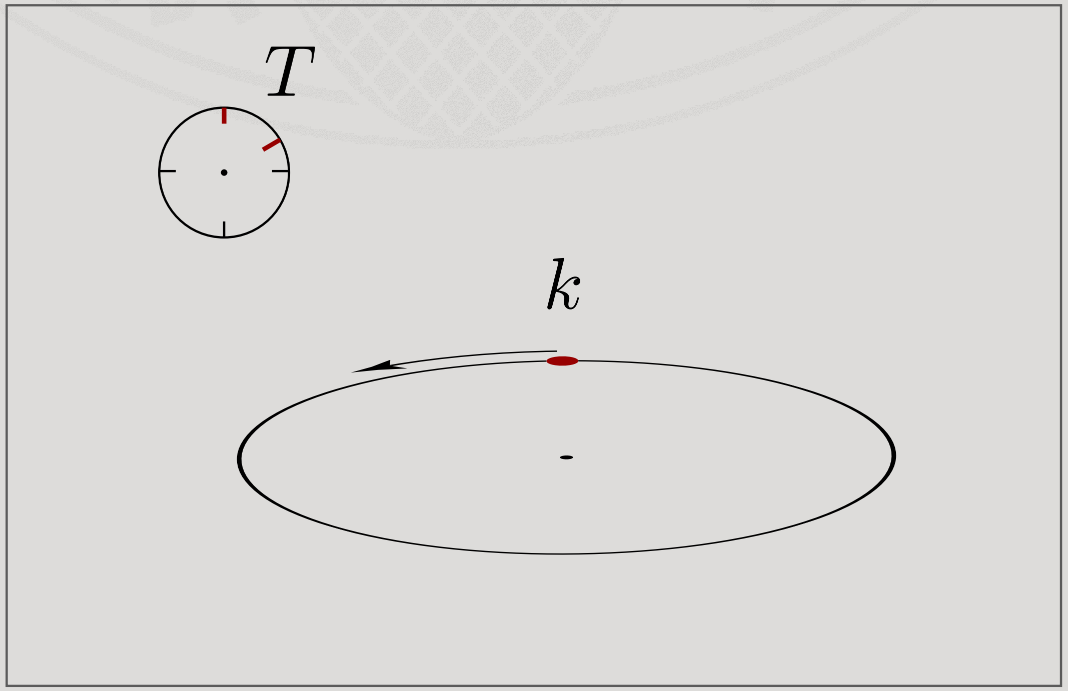
$$\boxed{\hat{H}(t) = \frac{\hat{l}^2}{2} + k\cos(\hat{\theta})\sum_m \delta(t - mT)}$$

Chirikov

- classical system characterized by a single parameter $K = kT$.
- quantum system sensitive to value of $\tilde{h} \equiv \hbar T$.
- here:

$\tilde{h} \ll K$	semiclassical dynamics (careful, though!)
$K \gg 1$	chaotic dynamics

- system exhibits correlations in angular momentum space



standard map	disordered quantum systems
chaotic scattering	impurity scattering
correlations in angular momentum space	velocity correlations

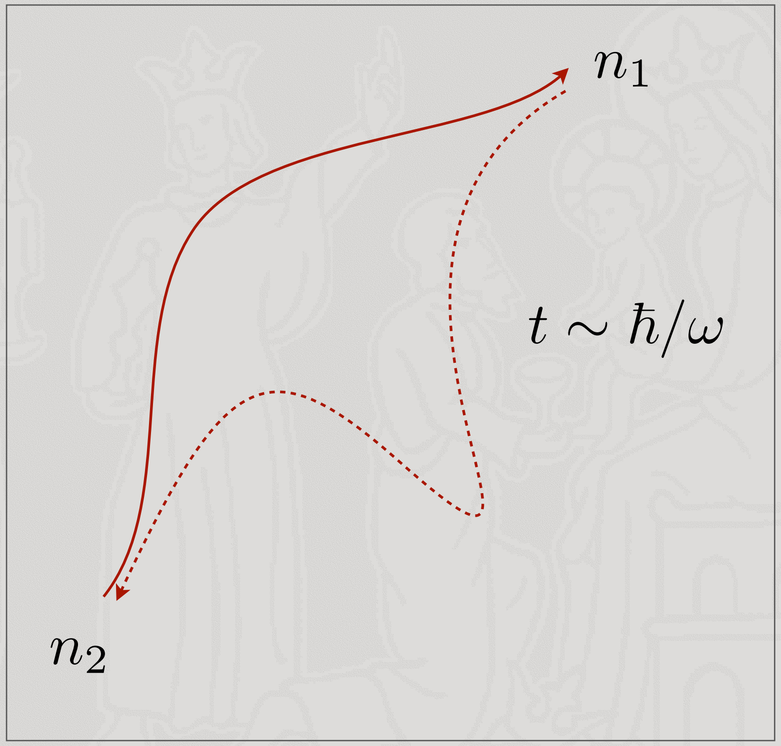
- ▷ Rechester-White (1980) corrections ($1/K$ corrections to diffusion constant)
- ▷ ϵ -classics, reentrant classical behaviour at $\tilde{h} = 4\pi + \epsilon$ (Fishman, 2004)
- ▷ $\tilde{h}$-resonances, absence of localization at $\tilde{h} = 4\pi p/q$ (Casati et al., 1979)
- ▷ accelerator modes, regular islands at special values of K (Karney, 1983)

diffusion	diffusion
localization	localization

Construct quasiclassical field theory that resolves analogies as well as differences

- describe QKR by correlation function

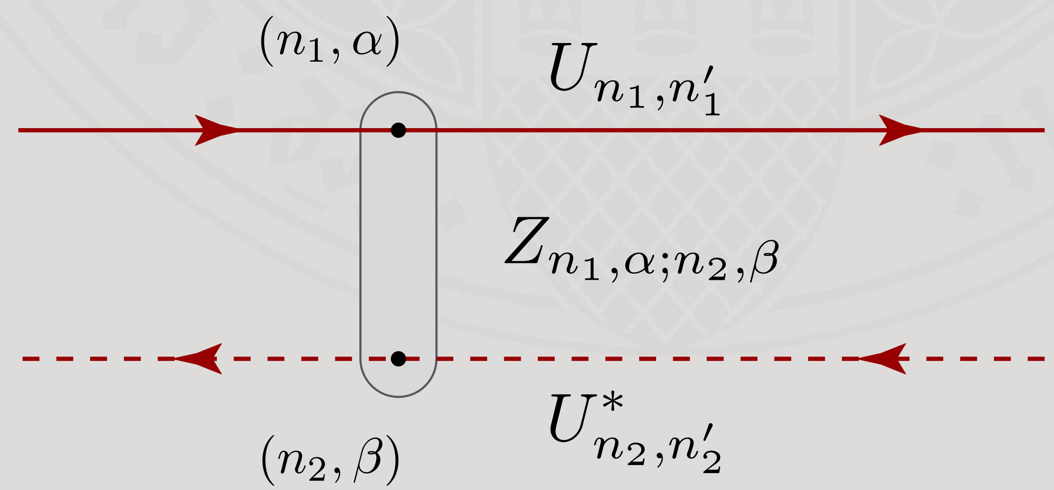
$$K_\omega(n_1,n_2) = \Big\langle \langle n_1|\hat G^+(\omega_+)|n_2\rangle\langle n_2|\hat G^-(\omega_-)|n_1\rangle \Big\rangle_{\omega_0}$$



$$\left[\begin{array}{l} \hat G^\pm(\omega_\pm) = \dfrac{1}{1-(e^{i(\omega_0\pm\omega/2)}\hat U)^{\pm 1}} \\[2em] \hat U = \exp\left(\dfrac{i\hat l^2}{4\tilde h}\right)\exp\left(\dfrac{iK\cos\hat\theta}{\tilde h}\right)\exp\left(\dfrac{i\hat l^2}{4\tilde h}\right). \end{array}\right.$$

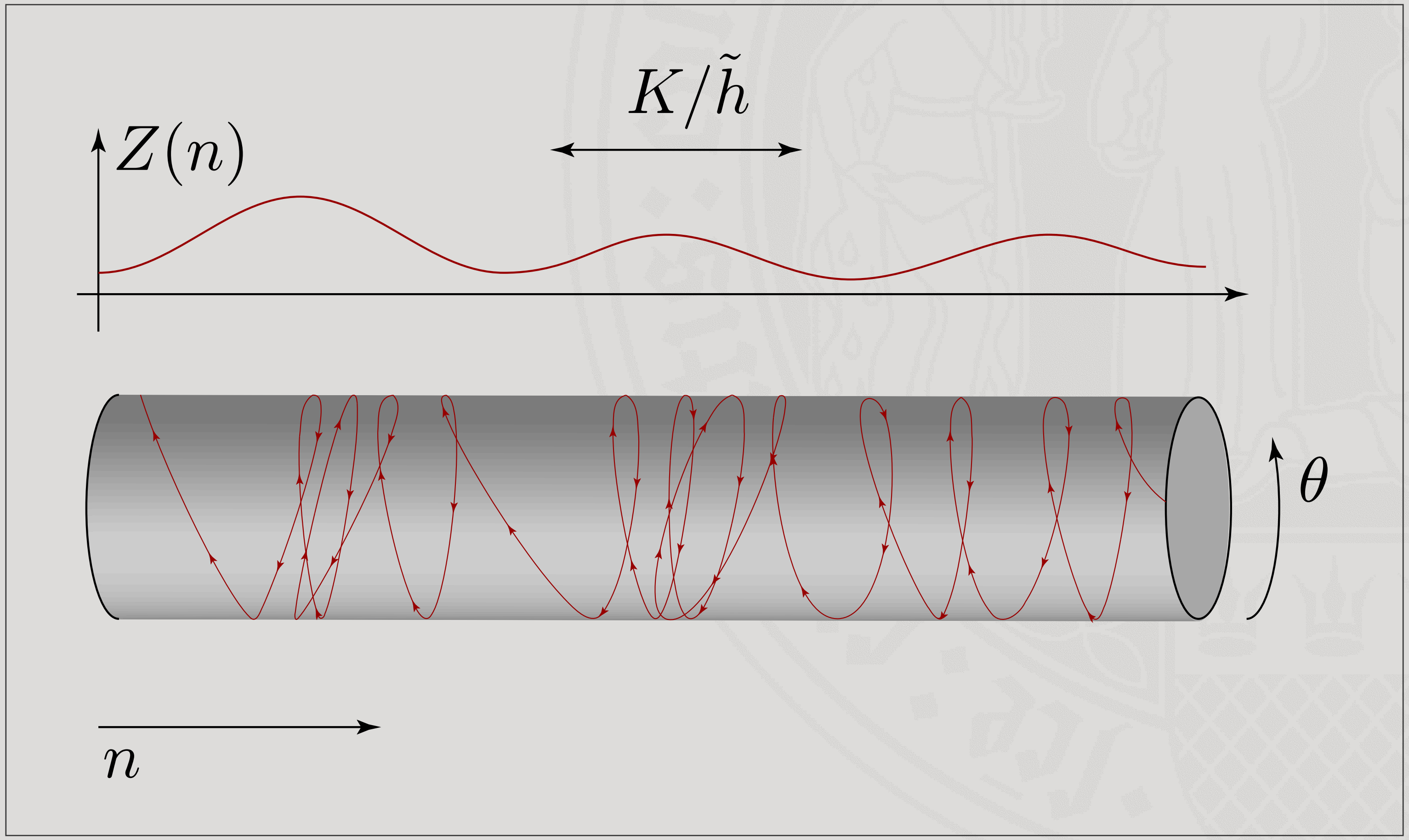
$$K_\omega(n_1, n_2) = \int dZ\, e^{-S[Z]}(\dots),$$

$$S[Z] = -\frac{1}{2}\text{str } \ln(1 - Z\tilde{Z}) + \frac{1}{2}\text{str } \ln(1 - e^{i\omega}\hat{U}^\dagger Z \hat{U}\tilde{Z})$$



soft modes: fluctuations whose action can be continuously tuned to zero. For generic irrational values of $\tilde{h}$

$$Z_{n_1,\alpha;n_2,\beta} \longrightarrow Z_{\alpha,\beta}(n,\theta) \longrightarrow B_{\alpha,\beta}(n), \qquad \Delta n \gtrsim K/\tilde{h}$$



all other field configurations ($C_{\alpha,\beta}(n,\phi)$) are massive (a linear decomposition $Z = B + C$)

$$\boxed{e^{-S[B]} = \int DC\, e^{-S[B+C]}}$$

Effective action cont'd

quadratic integration over massive modes:

$$S_{\mathrm{fl}}[B] = \frac{1}{2}\mathrm{str}\left(\tilde{B}\left(1 - \mathrm{ad}_U \frac{1}{1 - \hat{\pi}\,\mathrm{ad}_U}\right) B\right)$$

projector on massive modes

$\mathrm{Ad}_U(B) \equiv \hat{U} B \hat{U}^\dagger$

explicit representation

$$S_{\mathrm{fl}}[B] = -\frac{D_{\mathrm{eff}}}{2}\int dn\, \mathrm{str}(B(n)\partial_n^2 \tilde{B}(n))$$

$$D_{\mathrm{eff}} \simeq D^0(1 - 2J_2(x) - 2J_1^2(x) + 2J_3^2(x)), \qquad x = \frac{2K}{\tilde{h}}\sin\left(\frac{\tilde{h}}{2}\right)$$

Shepelyansky (1987)

Effective action (cont'd)

generalization beyond quadratic order

$$S[Q] = \frac{1}{16}\int dn\,\mathrm{str}\left(-D_q \partial_n Q \partial_n Q - 2i\omega Q \sigma^3_{\mathrm{ar}}\right)$$

$$Q \equiv g \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} g^{-1}, \qquad g \equiv \begin{pmatrix} 1 & B \\ \tilde{B} & 1 \end{pmatrix}.$$

discussion:

- ▷ effective low energy theory of disordered multi-channel quantum wire (Efetov & Larkin, 80)
- ▷ diffusion at length scales $\frac{K}{\tilde{h}} < \Delta n < \xi \equiv \frac{D_q}{2} \sim \left(\frac{K}{\tilde{h}}\right)^2$
- ▷ localization at length scales $\Delta n > \xi$
- ▷ at rational values $\tilde{h} = 4\pi\frac{p}{q}$: system effectively acquires ring topology. Circumference $\sim q$; (nonabelian) gauge flux.
- ▷ theory sensitive to ϵ-classics, reentrant classical behaviour at $\tilde{h} = 4\pi + \epsilon$

summary

- quasiclassical methods can be powerful alternative to semiclassics
- often implemented in field theoretical framework
- good in non-perturbative settings (localization, gap formation, non-perturbative correlations)
- and not-so-good in others (repetitive dynamics, low order perturbation theory)

time

It is good to know both, semiclassical and quasiclassical concepts