TIME COMPLEXITY ANALYSIS OF GENERALIZED DECOMPOSITION ALGORITHM

T. Kalganova, V. Pshibytko

Belarusian State University of Informatics and Radioelectronics, Laboratory of Image Processing and Pattern Recognition. Brovky St., 6, 220600, Minsk, Republic of Belarus, Phone: (*0375172*) 491-981, Fax: (*0375172*) 495-106, E-mail: jack@expert.belpak.minsk.by or pottosina@risq.belcaf.minsk.by

ABSTRACT: The time complexity of the fast algorithm for generalized disjunctive decomposition of an r-valued function is studied.The considered algorithm to find the best decomposition is based on the analysis of multiple-terminal multiple-valued decision diagrams. It is shown that the time complexity for random r-valued functions depends on the such restriction as the number n_1 of inputs in the first level circuit. In the case where the best partition of input variables is searched with restriction the time complexity is reduced in several times. The algorithm was simulated on a digital computer. The experimental results are in agreement with the theoretical predictions.

KEY WORDS: Decomposition algorithm, r-valued functions, generalized disjunctive decomposition.

1. INTRODUCTION

The decomposition of discrete functions is employed in the design and testing of logic combinational circuits based on the such logic devices as look-up table type FPGA (field program-mable gate array), MV PLA (multiple-valued programmable logic array), MV PAL (multiple-valued programmable array logic), T-date (multiplexer) and etc.

The decomposition method proposed solves the problems of the computation time and the memory requirements. Finding of this problem is to apply the multiple-terminal multiple-valued decision diagrams (MTMDD) instead of decomposition table. Here we can consider the decomposition where the number of variables in X_1 is changed. By one decision diagram, we can evaluate the column multiplicity for $(n-2)$ variants of partition of X. These will reduce the computational time drastically.

In this paper the time required by the decomposition algorithm using MTMDD is studied. In order to make the study of the algorithm meaningful a comparison should be made with the time required by some standard decomposability test.

The complexity of the decomposability algorithm is dependent on the function under test and the number of variables in bount set X_2. To estimate the time required by the algotithm we assume that the r-valued functions under test are random functions.

The rest of paper is organized as follows. In section 2 we start with describing the basic notations and definitions. We then discuss the algorithm to find the good decomposition using MTMDD. The time complexity of the decomposition algorithm is analyzed in Section 3. The experimental results is considered in Section 4. Section 5 is summary of this paper.

2. DEFINITION AND NOTATIONS

To represent r-valued functions with minimal expressions, the following notations are adopted:

E^r : the set of constants is an r-valued Rosser algebra, $E^r=\{0, 1, ..., r\text{-}1\}$

X : the set of n r-valued variables $\{x_1,x_2,...,x_n\}$, where n is the number of elements of X.

$d(X)$: the number of elements of the set X.

σ : $(\sigma_1\sigma_2...\sigma_n)$, where $\sigma_i\in E^r$, i=1,2, ..., n.

τ : $(\tau_1\tau_2...\tau_{n_1})\in\sigma$, where $\tau_i\in E^r$, i=1,2,...,n_1, $d(X_1)=n_1$.

η : $(\eta_1\eta_2...\eta_{n_2})\in\sigma$, where $\eta_i\in E^r$, i=1,2,...,n_2, $d(X_2)=n_2$.

Definition 1. An r-valued function $f(X)$ is said to have a *simple disjunctive decomposition* with respect to set X_1 if there exist r-valued functions h and g such that

$$f(X)=g(h(X_1), X_2) \quad (1)$$

where $\{X_1, X_2\}$ is a partition of X.

Definition 2. An r-valued function $f(X)$ is said to have a *generalized disjunctive decomposition* with respect to X_1 if there exist r-valued functions $h_1,h_2,...,h_k$ and g such that

$$f(X)=g(h_1(X_1), h_2(X_1),..., h_k(X_1), X_2) \quad (2)$$

where $\{X_1, X_2\}$ is a partition of X.

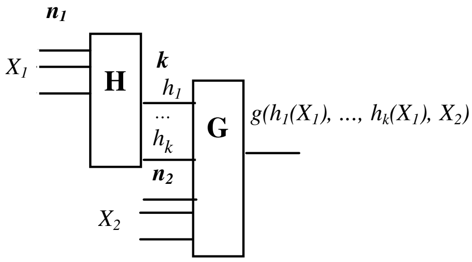


Fig. 1. Circuit diagram of
$f(X)=g(h_1(X_1), h_2(X_1),..., h_k(X_1), X_2)$.

Definition 3. The number of different column patterns in the decomposition table is called as a *column multiplicity* of the decomposition, and denoted by μ.

Note that the column multiplicity defines the type of disjunctive decomposition. An r-valued function $f(X)$ is said to have a simple disjunctive decomposition if μ is less or equal to r [1, 2, 3] and a generalized disjunctive decomposition if μ is less or equal to r^{n_1}, where n_1 is the number of elements in set X_1 [3].

A *multiple-valued decision diagram* (MDD) is a generalization of a binary decision diagram, in which an internal node may have more than two children. An MDD having more than two kinds of terminal nodes (e.g.0,1,...,r-1) is called a *multiple-terminal MDD* [4, 5].

An *ordered MTMDD* is a MDTMD such that the input variables appear in a fixed order in all the paths of the graph, and that no variable appears more than once in a path.

Definition 4. An ordered MTMDD is a *Quasi-Reduced MTMDDD* if every path from the root to the terminal nodes involves all variables, and has no isomorphic sub-graphs in the same level.

Definition 5. The *path function* of a node in a MTMDD is r-valued input 2-valued output function and represents the conditions that there is a path from root to the node. The *sub-path function* of a node in a MTMDD is 2-valued input r-valued output function and represents the conditions that there is the path from a node to the terminal node. A sub-path function of a node is the projection of $f(X)$ over X_1=τ, $f(\tau, X_2)$.

3. THE DECOMPOSITION SEARCHING ALGORITHM

In this section we will describe an algorithm to find all the generalized disjunctive decompositions for r-valued functions. The algorithm consists of the two phases:

1. A MTMDD for given function is generated.
2. This MTMDD are tested for good decomposition.

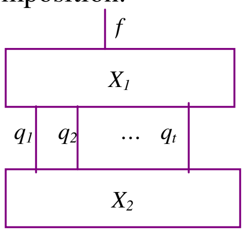


Fig. 2 Partition of a MDD

Lemma. Let $\{X_1, X_2\}$ be a partition of X and the quasi-reduced MTMDD for $f(X)$ is partitioned into two blocks as shown in Fig. 2. Let w_i (i=1, ...,t) be the path functions of the nodes in the lower block that are adjacent to the boundary of the blocks. Then, $w_i \cdot w_j=0$ ($i \neq j$) and $w_i \mathbf{V} w_j=r-1$.

Theorem. Let $\{X_1, X_2\}$ be a partition of X. Suppose that the reduced ordered MTMDD for $f(X)$ is partitioned into two blocks such as shown in Fig. 2. Let t be the number of the nodes in the lower block that are adjacent to the boundary of the two blocks, and μ be the column multiplicity of the decomposition $f(X)=g(h_1(X_1),..., h_k(X_1),X_2))$. Then, t=μ.

Example. Fig. 3 shows the three different partitions:

1) $X_1=\{x_1, x_2\}$, $X_2=\{x_3, x_4, x_5\}$
2) $X_1=\{x_1, x_2, x_3\}$, $X_2=\{x_4, x_5\}$
3) $X_1=\{x_1, x_2, x_3, x_4\}$, $X_2=\{x_5\}$

By Theorem, the column multiplicity for these decompositions are four, three and two, respectively. The first partition gives the generalized disjunctive decomposition, the second and third partitions - simple disjunctive decomposition.

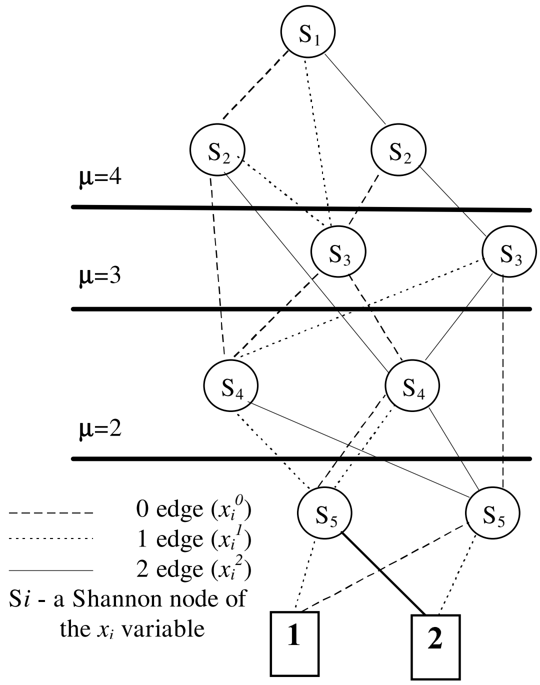


Fig. 3 Determination of column multiplicity using MTMDD

4. An analysis of decomposition algorithm with restrictions

Let us assume that r-valued function have a generalized disjunctive decomposition with the different number of outputs of H. Define the best partition of X when the multiple-valued combinational circuit contains the logical devices implemented the one variable functions (T-gates, PAL, look-up tables type FPGA and etc.). Let q be the time that is needed to process the first level of MTMDD. Because on the second level r nodes are placed, the processing time is $q_2=q_1\cdot r$, on the third level - $q_3=q_1\cdot r^2$,..., on the $(n-1)$-th level-$q_{n-1}=q_1\cdot r^{n-2}$.

The all levers of MTMDD are processed in T_1 denoted as

Definition and notations

$T_1=q_2+q_3+...+q_{n-1}=q_1\cdot(r+r^2+...+r^{n-2})=q_1\cdot(r_{n-1})/(r-1)$.

Note that in this case $n_1=1$. This time is the same as the time needed to processed the decomposition table but here we obtain (n-2) partitions of X.

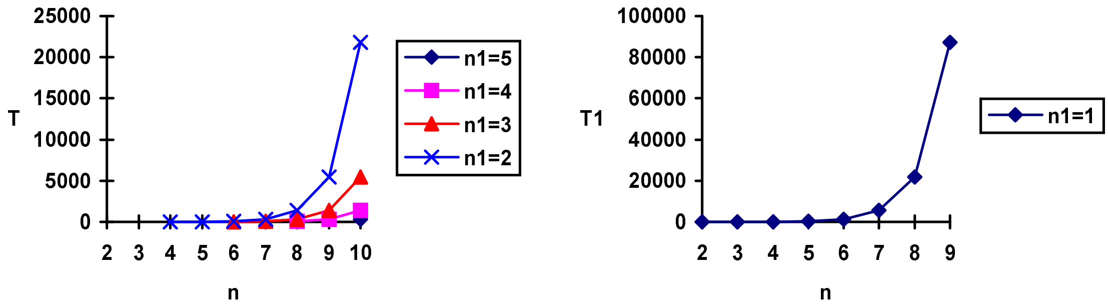
Let us assume that the multiple-valued combinational circuit will consists of logic blocks implemented n_1 variable r-valued function (such logical device can be the MV PLA). Here we can examine from n_1 to $(n-n_1)$ levels of MTMDD. An algorithm to built the quasi-ordered MTMDD is sequantly consider all levels. But, taking into consideration the restriction due to n_1, we can examine only $(n-n_1)$ levels. Thus, the processed time for this case is

$T_{n_1}=q_2+q_3+...+q_{n_1-1}=q_1\cdot(r+r^2+...+r^{n_1-1})=q_1\cdot(r^{n_1}-1)/(r-1)$.

The ratio of T_1/T_{n_1} defines in what times the processing time is reduces when we examine the generalized decomposition with restriction.

$$\frac{T_1}{T_{n_1}} = \frac{(r^n-1)\cdot}{(r-1)}\frac{(r-1)}{(r^{n-n1}-1)} = \frac{(r^n-1)}{(r^{n-n1}-1)} \approx \frac{r^n}{r^{n-n_1}} = r^{n_1}$$

The ratio of $\Delta = T_1/T_{n_1}$ (delta) determines the time complexity of the deccomposition algoritm. Fig. 1 and Fig. 2 show the dependence of the number of input variable on the processing time and on the time complexity, respectively when $r=4$ is constant. This figures illustrate the theoretical predictions. In the case where the number of inputs of H has a restriction the time complexity of proposed algorithm is reduced of the order r^{n_1}.



(a) (b)

Fig. 1 Theoretical predictions. Dependence of the number of input variable on the processing time (a) for n_1=2,3,4,5 (b) for n_1=1

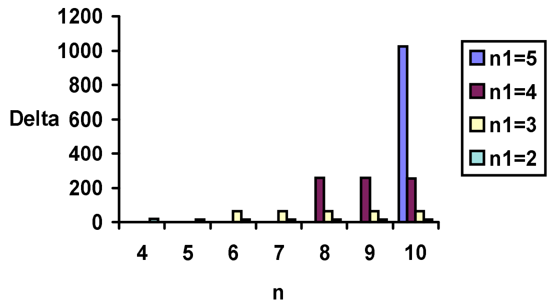


Fig. 2 Theoretical predictions. Dependence of the number of input variable on the time complexity

5. COMPUTER SIMULATION OF THE ALGORITHM

A computer program that simulates the algorithm was run on a Pentium 100 .The processing time for random *4*-valued functinons is protted in Fig.3 and their time complexity is shown in Fig.4.

To show that the processing time agrees with our predictions, the substraction of time complexity obtaned by theoretical way from the time complexity of simulated algorithm is protted in Fig.5. The curve of experimental time complexity is approximately the same as the theoretical time complexity curve. This is proved our theoretical predictions.

The error of experimental investigation is determined by following equition:

$\Delta\sigma = (T_{theoretical} - T_{experimental}) / \Delta T$

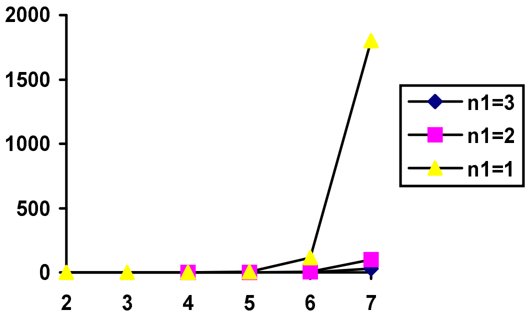


Fig. 3 Theoretical predictions. Dependence of the number of input variable on the processing time for n_1=1,2,3

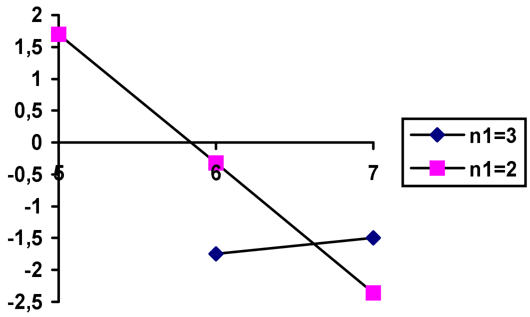


Fig. 4 Experimental investigations. Dependence of the number of input variable on the time complexity

6. Conclusion

The decomposition algorithm based on the MTMDD analysis is proposed for *r*-value functions. In the case where the decomposition is applied to synthesize the MV PLA-based combinational circuits and the thirst MV PLA has n_1 inputs, the time complexity of algorithm becomes of the order r^{n1}. It allows to executethe decomposition of *r*-valued functions for the large number of input variables.

References

1. Specialized multiple-valued analyzers. Editor: M.A. Rakov, *Naukova dumka*, Kiev, 1977.

2. Abugharieh S.B., S.C. Lee, A Fast Algorithm For The Disjunctive Decomposition Of *M*-Valued Functions. Part 1. The Decomposition Algorithm. *IEEE Trans. on Computers*. 1995, pp. 118-125.

3. Kalganova T., Functional Decomposition Methods for Multiple-valued Logic Functions and its System. *Proc. of the Third International Conference on Application of Computer Systems,* Szczecin, Poland, 1996., ñòð.75-82.

4. Sasao T., J.T. Butler, A Method To Represent Multiple-Output Switching Functions by Using Multi-Valued Decision Diagrams. *Proc. of the 26th Int. Symposium on Multiple-Valued Logic,* Spain, 1996, pp.248-254.

5. Shin-ichi Minato, Graph-Based Representations of Discrete Functions. *Proc. of Int. Workshop on Reed-Miller Expansion,* Japan, 1995, pp.1-10.