Systems Modelling and Simulation (5)

Simulation Modelling Environment

1. Chapter 4 course book
2. Simulation environment as a time base dynamic system analysis tool
3. Moving from previous theoretical discussions to practical context

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Today's discussions

- Steps to be taken in a simulation study
- Simulation input / output data
- Main techniques for experimental design
- Model verification, validation and testing

Discrete Event Simulation (DES)

Problem solving technique

Two driving factors:

1. Advances in computer technology
2. Restricted budgets and affordability

Simulation Techniques

- **Monte Carlo**: inherently non-probabilistic problem is solved by stochastic (Random) process – explicit representation of time is not required.

- **Continuous:** The variables within the simulation model are continuous functions and change over time – State of the variables change over time (e.g. differential equations); example: rates and levels

- **Discrete Event:** Changes to parameters (variables) within the model change at precise moments of simulation time; example Meantime between failure or customer inter-arrival.

Time and State

Within the context of DES *Time* and *State* are of paramount importance.

The underpinning elementary concepts that define the relationship between Time and State (Nance* 1987):

(1) Instant (2) Interval (3) Span (4) Object state

* Nance R.E. and Overstreet C.M. (1987), "Diagnostic assistance using digraph representations of DES model specifications", *Transactions of the Society for Computer Simulation,* Vol. 4, No. , pp. 33-57

Elementary Modelling Concepts

1. **Instant:** the value of system time in which an attribute of a system component changes

2. **Interval:** duration between two successive *instances*

3. **Span:** contiguous succession of one or more *interval*

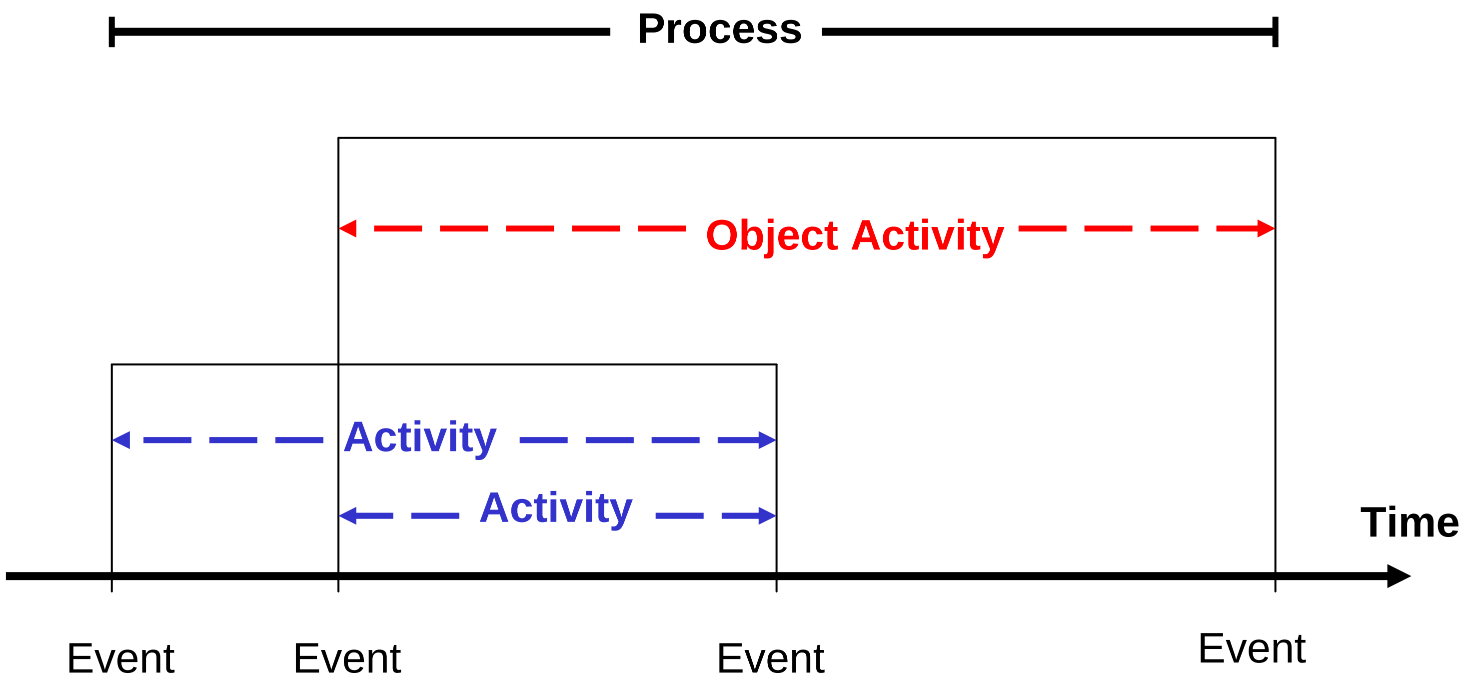
4. **Object State:** enumeration of all attribute values associated with an object at a particular *instant*

Leading to …

Basic Simulation Concepts

- **Activity:** state of an object over an interval
- **Event:** occurs at an instant, and represents a change in object state, it instantiates an activity. A condition expressed as function of time needs to be met for an event to occur.
- **Object Activity:** is the state of an object between two *events*, describing successive state changes for that object.
- **Process:** succession of states of an object over a time span

Basic Simulation Concepts cont.



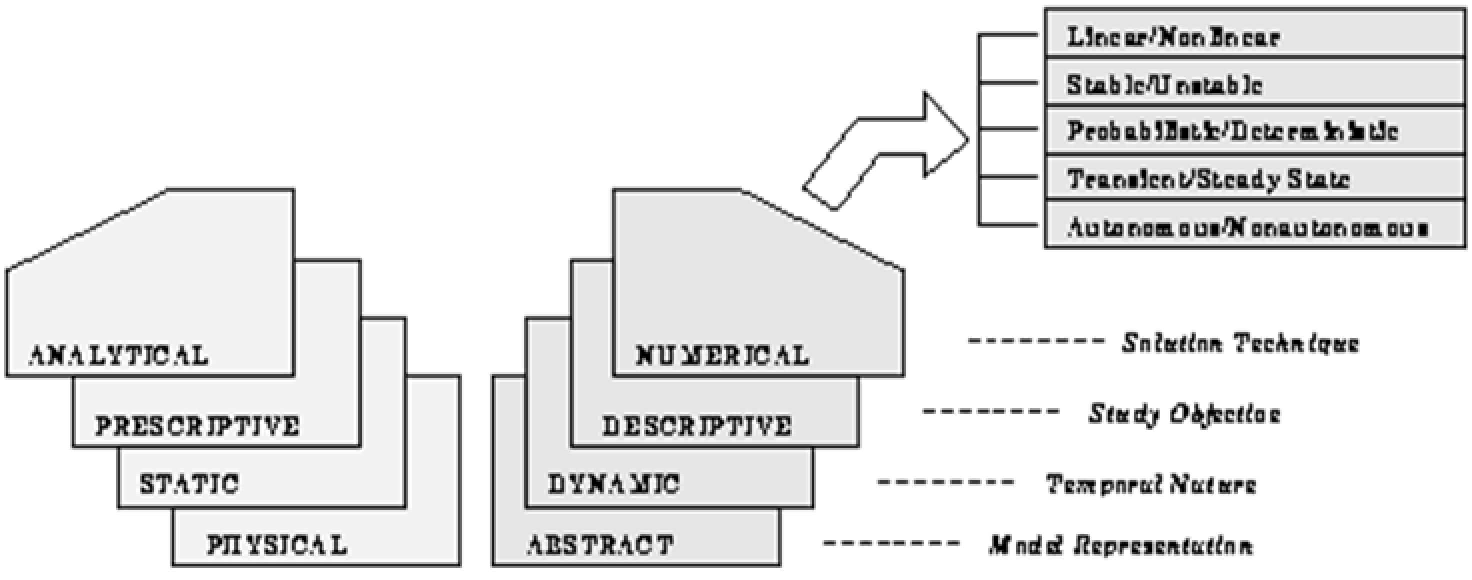
Therefore …

Simulation modelling is the process of describing a system with the goal of experimenting with the model to get a better insight to the behaviour of that system.

To reflect the system, the designed model contains the constituents of the system (i.e. objects) with their associated attributes and the rules of interaction with other objects (*model logic*).

Model Classification Scheme

In Modelling understanding the concept of the model is very important.



*** Source: Balci O. (1987), Modelling and simulation class notes, Department of Computer Science, Virginia Tech., VA pp. 10-13, Spring**

The four dimensions for model classification

According to Balci (1987):

1. Mathematical Model representation and formulation (also see lecture note 1 *Successful Simulation* section)
2. Definition of Model Objectives without preliminary value judgement
3. Temporal properties of the model (static vs. dynamic)
4. Solution technique (analytical vs. numerical)

DES

Discrete Event Simulation can be classified as: abstract, dynamic, descriptive and numerical models.

DES Models can be …

- relationship between its variables to be either **linear or non-linear**
- **Stable or unstable**
- **Steady-state** (behaviour remains similar in time) or **Transient** (behaviour changes in time)
- **Probabilistic** or **Deterministic**
- **Autonomous** (initial input) or **non-Autonomous** (input at any time during the simulation)

Also see lecture notes 1-4

Simulation facts

A. Input Data by nature is random e.g. time between arrivals, machine breakdown, …
B. The interrelationship between system elements (i.e. *prevailing logic)* are complicated
C. The result of the simulation should be a close approximation of the real system

A. Random Input variable

- A purely random input variable to a simulation could be considered as a ***Stochastic Process.***
- A stochastic process is defined by collection of random variables:

$\{X(t),\ t \text{ in } T\}$

Random Variable — Set of processes

- In DES:

$\{X_K,\ k = 1,2,3,\ \ldots\}$ *k: is the order of the variable*

With probability distribution function:

$$F_k(x) = Pr\ \{X_k <= x\}$$

A. Univariate vs. Multivariate

Univariate Process:

Where each random variable *X* represents a single quantity such as customer service time. So X_k would be the service time for customer *kth*.

Multivariate Process:

Where *X* represents a number of quantities such as service time for different type of products. So that

$X_k = (A_k, B_k, \ldots)$ where A, B, … represent the type of product

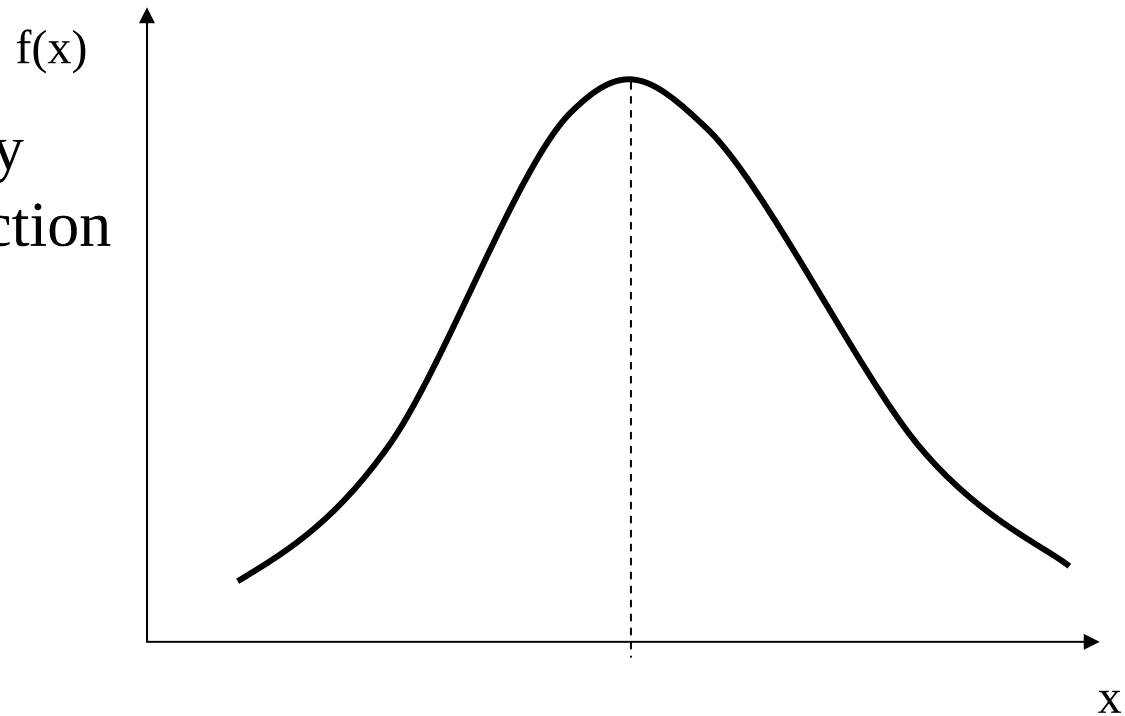
A. Input data in practice

1. Data collection
2. Statistical experimental design
3. Statistical Distribution analysis and Curve Fitting
4. Input data validation and verification (K-S and Chi-square tests)

Examples of Distributions: Uniform, Normal, LogNormal Exponential, Poisson, …

A. Example – Normal (Mean, StdDev)

Probability
Density Function



B. Process Definition (System Logic)

- Resource Attributes: Name, process time, schedule, capacity, MTBF, buffer size, priority rules, seize-delay-release rules, transport rules, …
- Queues: Location, capacity and other rules, …
- System layout: Location of resources, …
- Process Sequence: Process plan, process flow, transportation between resources, …
- Transporters: Type, capacity, speed, …

C. Simulation Analysis of Results

- Performance Evaluation and What-if scenarios
- Sensitivity analysis
- Optimisation
- Gradient Estimation Applications (test and measure sensitivity information)
- Report generating

C. Analysis Conditions

To perform statistical analysis of the simulation output we need to establish that output data is a **covariance stationary process**. That is the data collected over n simulation runs.

Stationary Process (strictly stationary)

- A stationary process is a stochastic process which probability distribution at a fixed time or position $\{ X(t), t \in \mathrm{T} \}$ is the same for all times and positions.
- Consequently, parameters *mean* and *variance* does not change over the same time and position

Example: Operator service time, mean time between arrivals, …

Covariance Stationary process

- A covariance stationary process is a stochastic process $\{ X(t), t \in T \}$ having finite second moments. That is: $[X(t)]^2$ to be finite.

- Two stochastic processes (covariance stationary):

1. **A sequence of Y_0, Y_1, … of random-value sequence of independent variables that are identically distributed having finite variance**

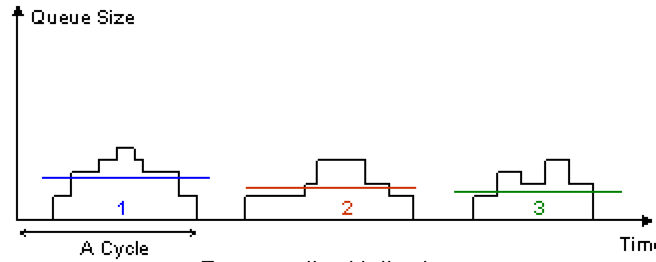
2. **Z being random variable with known distribution function, $Z_0 = Z_1, \ldots = Z$, the process $\{Z_0, Z_1, \ldots\}$ is stationary if Z has a finite variance.**

Steady State Simulation

- Gathering steady state simulation output requires statistical assurance.
- The difficulty is to obtain independent simulation runs without the transient period
- Most popular techniques:

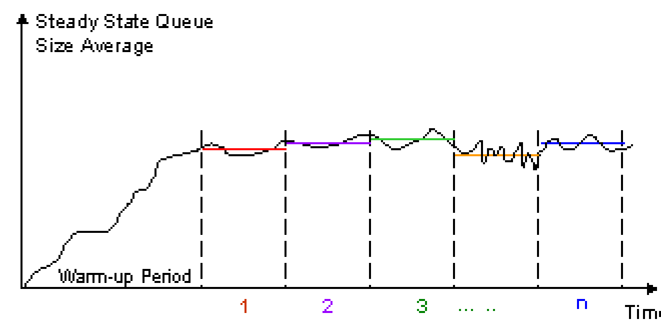
Steady State Simulation

- Regenerative Method



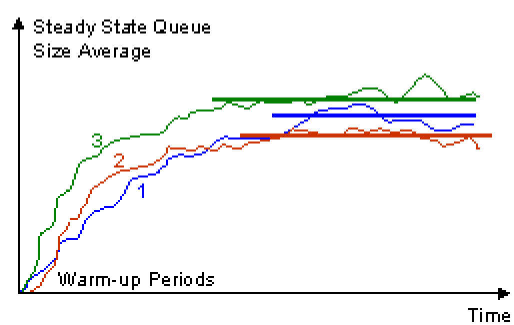
Regenerative Method

- Batch Means



Method of Batch Means

- Independent Replications



Method of Independent Replications

Steady State Simulation

- **Regenerative not really used but has good properties. But suppose you have a regenerative simulation consisting of m cycles of <u>n1, n2, …, nm</u> size – the mean value is total divided by number of <u>n</u>.**

- **The batch means consists of a very long simulation which is suitably divided into initial transient period of <u>n</u> batches. Each batch is then treated as an independent run, whilst no observation is made during the warm up period.**

- **Independent replication is most popular for systems with short transient period. Method requires independent simulation runs (number of replications) using different random seeds for simulation random number generator.**

Simulation Experiment and Design

- **Factors**: Input parameters and structural assumptions composing a model
- Decisions on which parameters in the model are fixed and which are experimental depends on the system and the system analyst
- Factors can be *quantitative (*i.e. numerical value) or *qualitative* (i.e. not quantifiable)
- Factors can be controllable or uncontrollable
- The experimental design in simulation provides for a platform to ascertain particular configurations to achieve expected results

Simulation Models

- Are more than just a combination of mathematical formulas which require input parameters and release output performance measures
- Could conduct sensitivity analysis (Gradient estimation) – which parameters have the most impact on the system

Simulation Model Validation, Verification and Testing

- Validation: demonstrate that the model behaves in a satisfactory way consistent with the objectives

- Verification: substantiating that a model has transformed from one condition to another as intended and as accurate as possible

- Testing: assess if inaccuracies exist in the model.