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Testing PPP for the South African Rand/ US Dollar Exchange Rate at Different Frequencies

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TESTING PPP FOR THE SOUTH AFRICAN RAND/ US DOLLAR REAL EXCHANGE RATE AT DIFFERENT DATA FREQUENCIES

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Abstract

This paper tests the PPP hypothesis for the South African rand/ US dollar real exchange rate using a fractional integration framework. The results suggest that the real exchange rate of the South African rand with respect to the US dollar is a highly dependent variable with an order of integration very close to 1. This finding is not affected by the data frequency considered (daily, weekly or monthly). Also, there appears to be a single break in December 2001 (possibly corresponding to a change in the monetary policy framework), with the unit root null being rejected in favour of $d > 1$ for the periods before the break, but not afterwards. Thus, our results strongly reject the PPP hypothesis for the South African rand/ US dollar rate across data frequencies.

\textbf{Keywords:} PPP; real exchange rate; mean reversion.  
\textbf{JEL Classification:} C12, C22, F31.

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1. Introduction

Purchasing Power Parity (PPP) is a central tenet in international economics. It is assumed to hold continuously in flexible-price models of the exchange rate, whilst in sticky-price ones it is a long-run property, temporary deviations from the long-run equilibrium being possible. In the new open economy models it is a condition for market completeness (Chortareas and Kapetanios, 2009). Establishing whether PPP holds is also crucial in order to assess the effects of a devaluation.

Empirical studies have used different methods to examine the validity of PPP. Some of them have tested for cointegration between nominal exchange rates and prices (Kim, 1990; McNown and Wallace, 1989; 1994; Serletis and Goras, 2004; Gouveia and Rodrigues, 2004; etc.). Others have applied unit root tests to real exchange rate (these are the so-called “stage-two” tests - see Froot and Rogoff, 1995). However, such tests have been found to be unable to distinguish between random-walk behaviour and very slow mean-reversion to the long-run equilibrium level (see, e.g., Frankel, 1986; Lothian and Taylor, 1997), as in small samples they have very low power against alternatives such as trend-stationary models (DeJong et al., 1992), structural breaks (Campbell and Perron, 1991), regime-switching (Nelson et al., 2001), or fractionally integration (Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996). Moreover, at times they exhibit erratic behaviour, suggesting the presence of endemic instability (see Caporale et al., 2003 and Caporale and Hanck, 2009, the latter finding that this also characterises cointegration tests, and therefore is not due to arbitrarily imposed symmetry/proportionality restrictions); however, adjusting the residuals for non-normality and heteroscedasticity using a wild bootstrap method attenuates this type of behaviour considerably (see Caporale and Gregoriou, 2009), and the latter disappears
almost completely if panel tests are performed, the evidence for PPP becoming much stronger (see Caporale and Hanck, 2010).

The aforementioned time series studies all restrict themselves to the cases of stationarity I(0) and nonstationary I(1) processes. The more recent literature has stressed the importance of considering the possibility of non-integer values for the degree of integration. In this case, PPP is satisfied if the fractional differencing parameter \( d \) is strictly smaller than 1, although the higher \( d \) is, the longer it takes for the adjustment to the long-run equilibrium to be completed. Alternatively, panel methods have been used to increase the power of tests for PPP (see, e.g., Chortareas and Kapetanos, 2009 and some of the references therein).

In the present paper we adopt a fractional integration framework which allows for long memory and also for a much richer dynamic specification. Earlier studies of this type have normally focused on the developed countries and analysed some of the major currencies. For instance, Booth et al. (1982) found positive memory (\( d > 0 \)) during the flexible exchange rate period (1973-1979) but negative one (\( d < 0 \), i.e., anti-persistence) during the fixed exchange rate period (1965-1971) for the British pound, French franc and Deutsche mark, Cheung (1993) also found evidence of long memory behaviour during the managed floating regime. On the other hand, Baum et al. (1999) estimated ARFIMA models and found no evidence of long-run PPP in the post-Bretton Woods era (see also Fang et al., 1994, Crato and Ray, 2000, and Wang, 2004). Caporale and Gil-Alana (2010) also provide some evidence for the Latin American countries. By contrast, the present study conducts the analysis for the exchange rate of the South African rand vis-à-vis the US dollar.

The outline of the paper is as follows. Section 2 describes the data. Section 3 presents the empirical results, and Section 4 offers some concluding remarks.
2. Data

The series used for the analysis is the daily real exchange rate for the South African rand vis-à-vis the US dollar, for the time period January 2nd, 1990 – December 31st, 2008, obtained from the “Statistics South Africa” (http://www.statssa.gov.za), and the Federal Reserve Bank of St. Louis database. Thus, we focus on the post-apartheid period.

[Insert Figure 1 about here]

Figure 1 plots the series at the daily frequency. At first sight, it appears to be stationary, but to exhibit some degree of dependence. However, the correlogram and the periodogram, plotted in Figures 2 and 3 respectively, both clearly indicate that the series is nonstationary: the former displays values decaying very slowly to zero and the later has its highest value at the zero frequency.

[Insert Figures 2 and 3 about here]

Figures 4 – 6 show similar plots for the first differenced data. These exhibit higher values towards the end of the sample, which may be consistent with conditional heteroscedastic models. In this paper, however, we focus on the degree of dependence and use a procedure that is robust to heteroscedastic errors. The correlogram and the periodogram of the differenced data indicate that the series may be stationary or I(0).

3. Empirical results

As a first step we carry out standard unit root tests, specifically Dickey-Fuller (ADF, 1979), Phillips-Perron, (PP, 1988), and Kwiatkowski et al. (KPSS, 1992) tests to determine whether the series is nonstationary I(1) or stationary I(0). The results (not reported for reasons of space) strongly support the presence of a unit root. However,
they should be taken with caution, as these methods have extremely low power if the alternatives are of a fractional form.

Next we consider the possibility of fractional integration and examine first a model of form:

\[ y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \] (1)

where \( y_t \) is the time series observed, \( \alpha \) and \( \beta \) are the coefficients on the deterministic terms (an intercept and a linear time trend respectively), and \( x_t \) is assumed to be I(d), where d can be any real number. Different assumptions will be made about the error term \( u_t \) in (1).

We estimate d in (1) using a Whittle function in the frequency domain (Dahlhaus, 1989) also employing a testing procedure developed by Robinson (1994) that is very general in the sense that we can test any real value d in (1) (i.e., including nonstationarity, \( d \geq 0.5 \)) with a standard normal limit distribution, which holds independently of the way of modelling the I(0) error term.

[Insert Table 1 about here]

First we assume that the disturbances (\( u_t \) in (1)) are white noise. We report the estimates of d along with the 95% confidence band of the non-rejection values using Robinson’s (1994) parametric approach. The results are displayed in the first row in Table 1. It can be seen that if regressors are not included (i.e., \( \alpha = \beta = 0 \) in (1)) the estimated value of d is 0.990, and the unit root null cannot be rejected at the 5% level. However, when including an intercept or an intercept with a linear time trend, the estimated value of d is around 0.978 and the unit root null is rejected in favour of slow mean reversion. Next, we allow for weak autocorrelation in the error term and assume that \( u_t \) in (1) is an AR(1) process. Higher AR orders were also considered obtaining very similar results. In this case, if regressors are not included the estimated value of d is
significantly above 1; however, with an intercept and/or a linear trend the estimated \(d\) is below unity and the unit root null cannot be rejected in these two cases.

The results presented so far seem to indicate that the exchange rate of the South African rand with respect to the US dollar is a highly dependent variable with an order of integration very close to 1. Next we check if the above result holds for different data frequencies. For this purpose we employ weekly and monthly data over the same sample period (January, 1990 – December, 2008). The results on a weekly basis are reported in Table 2, while those based on monthly data are given in Table 3.

[Insert Tables 2 and 3 near here]

Starting with the weekly case, it can be seen that in the case of white noise residuals the unit root null cannot be rejected, even though the estimate of \(d\) is smaller than 1 in the case of no regressors and above 1 with an intercept and/or a trend. If \(u_t\) is autocorrelated the results are similar to the daily case, finding evidence of \(d > 1\) for the case of no regressors and I(1) with deterministic terms. Finally, using monthly data (see Table 3) the same conclusions hold, i.e. the unit root cannot be rejected for the case of uncorrelated errors, \(d\) is above 1 with AR(1) \(u_t\) with no regressors, and the I(1) hypothesis cannot be rejected when including an intercept and/or a linear trend. Therefore, at least for the data analysed here, the results seem to be robust across data frequencies.

On the basis of LR tests and the t-values for the deterministic terms, a model with an intercept and AR(1) disturbances appears to be the most adequate specification for each series. Thus, the orders of integration are 0.980, 1.059 and 0.961 respectively for the daily, weekly and monthly rates, and the unit root cannot be rejected in any of the three series.
The potential presence of structural breaks should also be investigated. Note that fractional integration and structural breaks are closely related issues. For example, Bhattacharya et al. (1983), Teverovsky and Taqqu (1997), Diebold and Inoue (2001), Granger and Hyung (2004) and Ohanissian et al. (2008) among others show that fractional integration may be a spurious phenomenon caused by the existence of breaks in short-memory I(0) contexts. Similarly, Kuan and Hsu (1998), Wright (1998) and Krämer and Sibbertsen (2002) showed that evidence of structural change might be spurious since most commonly employed tests for breaks are biased towards an over-rejection of the null of no change when the process exhibits long memory.

In this paper we employ a recent technique developed by Gil-Alana (2008) that allows to consider breaks at unknown periods of time with different orders of integration across subsamples. In its simplest form (i.e., with a single break) it takes the following form:

\[
y_t = \beta_1^T z_t + x_t; \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1, \ldots, T_b, \tag{2}
\]

and

\[
y_t = \beta_2^T z_t + x_t; \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b + 1, \ldots, T, \tag{3}
\]

where the \( \beta \)'s are the coefficients of the deterministic terms, \( d_1 \) and \( d_2 \) can take real values, \( u_t \) is I(0), and \( T_b \) is the unknown break date. This method is based on minimising the residuals sum of squares in the two subsamples and can be easily extended to the case of two or more breaks (see Gil-Alana, 2008).

[Insert Tables 4 and 5 about here]

The results based on the above approach are displayed in Tables 4 and 5 respectively for the two cases of uncorrelated and AR(1) errors. In all cases the model is specified as to include an intercept but not a linear trend. The first noticeable feature is the presence of a single break in December 2001 in all three series. This might be
interpreted as a consequence of the change in the monetary policy framework which took place the year before, when inflation targeting was introduced, and is consistent with other studies focusing on inflation and interest rate expectations data and forward interest rate data to demonstrate the increased credibility and reasonable predictability of monetary policy since the adoption of inflation targeting in 2000 (see Aron and Mullbauer, 2007). Also, the order of integration decreases slightly after the break. Starting with the case of white noise disturbances (Table 4), the estimated orders of integration for the pre-break periods are 1.003, 1.087 and 1.161 for daily, weekly and monthly exchange rates respectively, the unit root null being rejected in the last two cases in favour of values of d above 1. Following the breaks the orders are 0.986, 0.964 and 0.972, and the unit root null is never rejected. In the case of AR(1) disturbances (see Table 5) the values are slightly different but the same conclusions hold. Thus, the unit root null is rejected in favour of d > 1 for weekly and monthly exchange rates for the periods before the break, but it cannot be rejected for any of the three series for the post-break periods.

4. Conclusions

This paper has tested the PPP hypothesis for the South African rand / US dollar real exchange rate using a fractional integration framework. The results suggest that the real exchange rate of the South African rand with respect to the US dollar is a highly dependent variable with an order of integration very close to 1. This finding is not affected by the data frequency considered (daily, weekly or monthly). Also, there appears to be a single break in December 2001 (possibly corresponding to a change in the monetary policy framework), with the unit root null being rejected in favour of d > 1 for the periods before the break, but not afterwards. Thus, although the degree of
dependence is lower after the break, no evidence of mean reversion is found, implying that PPP does not hold in this context.
References


Figure 1: Real exchange rate (South Africa Rand / US Dollar)

Figure 2: Correlogram of the real exchange rate

The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.

Figure 3: Periodogram of the real exchange rate (South Africa Rand / US Dollar)

The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$. 
Figure 4: First differences of the real exchange rates

Figure 5: Correlogram of the first differenced data

The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.

Figure 6: Periodogram of the first differenced data

The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$. 
Table 1: Estimates of \( d \) using daily data

<table>
<thead>
<tr>
<th></th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>White noise</td>
<td>0.990</td>
<td>0.978</td>
<td>0.978</td>
</tr>
<tr>
<td></td>
<td>(0.973, 1.009)</td>
<td>(0.961, 0.996)</td>
<td>(0.961, 0.996)</td>
</tr>
<tr>
<td>AR (1) errors</td>
<td>1.300</td>
<td>0.980</td>
<td>0.980</td>
</tr>
<tr>
<td></td>
<td>(1.257, 1.344)</td>
<td>(0.956, 1.006)</td>
<td>(0.957, 1.006)</td>
</tr>
</tbody>
</table>

The values in parentheses refer to the 95% confidence band of the non-rejection values of \( d \).

Table 2: Estimates of \( d \) using weekly data

<table>
<thead>
<tr>
<th></th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>White noise</td>
<td>0.997</td>
<td>1.033</td>
<td>1.033</td>
</tr>
<tr>
<td></td>
<td>(0.959, 1.041)</td>
<td>(0.995, 1.078)</td>
<td>(0.995, 1.078)</td>
</tr>
<tr>
<td>AR (1) errors</td>
<td>1.319</td>
<td>1.059</td>
<td>1.059</td>
</tr>
<tr>
<td></td>
<td>(1.232, 1.415)</td>
<td>(0.982, 1.150)</td>
<td>(0.982, 1.150)</td>
</tr>
</tbody>
</table>

The values in parentheses refer to the 95% confidence band of the non-rejection values of \( d \).

Table 3: Estimates of \( d \) using monthly data

<table>
<thead>
<tr>
<th></th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>White noise</td>
<td>0.991</td>
<td>1.035</td>
<td>1.035</td>
</tr>
<tr>
<td></td>
<td>(0.914, 1.088)</td>
<td>(0.953, 1.143)</td>
<td>(0.953, 1.143)</td>
</tr>
<tr>
<td>AR (1) errors</td>
<td>1.336</td>
<td>0.961</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>(1.182, 1.507)</td>
<td>(0.783, 1.149)</td>
<td>(0.787, 1.149)</td>
</tr>
</tbody>
</table>

The values in parentheses refer to the 95% confidence band of the non-rejection values of \( d \).
### Table 4: Estimates of $d$ with a single break and white noise disturbances

<table>
<thead>
<tr>
<th>Data frequency</th>
<th>Break date</th>
<th>$d_1$</th>
<th>$d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily</td>
<td>Dec. 20, 2001</td>
<td>1.003</td>
<td>0.986</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.984, 1.026)</td>
<td>(0.957, 1.018)</td>
</tr>
<tr>
<td>Weekly</td>
<td>Dec. 14, 2001</td>
<td>1.087</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.044, 1.139)</td>
<td>(0.880, 1.072)</td>
</tr>
<tr>
<td>Monthly</td>
<td>December 2001</td>
<td>1.161</td>
<td>0.972</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.026, 1.343)</td>
<td>(0.859, 1.138)</td>
</tr>
</tbody>
</table>

The values in parentheses refer to the 95% confidence band of the non-rejection values of $d$.

### Table 5: Estimates of $d$ with a single break and AR(1) disturbances

<table>
<thead>
<tr>
<th>Data frequency</th>
<th>Break date</th>
<th>$d_1$</th>
<th>$d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily</td>
<td>Dec. 20, 2001</td>
<td>1.021</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.992, 1.054)</td>
<td>(0.942, 1.038)</td>
</tr>
<tr>
<td>Weekly</td>
<td>Dec. 14, 2001</td>
<td>1.153</td>
<td>0.859</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.063, 1.269)</td>
<td>(0.683, 1.148)</td>
</tr>
<tr>
<td>Monthly</td>
<td>December 2001</td>
<td>1.177</td>
<td>0.917</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.963, 1.695)</td>
<td>(0.689, 1.153)</td>
</tr>
</tbody>
</table>

The values in parentheses refer to the 95% confidence band of the non-rejection values of $d$.