REAL EXCHANGE RATES IN LATIN AMERICA:
THE PPP HYPOTHESIS AND FRACTIONAL INTEGRATION

Guglielmo Maria Caporale
Brunel University, London

and

Luis A. Gil-Alana
University of Navarra

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Abstract
This paper tests for PPP in a group of seventeen Latin American (LA) countries by applying fractional integration techniques to real exchange rate series. Compared to earlier studies on these economies, this approach has the advantage of allowing for non-integer values for the degree of integration, and thus for the possibility of PPP not holding continuously but as a long-run equilibrium condition. Further, breaks in the series are endogenously determined using a procedure based on the least-squares principle. This is particularly crucial in the Latin American countries, which have been affected by several exchange rate crises and policy regime changes. The results, based on different assumptions about the underlying disturbances, are in the majority of cases inconsistent with PPP, even more so when breaks are incorporated: Argentina is the only country for which clear evidence of mean reversion is found in the model including a break, albeit only in the second subsample.

Keywords: Real Exchange Rates, Purchasing Power Parity, Fractional Integration, Structural Breaks

JEL Classification: C12, C22, F31

Corresponding author: Professor Guglielmo Maria Caporale, Centre for Empirical Finance, Brunel University, Uxbridge, Middlesex UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. Email: Guglielmo-Maria.Caporale@brunel.ac.uk

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1. Introduction

The theory of Purchasing Power Parity (PPP) plays a central role in international economics. It is a key building block in monetary models of exchange rate determination. In flexible-price models, it is assumed to hold continuously; in sticky-price ones, although temporary deviations from the long-run equilibrium are possible, PPP is still a maintained assumption for the long run. Therefore, whether PPP holds is a valuable piece of information for policymakers who want to assess the effects of a devaluation, for instance on competitiveness, since under PPP these will disappear in the long run. In the new open economy models PPP is a required condition for market completeness and the equalisation of the marginal utility of home and foreign currency that in turns allows for perfect risk sharing (Chortareas and Kapetanios 2008).

Empirical research has successively relied on various methodological approaches to test the validity of PPP. Some authors have employed cointegration tests for nominal exchange rates and prices (Kim, 1990; McNown and Wallace, 1989; 1994; Serletis and Goras, 2004; Gouveia and Rodrigues, 2004; etc.). Others have focused on the properties of the real exchange rates, and unit root tests have been widely employed to examine the validity of the PPP. Specifically, so-called “stage-two” tests (see Froot and Rogoff, 1995) focused on the null that the real exchange rate follows a random walk, the alternative being that PPP holds in the long run. However, such unit root tests were found not to be able to distinguish between random-walk behaviour and very slow mean-reversion in the PPP-consistent level of the real exchange rate (see, e.g., Frankel, 1986, and Lothian and Taylor, 1997), unless very long spans of data were used (see, e.g., Lothian and Taylor, 1996, and Cheung and Lai, 1994). The most frequently used tests were those of Fuller (1976) and Dickey and Fuller (ADF, 1979), but it is now well known that these and other unit root tests have very low power in small samples against
plausible alternatives such as trend-stationary models (Hakio, 1984; DeJong, Nankervis, Savin and Whiteman, 1992), structural breaks (Perron, 1989; Campbell and Perron, 1991), regime-switching (Nelson, Piger and Zivot, 2001), or even fractionally integrated alternatives (Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996). Moreover, Caporale et al. (2003) argued that the type of stationarity exhibited by the real exchange rate cannot be accommodated by the fixed-parameter autoregressive homoscedastic model underlying standard unit root tests. Using a dataset including 39 countries and spanning a period of up to two centuries, they analysed the behaviour of both WPI- and CPI-based measures of the real exchange rate. In particular, they computed a recursive t-statistic, and showed that it has an erratic behaviour, suggesting the presence of endemic instability, and of a type of non-stationarity more complex than the unit root one usually assumed. This was confirmed by Caporale and Hanck (2008), who report that erratic behaviour also characterises cointegration tests and hence it is not simply a consequence of arbitrarily imposed (symmetry/proportionality) restrictions. However, Caporale and Gregoriou (2008) find that, when the residuals are adjusted for non-normality and heteroscedasticity using a wild bootstrap method, the rejection percentages of the unit root null increase sharply, and the erratic behaviour of the t-statistic becomes less apparent, providing stronger evidence in favour of PPP, and suggesting that such a correction might at least go some way towards solving the “PPP puzzle”.

Other authors have used panel data unit root tests. Examples are the papers of Jorion and Sweeney (1996), Papell (1997), Papell and Theodoridis (1998), O’Connell (1998) and Koedijk, Schotman and Van Dijk (1998); however, the results based on panel data tests are very inconclusive. Among the papers that find stationarity of real exchange rates using panel approaches are Frankel and Rose (1996), MacDonald
(1996), Oh (1996), Papell (1997), Taylor and Sarno (1998), etc. Caporale and Hanck (2006) carry out a variety of panel unit root tests which are robust to cross-sectional dependence and report that evidence of erratic behaviour disappears, and empirical support is found for PPP. On the other hand, there is also evidence based on panel data that is less favourable to PPP (O’Connell, 1998; Papell and Theodoridis, 1998, 2001; etc.). However, several authors have pointed out some fundamental problems in using panel unit root tests. For instance, Mark, 2001, and Taylor and Sarno, 1998, note that the null is specified as a joint nonstationary hypothesis. Thus, cases may exist where the panel appears to be stationary but a number of individual series display unit roots. In fact, even one stationary series may suffice to reject the unit root null for the whole panel. Caporale and Cerrato (2006) highlight some more drawbacks affecting panel approaches. First, unit root tests suffer from severe size distortions in the presence of negative moving average errors. Second, the common demeaning procedure to correct for the bias resulting from homogeneous cross-sectional dependence is not effective; more worryingly, it introduces cross-correlation when it is not already present. Third, standard corrections for the case of heterogeneous cross-sectional dependence do not generally produce consistent estimators. Fourth, if there is between-group correlation in the innovations, the SURE estimator is affected by similar problems to FGLS methods, and does not necessarily outperform OLS. Finally, cointegration between different groups in the panel could also be a source of size distortions. They offer some empirical guidelines to deal with these problems, but conclude that panel methods are unlikely to solve the PPP puzzle.

Concerning the Latin American economies, most of the recent literature on long-run PPP is based on cointegration tests between the nominal exchange rate, domestic and foreign price indices as well as unit root testing on real exchange rates. The
evidence on PPP is again very inconclusive. For example, McNown and Wallace (1989) found support for PPP in the cases of Argentina, Brazil and Chile. Liu (1992) tested for PPP in a sample of ten Latin American countries finding evidence supporting it with respect to the US. On the other hand, Bahmani-Oskooee (1993, 1995) found that PPP does not hold in the majority of the Latin American countries.

The possibility of structural breaks is another issue that has been taken into account when examining the validity of the PPP hypothesis. This was motivated by the analysis of Perron (1989, 1993), who argued that the 1929 crash and the 1973 oil price shock could be the reason for the non-rejection of the unit root hypothesis in many macroeconomic series, and that when these were taken into account, deterministic models were preferable. In a recent paper, Breitung and Candelon (2005) analysed a panel of Latin American countries by applying a panel unit root test that is robust to structural breaks caused by currency crises. Their results do not provide empirical support to PPP, in contrast with the findings of Calderon and Duncan (2003). Diamantis (2003) studied the cases of Argentina, Brazil, Chile and Mexico and found some evidence in favour of PPP. Francis and Iyare (2006) applied the nonlinear stationary test of Kapetanios et al. (2003) to various Caribbean and Latin American real exchange rates, concluding that most of them were in fact stationary. Using a Markov regime-switching model, Holmes (2008) tested PPP in six Latin American economies; while standard unit root tests suggest nonstationarity, the regime-switching approach indicates the existence of two distinct stationary regimes.

A common feature of all the above literature is that it restricts itself to the cases of stationarity I(0) and nonstationary I(1) processes, and therefore does not consider the

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1 This question was also examined by Christiano (1992), Demery and Duck (1992), Krol (1992), Banerjee et al. (1992), Zivot and Andrews (1992), Mills (1994), Bai and Perron (1998), etc., some of these authors introducing procedures for endogenously determined breaks.
possibility of non-integer values for the degree of integration. In this paper we focus on univariate models and use long-range dependence techniques to analyse the validity of the PPP hypothesis in the Latin American countries. We focus on the real exchange rates and use fractional integration or I(d) models to examine if mean reversion takes place. Other authors have already used this methodology. Applying R/S techniques to daily rates returns for the British pound, French franc and Deutsche mark, Booth, Kaen and Koveos (1982) found positive memory (d > 0) during the flexible exchange rate period (1973-1979) but negative one (d < 0, i.e., anti-persistence) during the fixed exchange rate period (1965-1971). Later, Cheung (1993) also found evidence of long memory behaviour in foreign exchange markets during the managed floating regime. On the other hand, Baum, Barkoulas and Caglayan (1999) estimated ARFIMA models for real exchange rates in the post-Bretton Woods era and found almost no evidence to support long-run PPP. Additional papers on exchange rate dynamics using fractional integration are Fang, Lai and Lai (1994), Crato and Ray (2000) and Wang (2004). These papers focus on developed countries - to our knowledge, no attempt has been made so far to analyse PPP in the Latin American countries using from a fractional integration model as we do in the present study.

The outline of the paper is as follows: Section 2 describes the statistical model and the techniques employed for the analysis. Section 3 describes the data and discusses the empirical results. Section 4 contains some concluding remarks.

2. The statistical model

Throughout this paper we focus on fractional integration models. We first consider a very general specification that enables us to consider many cases of interest. We assume

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2 One exception is the paper of Alves et al. (2001). These authors found evidence of fractional cointegration between the US dollar exchange rate, domestic and foreign prices in Brazil.
that \( y_t \) is the time series we observe (in our case, the log of real exchange rates in the Latin American countries), which is specified as

\[
y_t = \alpha + \beta t + x_t, \quad t = 1, 2, ..., \tag{1}
\]

\[
(1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \tag{2}
\]

where \( u_t \) is supposed to be an I(0) process, defined as a covariance-stationary process with spectral density function that is positive and finite at any frequency, and \( d \) can be any real value. Thus, \( u_t \) may be a white noise process but also any class of weakly (e.g., ARMA) autocorrelated structure. We estimate the parameters in (1) and (2) using the Whittle function in the frequency domain (Dahlhaus, 1989), and use a parametric testing procedure developed by Robinson (1994) to determine the confidence bands for the fractional differencing parameter \( d \) (see Appendix A). We test the null hypothesis:

\[
H_0 : d = d_0 \tag{3}
\]

in (1) and (2) for any real value \( d_0 \). Thus, for example, if the null with \( d_0 = 0 \) cannot be rejected, this supports the trend-stationary representation adopted by many authors. On the other hand, failure to reject the null with \( d_0 = 1 \) supports a unit root specification, implying nonstationarity, and thus constituting evidence against PPP. However, since \( d \) can be a non-integer value, we also consider other stationary and nonstationary hypotheses. This is important in terms of PPP. If \( d < 1 \), the series is mean-reverting implying that shocks will disappear in the long run, as opposed to the case of \( d \geq 1 \) where no mean reversion occurs.\(^3\) In the former case, PPP will be satisfied in the long run, and the speed of the convergence process will depend on the value of \( d \). As long as

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\(^3\) In the case of nonstationary mean reversion, i.e., \( 0.5 \leq d < 1 \), the coefficients in the MA representation of the process decay, albeit at a slower rate than in the stationary case, \( d < 0.5 \) (Robinson, 2003). Nevertheless, some authors (e.g., Phillips and Xiao, 1999) argue that there cannot be mean reversion in this context since the variance explodes as \( t \) goes to infinity. The reason for this discrepancy might be that the latter authors employ the “Type I” definition for fractional integration while in this paper we use the Type II” definition that implicitly assumes the condition \( x_t = 0, t \leq 0 \) in (2).
d is smaller than 1, the bigger it is, the longer it takes for mean-reversion to the long-run PPP equilibrium condition to occur.

Given the difficulties in distinguishing between models with fractional orders of integration and those with broken deterministic trends, (Diebold and Inoue, 2001; Granger and Hyung, 2004; etc.), it is important also to consider estimation procedures that deal with fractional unit roots in the presence of broken deterministic trends.

Gil-Alana (2008) proposes a simple procedure for estimating fractional orders of integration with deterministic linear trends and structural breaks at unknown dates. In a model with a single break, he considers the following specification:

\[ y_t = \alpha_1 + \beta_1 t + x_t; \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1, ..., T_b, \]

(4)

and

\[ y_t = \alpha_2 + \beta_2 t + x_t; \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b + 1, ..., T, \]

(5)

where the \( \alpha \)'s and the \( \beta \)'s are the coefficients corresponding to the intercepts and the linear trends respectively, \( d_1 \) and \( d_2 \) can be real values, \( u_t \) is I(0), and \( T_b \) is the time of the break that is assumed to be unknown. This method is based on minimising the residuals sum squares for each subsample, and it is briefly described in Appendix B.\(^4\) \(^5\)

3. The empirical results

The series analysed are the real monthly exchange rates in seventeen Latin American countries. Nominal exchange rates are converted into real values using consumer price index deflators. The data sources are the International Financial Statistics of the International Monetary Fund and the Financial Statistics of the Federal Reserve Board.

\(^4\) In the present paper we focus on the case of a single break but the model can be easily extended to allow for multiple breaks.

\(^5\) Other procedures for fractional integration with breaks are Beran and Terrin (1996), Bos et al. (2001), etc. However, unlike Gil-Alana (2008), they assume that the order of integration is the same in each subsample.
The countries examined are Mexico, Guatemala, El Salvador, Honduras, Costa Rica, Panama, Jamaica, the Dominican Republic, Trinidad and Tobago, Colombia, Venezuela, Ecuador, Chile, Brazil, Paraguay, Uruguay and Argentina. The sample period goes in all cases from January 1970 to May 2008.

[Insert Figure 1 about here]

Figure 1 shows plots of the time series (in logarithm form). Evidence of structural breaks is found in practically all cases. We first present the results for the case of no breaks, estimating the fractional differencing parameter under the assumption that the underlying disturbances are white noise. Therefore, any association between the observations is captured by the degree of integration of the series.

[Insert Table 1 about here]

Table 1 reports the estimates of d along with the 95% confidence bands for the three standard cases of no regressors, an intercept, and an intercept with a linear time trend. We notice that the unit root null hypothesis (i.e. d = 1) cannot be rejected in practically any of the cases. Evidence of mean reversion (d < 1) in the real exchange rates, which would support the PPP hypothesis, is only obtained for Brazil, Uruguay and Argentina for the three cases of no regressors, an intercept, and an intercept with a linear trend, and also for Guatemala if deterministic terms are included. On the other hand, for another group of countries (El Salvador, Honduras, Costa Rica, Panama, Jamaica, Trinidad and Tobago and Chile) the unit root null is rejected in favour of higher orders of integration (d > 1) if an intercept or a linear trend is included in the
regression model. For the remaining countries, the intervals include the unit root in all cases.

So far we have considered a very simple model with no autocorrelation for the error term. The results for the case of autocorrelated disturbances with $u_t$ in (2) following an AR(1) process are displayed in Table 2. Evidence of mean reversion is found here for Uruguay and Argentina in all three cases and also for Guatemala, El Salvador, Trinidad and Tobago, Venezuela and Ecuador with deterministic terms. Higher AR orders were also employed and the results were very similar to those for the AR(1) case, though there are also some inconsistencies in the test results. For example, in some cases, the null cannot be rejected with $d_o = 0$ or 1, but it is rejected for cases with $d_o$ lying between these two values. Table 3 reports the results when we allow for a more general type of weak autocorrelation, which is based on the exponential spectral model of Bloomfield (1973). This is a non-parametric approach to describe weak dependence that resembles fairly well the case of ARMA processes. In his model, the disturbances are exclusively specified in terms of the spectral density function, which is given by:

$$
 f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} \exp \left( 2 \sum_{r=1}^{m} \tau_r \cos(\lambda r) \right),
$$

(6)

where $m$ indicates the number of parameters required to describe the short-run dynamics of the series. Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often found to be a fairly well-behaved function and thus can be approximated by a truncated Fourier series. He showed that (6) approximates the
spectral density of an ARMA(p, q) process well when p and q are small values, which is usually the case for most economic time series. Like the stationary AR(p) model, this has exponentially decaying autocorrelations and thus, using this specification, one does not need to rely on as many parameters as in the case of ARMA processes.\textsuperscript{6}

[Insert Table 3 about here]

The results based on the model of Bloomfield (1973) (displayed in Table 3) are very similar to those presented in Table 2 for the AR(1) case. Evidence of mean reversion is found for the three cases of no regressors, an intercept and an intercept with a linear trend in Venezuela, Brazil and Argentina, and for the cases of an intercept and an intercept with a linear trend in Guatemala, El Salvador, Ecuador and Trinidad and Tobago. In the remaining cases, the unit root null cannot be rejected and thus the evidence does not support PPP in these countries. Therefore, if we do not allow for structural breaks, evidence in favour of the PPP is only obtained for Argentina and Guatemala regardless of the way of modelling the I(0) disturbances, and, to a lesser extent, in the cases of Brazil, Uruguay, Venezuela, Ecuador and El Salvador.

However, as mentioned above, structural breaks are an important feature of many macroeconomic series, and particularly so in the case of Latin America. It is now well recognised that structural breaks and fractional integration are issues which are highly related (Diebold and Inoue, 2001; Granger and Hyung, 2004; etc.). Some authors, such as Lobato and Savin (1998), argue that structural breaks may be responsible for the long memory in return volatility processes. Engle and Smith (1999) investigate the relationship between structural breaks and long memory using a simple

\textsuperscript{6} Gil-Alana (2004) showed that the model of Bloomfield (1973) approximates well ARMA structures in the context of fractional integration.
unit root process which occasionally changes over time. Other authors, such as Beran and Terrin (1996) and Bos et al. (2001) proposed Lagrange Multiplier tests for fractional integration with breaks. In what follows, we implement the procedure developed by Gil-Alana (2008) assuming the existence of a single break at an unknown date in all series.\textsuperscript{7}

In Table 4 we display the estimates of the fractional differencing parameters and the intercepts for each subsample along with the break date in a model with no linear trends and uncorrelated disturbances. In other words, for each series, we consider the following model:

\[
y_t = \alpha_1 + x_t, \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1,\ldots,T_b,
\]

\[
y_t = \alpha_2 + x_t, \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b + 1,\ldots,T,
\]

with uncorrelated \(u_t\).\textsuperscript{8} We focus on this model since the time trend coefficients for the cases displayed in Tables 1 – 3 (not reported) were found not to be significantly different from zero in virtually all cases.

It can be seem that the break dates are very different across countries. In four cases (Panama, Uruguay, Chile and Mexico) the break takes place in the 70s; in another group of four countries (Honduras, Brazil, Trinidad and Tobago and Colombia), it occurs in the early 90s, while in the remaining nine countries in the 80s. It can be noticed that the break dates reported in Table 4 coincide with abrupt changes in the real

\textsuperscript{7} In some cases two breaks could be more appropriate. However, when a second break occurs, it takes place in all cases towards the beginning or the end the sample, with the result that one of the subsamples only comprises a few observations, which is clearly inappropriate in the context of fractional integration.

\textsuperscript{8} When allowing for autocorrelated disturbances, the break dates were found to be exactly the same as in the case of uncorrelated \(u_t\), and the orders of integration were practically the same as those reported in Table 4.
exchange rates which are apparent in Figures 1a – 1q, and correspond to clearly identifiable policy changes or financial crises. For instance, in Mexico the fixed-exchange-rate regime was abandoned in 1976, which resulted in a Barro-Gordon type inflation bias caused by the inability of policy-makers to commit to low inflation (see Li et al, 2002). In the other countries of Central America (Guatemala, El Salvador, Honduras, Costa Rica, Dominican Republic, Panama) the breaks can also be interpreted in terms of policy changes and developments in domestic credit creation and fiscal deficit, as analysed in Edwards (1995), and similar considerations apply to the Caribbean countries (Jamaica, Trinidad and Tobago). In Colombia the endogenously determined break is found to occur with a time lag relative to the switch from a crawling peg to a “crawling band” peg regime of December 1991 (see Milas and Otero, 2003 for more institutional details). In Venezuela free floating has been replaced by a dollar peg and capital controls have been imposed. In Ecuador in 2000 dollarisation was eventually adopted. In Chile developments in the exchange rate in early years reflected policies aimed at encouraging exports which were central to economic development. In Brazil in March 1990, at the beginning of the Collor Administration, the floating exchange rate and the retention of assets in local currency were adopted, restricting the demand for external currency. Throughout the 80s exchange rate policy in Paraguay was characterised by numerous devaluations. In Uruguay after a period of free floating the monetary authorities carried out a "dirty float," repeatedly entering the currency market to lower the exchange rate of the peso. Devaluation translated into increased competitiveness. Argentina opted for a currency board in the 1990s after a troubled period.

The intercept is found to be statistically significant in almost all cases. Estimates of the differencing parameters below 1 are obtained in Mexico and Brazil for the two
subsamples; in Honduras, Jamaica, Colombia, Paraguay and Uruguay for the first subsample; in Guatemala, Ecuador and Argentina for the subsample after the break. However, in the majority of cases the unit root null hypothesis cannot be rejected. In fact, definite evidence of mean reversion (i.e. with values significantly below 1) is only obtained for Argentina during the second subsample. For this country $d_1$ is strictly above 1 while $d_2$ is below 1 after the break of August 1981, implying that PPP is satisfied after that date. On the other hand, there are some cases with strong evidence against PPP, namely Panama and Trinidad and Tobago in the two subsamples; Ecuador, Chile and Argentina in the first subsample, and a group of nine countries (El Salvador, Honduras, Costa Rica, Panama, Jamaica, Dominican Republic, Trinidad and Tobago, Colombia and Paraguay) in the second subsample. In the remaining cases, the results are ambiguous, and, although some estimates are found to be below unity, the unit root null hypothesis cannot be rejected at the 5% level.

4. Conclusions

This paper tests for PPP in a group of seventeen Latin American (LA) countries by applying fractional integration techniques to real exchange rate series. Compared to earlier studies on these economies, this approach has the advantage of allowing for non-integer values for the degree of integration, and thus for the possibility of PPP not holding continuously but as a long-run equilibrium condition. Further, breaks in the series are endogenously determined using a procedure based on the least-squares principle (see Gil-Alana, 2008). This is particularly crucial in the Latin American countries, which have been affected by several exchange rate crises and policy regime changes. The results, based on different assumptions about the underlying disturbances, are in the majority of cases inconsistent with PPP, even more so when breaks are
incorporated: Argentina is the only country for which clear evidence of mean reversion is found in the model including a break, albeit only in the second subsample.

This paper could be extended in several ways. For example, possible non-linearities could be taken into account within a model with fractional integration and breaks. Similarly, regime-switching models with long-memory properties could also be considered. Finally, the possibility of fractional cointegration between nominal exchange rates, and domestic and foreign prices is another avenue for further research.
Appendix A

The LM test of Robinson (1994) for testing $H_0$ (3) in (1) and (2) is

$$\hat{r} = \frac{T^{1/2}}{\sigma^2} \hat{A}^{-1/2} \hat{a},$$

where $T$ is the sample size and:

$$\hat{a} = -\frac{2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\epsilon}(\lambda_j) \right) \times \left( \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j) \hat{\epsilon}(\lambda_j) \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j) \psi(\lambda_j)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|, \quad \hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j = \frac{2\pi j}{T}; \quad \hat{\tau} = \arg \min \sigma^2(\tau).$$

$\hat{a}$ and $\hat{A}$ in the above expressions are obtained through the first and second derivatives of the log-likelihood function with respect to $d$ (see Robinson, 1994, page 1422, for further details). $I(\lambda_j)$ is the periodogram of $u_t$ evaluated under the null, i.e.:

$$\hat{u}_t = (1 - L)^{d_0} y_t - \hat{\beta}' w_t; \quad \hat{\beta} = \left( \sum_{i=1}^{T} w_i w_i' \right)^{-1} \sum_{i=1}^{T} w_i (1 - L)^{d_0} y_i; \quad w_t = (1 - L)^{d_0} z_t,$$

where $z_t$ refers to the deterministic terms, and $g$ is a known function related to the spectral density function of $u_t$:

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi.$$

Appendix B

The model in (4) and (5) can also be written as:

$$(1 - L)^{d_1} y_t = \alpha_1 \tilde{I}(d_i) + \beta_1 \tilde{u}(d_i) + u_t, \quad t = 1, ..., T_b,$$

$$(1 - L)^{d_2} y_t = \alpha_2 \tilde{I}(d_2) + \beta_2 \tilde{u}(d_2) + u_t, \quad t = T_b + 1, ..., T,$$

where $\tilde{I}(d_i) = (1 - L)^{d_1} 1$, and $\tilde{u}(d_i) = (1 - L)^{d_i} t$, $i = 1, 2$. 
The procedure is based on the least square principle. First we choose a grid for the values of the fractionally differencing parameters $d_1$ and $d_2$, for example, $d_{io} = 0, 0.01, 0.02, \ldots, 1, i = 1, 2$. Then, for a given partition $\{T_b\}$ and given initial $d_1$, $d_2$-values, $(d_{io}^{(1)}, d_{io}^{(2)})$, we estimate the $\alpha$'s and the $\beta$'s by minimising the sum of squared residuals,

$$
\text{min}_{\{\alpha_1, \alpha_2, \beta_1, \beta_2\}} \sum_{t=1}^{T_b} \left[ (1-L)d_{io}^{(1)} y_t - \alpha_1 \tilde{t}_1 (d_{io}^{(1)}) - \beta_1 \tilde{t}_1 (d_{io}^{(1)}) \right]^2 + \sum_{t=T_b+1}^{T} \left[ (1-L)d_{2o}^{(1)} y_t - \alpha_2 \tilde{t}_2 (d_{2o}^{(1)}) - \beta_2 \tilde{t}_2 (d_{2o}^{(1)}) \right]^2.
$$

Let $\hat{\alpha}(T_b; d_{io}^{(1)}, d_{2o}^{(1)})$ and $\hat{\beta}(T_b; d_{io}^{(1)}, d_{2o}^{(1)})$ denote the resulting estimates for the partition $\{T_b\}$ and initial values $d_{io}^{(1)}$ and $d_{2o}^{(1)}$. Substituting these estimated values into the objective function, we have $\text{RSS}(T_b; d_{io}^{(1)}, d_{2o}^{(1)})$, and minimising this expression for all values of $d_{io}$ and $d_{2o}$ in the grid we obtain $\text{RSS}(T_b) = \arg\min_{d_{io}, d_{2o}} \text{RSS}(T_b; d_{io}^{(1)}, d_{2o}^{(1)})$. Then, the estimated break date, $\hat{T}_k$, is such that $\hat{T}_k = \arg\min_{i=1,\ldots,m} \text{RSS}(T_i)$, where the minimisation is carried out over all partitions $T_1, T_2, \ldots, T_m$, such that $T_i - T_{i-1} \geq |\varepsilon|T$. Then, the regression parameter estimates are the associated least-squares estimates of the estimated k-partition, i.e., $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$, $\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$, and their corresponding differencing parameters, $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$, for $i = 1$ and 2.
References


Figure 1: Real exchange rates in Latin American countries (in logs)

1a: Mexico

1b: Guatemala

1c: El Salvador

(cont.)
1g: Jamaica

1h: Dominican Rep.

1i: Trinidad and Tobago

(cont.)
1j: Colombia

1k: Venezuela

1l: Ecuador

(cont.)
1m: Chile

1n: Brazil

1o: Paraguay

(cont.)
Table 1: Estimates of $d$ (and 95% confidence bands) in a model with white noise disturbances

<table>
<thead>
<tr>
<th>Series</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>[0.95 (1.00) 1.06]</td>
<td>[0.95 (1.00) 1.06]</td>
<td>[0.95 (1.00) 1.06]</td>
</tr>
<tr>
<td>Guatemala</td>
<td>[0.92 (0.97) 1.04]</td>
<td><strong>[0.85 (0.91) 0.98]</strong></td>
<td><strong>[0.85 (0.91) 0.98]</strong></td>
</tr>
<tr>
<td>El Salvador</td>
<td>[0.95 (1.00) 1.06]</td>
<td>[1.09 (1.19) 1.31]</td>
<td>[1.09 (1.19) 1.31]</td>
</tr>
<tr>
<td>Honduras</td>
<td>[0.94 (0.99) 1.06]</td>
<td>[1.11 (1.19) 1.28]</td>
<td>[1.11 (1.19) 1.28]</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>[0.94 (0.99) 1.06]</td>
<td>[1.01 (1.08) 1.16]</td>
<td>[1.01 (1.08) 1.17]</td>
</tr>
<tr>
<td>Panama</td>
<td>[0.97 (1.01) 1.07]</td>
<td>[1.08 (1.13) 1.20]</td>
<td>[1.08 (1.13) 1.20]</td>
</tr>
<tr>
<td>Jamaica</td>
<td>[0.93 (0.99) 1.05]</td>
<td>[1.06 (1.13) 1.21]</td>
<td>[1.06 (1.13) 1.21]</td>
</tr>
<tr>
<td>Dominican Rep.</td>
<td>[0.93 (0.98) 1.04]</td>
<td>[0.87 (0.93) 1.00]</td>
<td>[0.88 (0.93) 1.00]</td>
</tr>
<tr>
<td>Trinidad + Tobago</td>
<td>[0.94 (0.99) 1.06]</td>
<td>[1.03 (1.11) 1.21]</td>
<td>[1.03 (1.11) 1.21]</td>
</tr>
<tr>
<td>Colombia</td>
<td>[0.94 (0.99) 1.05]</td>
<td>[1.00 (1.05) 1.12]</td>
<td>[1.00 (1.05) 1.12]</td>
</tr>
<tr>
<td>Venezuela</td>
<td>[0.85 (0.92) 1.01]</td>
<td>[0.90 (0.96) 1.04]</td>
<td>[0.90 (0.96) 1.04]</td>
</tr>
<tr>
<td>Ecuador</td>
<td>[0.94 (0.99) 1.05]</td>
<td>[0.91 (0.97) 1.05]</td>
<td>[0.90 (0.97) 1.05]</td>
</tr>
<tr>
<td>Chile</td>
<td>[1.07 (1.10) 1.14]</td>
<td>[1.07 (1.10) 1.14]</td>
<td>[1.07 (1.10) 1.14]</td>
</tr>
<tr>
<td>Brazil</td>
<td><strong>[0.84 (0.88) 0.94]</strong></td>
<td><strong>[0.84 (0.89) 0.95]</strong></td>
<td><strong>[0.84 (0.89) 0.95]</strong></td>
</tr>
<tr>
<td>Paraguay</td>
<td>[0.93 (0.98) 1.05]</td>
<td>[0.94 (0.99) 1.06]</td>
<td>[0.95 (0.99) 1.06]</td>
</tr>
<tr>
<td>Uruguay</td>
<td><strong>[0.69 (0.72) 0.75]</strong></td>
<td><strong>[0.68 (0.71) 0.74]</strong></td>
<td><strong>[0.68 (0.71) 0.74]</strong></td>
</tr>
<tr>
<td>Argentina</td>
<td><strong>[0.83 (0.89) 0.97]</strong></td>
<td><strong>[0.83 (0.89) 0.97]</strong></td>
<td><strong>[0.83 (0.89) 0.97]</strong></td>
</tr>
</tbody>
</table>

In bold: Values of $d$ which are found to be strictly smaller than 1 at the 95% level.
<table>
<thead>
<tr>
<th>Series</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>0.80 (0.99) 1.13</td>
<td>0.92 (1.01) 1.11</td>
<td>0.92 (1.01) 1.11</td>
</tr>
<tr>
<td>Guatemala</td>
<td>0.98 (1.11) 1.35</td>
<td><strong>0.68 (0.79) 0.92</strong></td>
<td><strong>0.74 (0.80) 0.92</strong></td>
</tr>
<tr>
<td>El Salvador</td>
<td>1.25 (1.36) 1.48</td>
<td><strong>0.76 (0.80) 0.92</strong></td>
<td><strong>0.67 (0.73) 0.82</strong></td>
</tr>
<tr>
<td>Honduras</td>
<td>1.25 (1.36) 1.49</td>
<td>0.81 (0.90) 1.05</td>
<td>0.80 (0.91) 1.06</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>1.28 (1.38) 1.51</td>
<td>0.77 (0.82) 1.04</td>
<td>0.80 (0.88) 1.04</td>
</tr>
<tr>
<td>Panama</td>
<td>1.27 (1.37) 1.49</td>
<td>0.98 (1.04) 1.12</td>
<td>0.97 (1.04) 1.12</td>
</tr>
<tr>
<td>Jamaica</td>
<td>1.25 (1.35) 1.48</td>
<td>0.81 (0.93) 1.06</td>
<td>0.85 (0.94) 1.06</td>
</tr>
<tr>
<td>Dominican Rep.</td>
<td>1.16 (1.31) 1.45</td>
<td>0.74 (0.89) 1.04</td>
<td>0.80 (0.90) 1.04</td>
</tr>
<tr>
<td>Trinidad + Tobago</td>
<td>1.23 (1.34) 1.48</td>
<td><strong>0.77 (0.84) 0.93</strong></td>
<td><strong>0.80 (0.85) 0.94</strong></td>
</tr>
<tr>
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<td>1.28 (1.38) 1.50</td>
<td>0.93 (1.02) 1.13</td>
<td>0.95 (1.02) 1.13</td>
</tr>
<tr>
<td>Venezuela</td>
<td>0.77 (0.94) 1.10</td>
<td><strong>0.66 (0.81) 0.95</strong></td>
<td><strong>0.69 (0.82) 0.95</strong></td>
</tr>
<tr>
<td>Ecuador</td>
<td>1.27 (1.37) 1.49</td>
<td><strong>0.76 (0.83) 0.97</strong></td>
<td><strong>0.78 (0.83) 0.97</strong></td>
</tr>
<tr>
<td>Chile</td>
<td>1.18 (1.24) 1.31</td>
<td>1.18 (1.24) 1.31</td>
<td>1.18 (1.24) 1.31</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.81 (0.89) 1.00</td>
<td>0.81 (0.90) 1.00</td>
<td>0.81 (0.90) 1.00</td>
</tr>
<tr>
<td>Paraguay</td>
<td>1.26 (1.37) 1.49</td>
<td>0.82 (0.96) 1.08</td>
<td>0.90 (0.97) 1.08</td>
</tr>
<tr>
<td>Uruguay</td>
<td><strong>0.83 (0.87) 0.93</strong></td>
<td><strong>0.81 (0.86) 0.91</strong></td>
<td><strong>0.81 (0.86) 0.91</strong></td>
</tr>
<tr>
<td>Argentina</td>
<td><strong>0.56 (0.78) 0.98</strong></td>
<td><strong>0.57 (0.78) 0.98</strong></td>
<td><strong>0.58 (0.78) 0.98</strong></td>
</tr>
</tbody>
</table>

In bold: Values of $d$ which are found to be strictly smaller than 1 at the 95% level.
Table 3: Estimates of $d$ (and 95% confidence bands) in a model with Bloomfield disturbances

<table>
<thead>
<tr>
<th>Series</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>[0.91 (0.99) 1.11]</td>
<td>[0.92 (1.00) 1.12]</td>
<td>[0.92 (1.00) 1.12]</td>
</tr>
<tr>
<td>Guatemala</td>
<td>[0.86 (0.94) 1.06]</td>
<td>[0.73 (0.81) 0.92]</td>
<td>[0.73 (0.81) 0.92]</td>
</tr>
<tr>
<td>El Salvador</td>
<td>[0.89 (0.99) 1.09]</td>
<td>[0.66 (0.73) 0.86]</td>
<td>[0.60 (0.72) 0.86]</td>
</tr>
<tr>
<td>Honduras</td>
<td>[0.88 (0.96) 1.09]</td>
<td>[0.85 (0.93) 1.04]</td>
<td>[0.85 (0.93) 1.04]</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>[0.88 (0.98) 1.09]</td>
<td>[0.84 (0.94) 1.07]</td>
<td>[0.84 (0.94) 1.07]</td>
</tr>
<tr>
<td>Panama</td>
<td>[0.95 (1.02) 1.13]</td>
<td>[0.97 (1.04) 1.11]</td>
<td>[0.97 (1.04) 1.12]</td>
</tr>
<tr>
<td>Jamaica</td>
<td>[0.88 (0.96) 1.08]</td>
<td>[0.86 (0.95) 1.06]</td>
<td>[0.86 (0.95) 1.06]</td>
</tr>
<tr>
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<td>[0.80 (0.90) 1.03]</td>
<td>[0.79 (0.91) 1.03]</td>
</tr>
<tr>
<td>Trinidad + Tobago</td>
<td>[0.88 (0.96) 1.08]</td>
<td>[0.76 (0.84) 0.96]</td>
<td>[0.77 (0.84) 0.96]</td>
</tr>
<tr>
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<td>[0.95 (1.02) 1.12]</td>
<td>[0.95 (1.02) 1.12]</td>
</tr>
<tr>
<td>Venezuela</td>
<td>[0.63 (0.72) 0.86]</td>
<td>[0.73 (0.84) 0.94]</td>
<td>[0.73 (0.84) 0.94]</td>
</tr>
<tr>
<td>Ecuador</td>
<td>[0.90 (0.98) 1.10]</td>
<td>[0.74 (0.86) 0.97]</td>
<td>[0.76 (0.85) 0.98]</td>
</tr>
<tr>
<td>Chile</td>
<td>[1.19 (1.27) 1.35]</td>
<td>[1.19 (1.26) 1.34]</td>
<td>[1.18 (1.26) 1.35]</td>
</tr>
<tr>
<td>Brazil</td>
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<td>[0.82 (0.90) 0.99]</td>
<td>[0.82 (0.90) 0.99]</td>
</tr>
<tr>
<td>Paraguay</td>
<td>[0.88 (0.95) 1.08]</td>
<td>[0.88 (0.96) 1.09]</td>
<td>[0.88 (0.96) 1.09]</td>
</tr>
<tr>
<td>Uruguay</td>
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<td>[0.87 (0.92) 1.00]</td>
<td>[0.87 (0.93) 1.00]</td>
</tr>
<tr>
<td>Argentina</td>
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<td>[0.71 (0.83) 0.96]</td>
<td>[0.71 (0.83) 0.96]</td>
</tr>
</tbody>
</table>

In bold: Values of $d$ which are found to be strictly smaller than 1 at the 95% level.
Table 4: Estimates of the fractional differencing parameters in the case of a single break

<table>
<thead>
<tr>
<th>Country</th>
<th>Break date</th>
<th>First sub-sample</th>
<th>Second sub-sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>d₁</td>
<td>β₁</td>
</tr>
<tr>
<td>Mexico</td>
<td>January 1977</td>
<td>0.98 [0.84, 1.15]</td>
<td>0.903 (8.437)</td>
</tr>
<tr>
<td>Guatemala</td>
<td>June 1986</td>
<td>1.02 [0.90, 1.06]</td>
<td>2.012 (85.945)</td>
</tr>
<tr>
<td>El Salvador</td>
<td>February 1986</td>
<td>1.01 [0.89, 1.10]</td>
<td>3.017 (134.07)</td>
</tr>
<tr>
<td>Honduras</td>
<td>March 1990</td>
<td>0.99 [0.87, 1.09]</td>
<td>2.674 (255.57)</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>October 1981</td>
<td>1.08 [0.96, 1.19]</td>
<td>5.609 (144.89)</td>
</tr>
<tr>
<td>Panama</td>
<td>July 1974</td>
<td>1.19 [1.02, 1.27]</td>
<td>-0.667 (-200.81)</td>
</tr>
<tr>
<td>Jamaica</td>
<td>December 1983</td>
<td>0.99 [0.88, 1.10]</td>
<td>4.000 (132.68)</td>
</tr>
<tr>
<td>Dominican Rep.</td>
<td>January 1985</td>
<td>1.03 [0.84, 1.11]</td>
<td>3.270 (153.51)</td>
</tr>
<tr>
<td>Trinidad +</td>
<td>April 1993</td>
<td>1.23 [1.11, 1.39]</td>
<td>2.100 (92.449)</td>
</tr>
<tr>
<td>Tobago</td>
<td>March 1994</td>
<td>0.98 [0.88, 1.02]</td>
<td>7.368 (236.55)</td>
</tr>
<tr>
<td>Venezuela</td>
<td>March 1989</td>
<td>0.97 [0.89, 1.10]</td>
<td>0.785 (9.670)</td>
</tr>
<tr>
<td>Ecuador</td>
<td>May 1981</td>
<td>1.40 [1.19, 1.66]</td>
<td>8.976 (517.40)</td>
</tr>
<tr>
<td>Chile</td>
<td>August 1976</td>
<td>1.14 [1.01, 1.20]</td>
<td>-0.204 (-0.908)</td>
</tr>
<tr>
<td>Brazil</td>
<td>December 1990</td>
<td>0.99 [0.92, 1.10]</td>
<td>0.036 (0.998)</td>
</tr>
<tr>
<td>Paraguay</td>
<td>March 1989</td>
<td>0.96 [0.85, 1.04]</td>
<td>8.030 (140.12)</td>
</tr>
<tr>
<td>Uruguay</td>
<td>June 1974</td>
<td>0.98 [0.79, 1.12]</td>
<td>-0.441 (-3.385)</td>
</tr>
<tr>
<td>Argentina</td>
<td>August 1981</td>
<td>1.21 [1.11, 1.28]</td>
<td>0.231 (2.497)</td>
</tr>
</tbody>
</table>

In bold: Values of d which are found to be strictly smaller than 1 at the 95% level.