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# Audit Probability Versus Effectiveness: The Beckerian Approach Revisited

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# Audit Probability Versus Effectiveness: The Beckerian Approach Revisited

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## Abstract

The Beckerian approach to tax compliance examines how a tax authority can maximize social welfare by trading-off audit probability against the fine rate on undeclared tax. This paper offers an alternative examination of the privately optimal behavior of a tax authority tasked by government to maximize expected revenue. The tax authority is able to trade-off audit probability against audit effectiveness, but takes the fine rate as fixed in the short run. I find that the tax authority's privately optimal audit strategy does not maximize voluntary compliance, and that voluntary compliance is non-monotonic as a function of the tax authority's budget. Last, the tax authority's privately optimal effective fine rate on undeclared tax does not exceed two at interior optima.

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# 1    Introduction

The economics of tax compliance has at its foundations the seminal analysis of Becker (1968) on optimal law enforcement: the influential portfolio model of tax compliance (Allingham and Sandmo, 1972; Christiansen, 1980; Srinivasan, 1973; Yitzhaki, 1974) can be seen as little more than a specific application of Becker's more general analysis.

The Beckerian approach considers the socially optimal enforcement strategy in respect of the trade-off between the fine rate and the probability of detection. A key insight is that a government concerned with maximizing the expected utility of a representative citizen should set the fine rate on undeclared tax as high as possible, and the audit probability as low as possible ('hang 'em with probability zero'). Subsequent literature analyzing this trade-off includes Kolm (1973), Stern (1978) and Polinsky and Shavell (1979).

The analysis here differs from the Beckerian approach in two important respects. First, whereas Becker considers the *socially* optimal enforcement strategy, I consider the tax authority's *privately* optimal enforcement strategy for a given objective function set by government. In this sense, there is no presumption that the equilibrium of the model is socially efficient: the model aims to be descriptive rather than prescriptive.

The second difference is that, whereas the Beckerian framework focuses on the trade-off between audit probability and the fine rate, I focus on the trade-off between audit probability and audit effectiveness (the proportion of non-compliance that an audit detects). Although this dimension of enforcement strategy has received little attention - the standard portfolio model assumes that the tax authority is able to perform investigations that are fully effective - I argue that it is of greater practical significance to the work of tax authorities than is the trade-off between audit probability and fine rates.

The ability of the tax authority to set fine rates is much more limited than is

typically recognized in the literature (Slemrod, 2007). Even though fine rates can be adjusted in the long run, the need for punishments to be proportionate to the perceived seriousness of the crime acts as a powerful constraint. For instance, Kirchler et al. (2003) find socially positive attitudes towards tax avoidance, suggesting that some types of non-compliance are socially acceptable. In a list of crimes, tax evasion is ranked as being only slightly more serious than stealing a bicycle (Song and Yarbrough, 1978); and as no more serious than minimum wage law violations (Burton et al., 2005). The policy relevance of Becker's 'hang 'em with probability zero' equilibrium has therefore been questioned, given its stark deviation from observed practice (Dhami and al-Nowaihi, 2006).

The few studies that do allow for imperfect audit effectiveness include Alm (1988), Alm and McKee (2006), Reinganum and Wilde (1986) and Snow and Warren Jr. (2005a,b). None of these studies, however, investigates the trade-off between audit effectiveness and audit probability. Reinganum and Wilde (1986) assume that audits are either fully effective or fully ineffective. However, it seems more realistic to allow for audits to be partially effective. Also, their approach implies that, if taxpayers are able to compute compound lotteries correctly, the compliance effect of a change in audit effectiveness is simply the same as the effect of an equivalent change in audit probability (Alm and McKee, 2006). Therefore, I adopt the approach of Snow and Warren Jr. (2005a,b), who allow audits to detect a proportion $q \in [0,1]$ of undeclared income. With this approach, audit effectiveness enters taxpayer utility in a different manner to audit probability, making the compliance effects of these two parameters distinct.

Similar to the model of Reinganum and Wilde (1985), I model the strategic interaction between taxpayers and the tax authority in a principal-agent setting where the tax authority (principal) commits to an audit strategy, then taxpayers (agents) maximize expected utility, taking as given the choice of the

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tax authority. However, income - a random variable in Reinganum and Wilde (1985) - is, in my model, an exogenous variable, equal across taxpayers. This simplification implies that random auditing is weakly optimal, which moves the focus of the model away from the problem of optimal audit selection towards the problem of how to set a common audit probability, given the reaction function of taxpayers and the trade-off between audit probability and effectiveness. By contrast, when taxpayers differ in income, Reinganum and Wilde (1985) show that there exist audit strategies which condition on taxpayers' reported incomes (such as a cutoff rule) that may dominate a random audit strategy.

Although I shall argue that my approach is consistent with that of Becker, I nevertheless demonstrate that it gives rise to a number of descriptively important differences in prediction. First, the expected-revenue maximizing audit strategy does not maximize voluntary compliance. Instead, the optimal audit probability exceeds that consistent with the maximization of compliance such that, in equilibrium, a marginal increase in the probability of audit reduces declared income.

Second, although the tax authority still has an incentive to raise the fine rate if it is able, Becker's 'hang 'em with probability zero' equilibrium does not emerge. Rather, at all interior solutions of the model, the optimal 'effective' fine rate on undeclared tax does not exceed two. Third, compliance is non-monotonic in the tax authority's budget.

As extensions to the basic model I investigate the implications for my results if taxpayers exhibit probability weighting of the form supposed by prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), and if taxpayers are uncertain as to the true audit probability or effectiveness.

The plan of the paper is as follows: Section 2 motivates the main aspects of my approach, while Section 3 outlines a model of taxpayers' compliance de-

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cision, and the tax authority's optimal audit strategy. Section 4 analyzes the main results, and Section 5 provides some extensions. Section 6 concludes.

## 2   Modelling the Tax Authority

In modern government, responsibility for the collection of taxes is often decoupled from the setting of fiscal policy - the former being considered an operational matter, the latter one of policy. For instance, in the US, responsibility for the collection of taxes resides with an operational bureau of the Department of the Treasury, the Internal Revenue Service (IRS), whereas responsibility for fiscal policy lies on the policy side of the Department - the Office of Tax Policy. This structure is mirrored in the UK between H.M. Treasury and its collection agency, H.M. Revenue and Customs (HMRC). Therefore, although tax rates are endogenous at the level of government, they are typically exogenous to the tax authority itself, which instead has a narrow operational remit.

The precise nature of the tax authority's objective function is typically negotiated by the tax authority with government. It remains debated as to the choice of objective function politicians seek to apply. From a law enforcement perspective, the relevant objective would be to maximize voluntary compliance. However, as well as law enforcement, politicians may have an instrumental concern for maximizing expected revenue (which comprises receipts and penalties from audit activity, in addition to voluntary compliance).

Consistent with the latter interpretation, the British tax authority, HMRC, is committed to a legal obligation to maximize expected revenue (Ratto, Thomas and Ulph, 2009). Although the best characterization of the IRS is less clear, Plumley and Steuerle (2004) state that IRS "enforcement programs have traditionally pursued the objective of maximizing the revenue that they produce from the taxpayers whom they contact, subject to their

budget constraint." Expected revenue maximization is also assumed as the tax authority's objective function in the literature on optimal audit rules (e.g. Graetz et al., 1986; Reinganum and Wilde, 1985, 1986). Accordingly, in what follows I assume the remit of the tax authority is to maximize expected revenue.

Tax authorities must compete with other government agencies for a budget settlement. Again, this implies that, although the tax authority's budget is endogenous at the level of government, it is largely exogenous to the tax authority itself - at least in the short run. The problem facing tax authorities is therefore to maximize tax revenue for a given budget. In this sense the concern of the paper is not how the tax authority's budget compares with any putative social optimum (see Slemrod and Yitzhaki, 1987), but on how the tax authority chooses to spend its pre-determined budget.

Without levers over fiscal policy and its overall budget, the principal tools available to tax authorities are the legal right to perform compliance audits, and to levy fines on detected non-compliance. However, following the discussion in the Introduction, I assume that the constraints on the setting of the fine rate are sufficiently strong that the tax authority treats it as fixed.

The tool which tax authorities can most readily use to maximize tax revenue is therefore the ability to perform audits. For a given audit technology, the tax authority's audit strategy can be summarized by the pair $(p, q)$ where $p$ is audit probability and $q$ is audit effectiveness. The tax authority can be modelled as choosing either $p$ or $q$, as for a given choice of one, the other is determined endogenously by the budget constraint: the tax authority therefore faces a trade-off between the number of audits it performs, and the effectiveness of each audit.

# 3 A Model

## 3.1 Preliminaries

My modelling of the fiscal environment is based on that of Yitzhaki (1974). In particular, there are $n$ taxpayers, each with an exogenous taxable income $y$ (which is known by the taxpayer but not by the tax authority). The government levies a proportional income tax at marginal rate $\theta$ on declared income $x$. A proportion $p$ of taxpayers are randomly selected for audit each year and, when performed, an audit detects a proportion $q$ of the true level of undeclared tax. Taxpayers face a fine at rate $f > 1$ on all detected undeclared tax, giving an 'effective' fine rate of $qf$.

The timing of the model is as follows: in the first stage, the tax authority publicly pre-commits to a pair $(p, q)$, and in the second stage, taxpayers choose an optimal level of declared income, taking as given the tax-authority's choice of $(p, q)$.

## 3.2 Taxpayers' Problem

Taxpayers are assumed to act as if they maximize expected utility, where utility, $U[\cdot]$, satisfies the following properties:

  A1.   $U[x]$ is continuous and twice differentiable for all $x \geq 0$.
  A2.   $U'[x] > 0$ and $U''[x] < 0$.
  A3.   $A[x] \equiv -U''[x]/U'[x]$ is decreasing in $x$.

Assumption A1 is a standard technical assumption. Assumption A2 implies that taxpayers are risk averse. Following Arrow (1965) and Allingham and Sandmo (1972), assumption A3 is decreasing absolute risk aversion (DARA). Taxpayers choose $x$, taking fiscal policy and the tax authority's audit strategy as given, yielding the problem

$$\max_x E\left[U\right] = (1-p)\,U\left[y - \theta x\right] + pU\left[y - \theta x - qf\theta\left(y - x\right)\right]. \qquad (1)$$

For notational convenience I define

$$W_g \equiv y - \theta x; \qquad W_b \equiv W_g - qf\theta\left(y - x\right); \qquad (2)$$

then differentiating expected utility in (1) with respect to $x$ gives

$$\frac{\partial E\left[U\right]}{\partial x} \equiv T\left[x, p\right] = \theta\left\{p\left(qf - 1\right)U'\left[W_b\right] - (1 - p)\,U'\left[W_g\right]\right\}. \qquad (3)$$

The first order condition for an interior maximum of (1) is therefore $T\left[x^*, p\right] = 0$, which implicitly defines a function $x^*\left[p, qf\right]$ that maps taxpayers' optimal income declaration as a function of the audit probability and the effective fine rate. The second derivative of expected utility is given by

$$\frac{\partial^2 E\left[U\right]}{\left(\partial x\right)^2} \equiv D\left[x, p\right] = \theta^2\left\{(1 - p)\,U''\left[W_g\right] + p\left(qf - 1\right)^2 U''\left[W_b\right]\right\}. \qquad (4)$$

The second order condition, $D < 0$, is satisfied by the assumption of strict concavity of the utility function. The conditions for the existence of an interior maximum are

$$\frac{U'\left[y\right]}{U'\left[y\left(1 - qf\theta\right)\right]} < \frac{p\left(qf - 1\right)}{1 - p} < 1. \qquad (5)$$

The first condition in (5) requires as a necessary condition that $qf > 1$, for if $qf < 1$ non-compliance pays even in the audit state. The second condition in (5) requires that $pqf < 1$, which is the standard condition that the tax gamble must be better than fair.

## 3.3   Audit Effectiveness

I assume that audit effectiveness is a function of the labor expended, $q = h\left[L\right]$, where $h\left[\cdot\right]$ has the following properties:

A4.   $h[L]$ is continuous and twice differentiable for all $L \geq 0$.

A5.   $h[0] = 0$ and $\lim_{L \uparrow \infty} h[L] = 1$.

A6.   $h'[\cdot] > 0$.

A7.   $h''[\cdot] < 0$.

Assumption A4 is a standard technical assumption. Assumption A5 is the idea that if the tax authority does not expend any resource on an audit, it will not detect any non-compliance, but a very resource-intensive audit can ultimately detect all non-compliance. Assumption A6 is that audit effectiveness increases as a function of labor. Last, assumption A7 is that audit effectiveness exhibits diminishing returns to labor. Diminishing returns in this context can arise as, unlike many other types of crime, non-compliance takes a great many shapes and forms, each of which differs according to the ease with which it can be detected. The most readily detectable forms of non-compliance may be exposed relatively cheaply, but it becomes increasingly labor consuming to detect further instances of non-compliance.[1]

## 3.4   Tax Authority's Problem

Let $0 \leq k \leq n$ be the number of audits performed by the tax authority. For a fixed budget allocation $b$, and normalizing the price of labor to $p_L = 1$, the budget constraint of the tax authority is given by $kL \leq b$ and the audit probability by

$$p \equiv \frac{k}{n}, \tag{6}$$

where $p \in [0, 1]$. If the tax authority's budget constraint is binding I have from (6) and the budget constraint that

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[1]In practice, the difficulty of proving some instances of non-compliance often implies that the final level of undeclared income is reached through a process of bargaining between the taxpayer and the tax authority. While a potentially interesting extension to the present analysis, for simplicity I do not develop this aspect of the model.

$$q = h[L] = h\left[\frac{\tau}{p}\right], \tag{7}$$

where $\tau \equiv b/n$ is the per-capita budget of the tax authority. The inverse relationship between $p$ and $q$ makes clear the trade-off in audit strategy between audit probability and effectiveness. Differentiating (7) I have that

$$\frac{\partial q}{\partial p} = \frac{\partial h[\tau/p]}{\partial p} = -\left(\frac{q}{p}\right)e_q < 0, \tag{8}$$

where $e_q[L] \equiv Lh'[L]/h[L]$ is the elasticity of audit effectiveness with respect to labor and satisfies $e_q \in (0,1)$.[2]

I am now able to bring together the budget constraint $q = h[\tau/p]$ and the taxpayer behavioral function $x^*[p, qf]$ to define a function $X[p, f] \equiv x^*[p, h[\tau/p]f]$ that describes the compliance behavior of taxpayers, taking explicit account of the endogeneity of the effective fine rate.

The problem facing the tax authority is to choose the audit probability so as to maximize expected revenue, subject to its budget constraint and its understanding of the behavioral response of taxpayers (as summarized by taxpayers' first order condition). Expected revenue is composed of that generated directly in fines from non-compliance detected at audit (direct effect), and that arising indirectly from voluntary compliance induced by the threat of audit (indirect effect), giving:

$$\max_p E[R] = n\{\theta X[p, f] + ph[\tau/p]f\theta(y - X[p, f])\}. \tag{9}$$

Differentiating $E[R]$ in (9) with respect to $p$ gives:

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[2]That $e_q < 1$ follows from assumption A7.

$$\frac{\partial E\left[R\right]}{\partial p} \equiv G\left[X,p\right] = n\left\{\left(W_g - W_b\right)\left(1 - e_q\right) + \theta\frac{\partial X\left[p,f\right]}{\partial p}\left(1 - ph\left[\tau/p\right]f\right)\right\},$$
$$\tag{10}$$

where, from (3),

$$\frac{\partial X\left[p,f\right]}{\partial p} = -\frac{\theta}{D}\left\{\begin{array}{l} U'\left[W_g\right] - U'\left[W_b\right]\left\{1 - fh\left[\tau/p\right]\left(1 - e_q\right)\right\} \\ +e_q\left(h\left[\tau/p\right]f - 1\right)\left(W_g - W_b\right)U''\left[W_b\right]\end{array}\right\}. \tag{11}$$

The tax authority's first order condition for an interior maximum is therefore $G\left[X^*,p\right] = 0$, which implicitly defines a function $X^*\left[p,f\right]$ that maps taxpayers' optimal income declaration given the fine rate and the tax authority's optimal choice of audit probability.

It is instructive to explore the region of $\tau$ that generates interior optima for compliance. In particular, there exist $(\underline{\tau},\overline{\tau})$ such that taxpayers' optimal income declaration can be written as:

$$X^*\left[p,f;\tau\right]\left\{\begin{array}{ll} = 0 & \tau \le \underline{\tau}, \\ \in (0,y) & \tau \in (\underline{\tau},\overline{\tau}), \\ = y & \tau \ge \overline{\tau}, \end{array}\right.$$

where $(\underline{\tau},\overline{\tau})$ are the unique solutions to

$$\frac{U'\left[y\right]}{U'\left[y\left(1 - h\left[\underline{\tau}/p\left[\underline{\tau}\right]\right]f\theta\right)\right]} = \frac{p\left[\underline{\tau}\right]\left(h\left[\underline{\tau}/p\left[\underline{\tau}\right]\right]f - 1\right)}{1 - p\left[\underline{\tau}\right]}; \qquad h\left[\overline{\tau}\right]f = 1. \tag{12}$$

The expression for $\overline{\tau}$ derives from the full-compliance outcome $(pqf = 1)$, which is always the equilibrium of the model if it is feasible. As $ph\left[\tau/p\right]$ is increasing in $p$, $pqf = 1$ is achieved at least cost by setting $p = 1$, from which the result follows. The expression for $\underline{\tau}$ is simply the first inequality in (5).

So far as I know, there are no tax authorities so lavishly funded as to have eliminated non-compliance, nor any so impoverished as to be unable to enforce any positive level of compliance. Therefore, were the model calibrated

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empirically, I would expect observed values of $\tau$ to be consistent with an interior solution for compliance. In this sense, while a corner solution for compliance remains a theoretical possibility, from a positive standpoint, analysis pertaining to interior equilibria of the model is of greater significance. This point is of importance in what follows, as the analysis makes strong predictions about behavior in all equilibria with an interior solution for compliance.

The problem in (9) is not a standard concave maximization problem in that the objective function is convex and the constraint function is neither globally concave nor convex (Figure 1). I am nevertheless able to state my first Proposition, establishing the existence of a unique optimal choice of $p$ by the tax authority (all proofs being in the Appendix):

**Proposition 1** *For $\tau \in (\underline{\tau}, \overline{\tau})$ there exists a unique $p \in (0, 1)$ such that $G[X^*, p; \tau] = 0$ and $X^*[p, f; \tau] \in (0, y)$ as the solution to the tax authority's problem.*

The proof of existence establishes that $G[X, p]$ switches sign on a sub-interval of $(0, 1)$ which guarantees the result by continuity. The proof of uniqueness is complicated by the fact that $X[p, f]$ is convex for $p$ close to zero, and concave thereafter. The former problem is overcome by noting that $X[p, f]$ is increasing on the convex interval, so this feature of the model does not generate multiple equilibria, while the possibility of the objective and constraint functions coinciding, except at a single point, on the concave interval is ruled out by consideration of the roots of the constraint and objective functions at $x = 0$.

In the event that the tax authority's budget does not lie on the interval $[\underline{\tau}, \overline{\tau}]$, however unlikely in practice, then compliance is a corner solution, and the properties of the equilibrium are as follows:

**Proposition 2** *If*

*i)* $\tau \leq \underline{\tau}$ *the equilibrium satisfies* $p = 1$, $q = h[\tau]$, $x = 0$;

*ii)* $\tau \geq \bar{\tau}$ *the equilibrium satisfies* $ph[\tau/p]f = 1$, $x = y$.

In part *(i)* of the Proposition, the tax authority is insufficiently resourced to generate a positive indirect effect, so seeks solely to maximize the direct effect. This is achieved by maximizing the value of $ph[\tau/p]$, which implies $p = 1$. By contrast, in part *(ii)*, the indirect effect is maximal, and the direct effect is zero.

# 4    Analysis

In this section, I explore the properties of interior solutions of the model in order to contrast the predictions flowing from the taxpayer behavioral function $x^*[p, qf]$, which has all the properties of the standard portfolio model, with the equilibrium predictions of the full model, as represented by $X^*[p, f]$.

## 4.1    Compliance

A well-known prediction of the standard model is that an increase in audit probability increases compliance, i.e. $\partial x^*[p, qf]/\partial p > 0$. However, the ceteris paribus condition under which $qf$ is held constant implicitly presupposes an accompanying increase in the tax authority's budget. Under the extension to balanced-budget analysis I obtain the following Proposition:

**Proposition 3** *At all interior equilibria an increase in audit probability decreases compliance:* $\frac{\partial X^*[p,f]}{\partial p} < 0$.

Proposition 3 follows immediately from the tax authority's first order condition in (10). The first term in (10) is the marginal change in the direct effect from an increase in $p$, while the second term captures the marginal change in the indirect effect. The former effect is always positive, while the latter takes the sign of $\partial X[p, f]/\partial p$. For $\partial X[p, f]/\partial p > 0$ both the indirect and

direct effect are increasing in $p$, so $\partial X\left[p, f\right]/\partial p > 0$ is never optimal. By similar reasoning, $\partial X\left[p, f\right]/\partial p = 0$ (the compliance maximizing choice of $p$), is never optimal. Instead, the optimal audit probaility must be such that $\partial X\left[p, f\right]/\partial p < 0$. At the optimal audit probability the marginal increase in the direct effect is fully offset by the marginal decrease in the indirect effect, so not only is the indirect effect negative at an interior optimum, it is also strong enough to offset the direct effect.

An implication of Proposition 3 is that audit probability is optimally set *higher* than the compliance maximizing level, and audit effectiveness is set *lower* than the compliance maximizing level. This suggests a tension between the role of the tax authority as a law enforcer (as envisaged by Becker), and as a revenue raiser: to maximize expected revenue the tax authority finds it optimal to tolerate a degree of non-compliance that it could, if it chose, prevent.

The Proposition relies both on the assumptions that the tax authority maximizes expected revenue and that audit effectiveness is endogenous. First, were the tax authority assumed to maximize compliance, then $\partial X\left[p, f\right]/\partial p = 0$ would, by assumption, define the optimal choice of $p$. Second, if audit effectiveness were to be assumed exogenous, which is equivalent to setting $e_q = 0$, there would be no trade-off between audit probability and effectiveness and the standard result of the Beckerian framework would re-emerge: $\partial X\left[p, f\right]/\partial p = \partial x^*\left[p, qf\right]/\partial p > 0$.

Intuition alone would convince most that, when audit effectiveness is endogenous and the tax authority is resource-constrained, an increase in audit probability might lead overall compliance to fall if audit effectiveness falls sufficiently fast and if taxpayer behavior is sufficiently responsive to audit effectiveness. The salience of Proposition 3 from a theoretical perspective is that it demonstrates that at *any* interior optima, it is necessarily the case

that these two conditions are met. The salience of the result from an empirical perspective is that it applies to the type of audit regimes observed empirically: those that generate less than full compliance.[3]

## 4.2 Effective Fine Rate

As a straightforward application of the envelope theorem, it can be shown that expected revenue is a (weakly) increasing function of $f$ (and strictly increasing for $\tau < \overline{\tau}$). As such, the model retains the basic insight behind Becker's 'hang 'em with probability zero' equilibrium: unless equilibrium non-compliance is already zero, if the tax authority is able to increase $f$, it has the incentive to do so.

However, in the present model, the tax authority is not able to choose $f$, but is able to choose the effective fine rate, $qf$, through its choice of $q$. What this approach reveals is that, even were the tax authority able to convince the relevant legislatures to approve a high $f$, it would in turn be optimal for the tax authority to reduce $q$ (and increase $p$), such that the effective fine rate turns out to be bounded at all interior equilibria.

**Proposition 4** *At all interior equilibria the effective fine rate on undetected tax satisfies $qf < 2$.*

Some intuition for Proposition 4 lies in the observation that the equilibrium $qf$ is not monotonically increasing as a function of $f$. To see this, first note that, analogous to $(\underline{\tau}, \overline{\tau})$, there exist $(\underline{f}, \overline{f})$, which denote the upper and lower bounds of $f$ consistent with an interior equilibria for compliance. Then:

$$qf \begin{cases} \leq 1 & f \leq \underline{f}; \\ > 1 & f \in \left( \underline{f}, \overline{f} \right); \\ = 1 & f \geq \overline{f}. \end{cases}$$

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[3]Proposition 3 also lies behind a number of other surprising results. For instance, in equilibrium, per-audit yield $(W_g - W_b)$ is an increasing function of audit probability. By contrast, in the standard model per-audit yield decreases in audit probability, as an increase in $p$ increases voluntary compliance.

For $f \leq \underline{f}$, we have $p = 1$ from Proposition 2, in which case to have $T[x, p] < 0$ in (3) requires $qf < 1$. For $f \in (\underline{f}, \overline{f})$ the condition is implied by the interior conditions for compliance in (5). For $f = \overline{f}$ the result is immediate from (12) as $p = pq\overline{f} = 1$.

The above arguments demonstrate that $qf$ is increasing in $f$ as $f \downarrow \underline{f}$, but decreasing as $f \uparrow \overline{f}$, so $qf$ attains a local maximum on the interval $f \in (\underline{f}, \overline{f})$. The proof of Proposition 4 demonstrates that all such interior equilibria satisfy $qf \in \left(1, \min\left[p^{-1}, (1-p)^{-1}\right]\right)$, which is a sub-interval of $(1, 2)$ for $p \in (0, 1)$. The non-monotonicity of $qf$ reflects the balanced budget trade-off between audit probability and the effective fine rate: for a fixed $f$, an increase in $qf$ requires a compensating reduction in $p$. Because $q$ is subject to diminishing returns, it follows that raising the effective fine rate indefinitely is not optimal.

A bounded effective fine rate therefore emerges as the optimal choice of the tax authority, rather than being artificially imposed. The result fits closely with empirical evidence: the Internal Revenue Code specifies $f = 1.75$ for fraudulent returns, while HMRC apply $f = 2$ for intentional non-compliance, both of which imply an effective fine rate of less than two (assuming $q < 1$).

## 4.3   Audit Expenditure

Suppose that the tax authority receives an exogenous increase in $\tau$, either as a result of an increase in $b$, or a fall in $n$.

**Proposition 5** *As $\tau \uparrow \overline{\tau}$ it holds that:*

$$\lim_{\tau \uparrow \overline{\tau}} \frac{\partial p}{\partial \tau} > 0; \qquad \lim_{\tau \uparrow \overline{\tau}} \frac{\partial q}{\partial \tau} < 0; \qquad \lim_{\tau \uparrow \overline{\tau}} \frac{\partial X^*[p, f]}{\partial \tau} < 0.$$

Simple intuition for the comparative static result for audit probability is as follows. I have from (12) that $p|_{\tau = \overline{\tau}} = 1$, but interior optima satisfy $pqf < 1$ and $qf > 1$, which together imply $p < 1/qf < 1$. Therefore audit

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probability must be increasing as $\tau \uparrow \bar{\tau}$. Similarly for $q$, I have from (12) that $q \,|_{\tau=\bar{\tau}} = 1/f$, but the interior conditions imply $q > 1/f$, so audit effectiveness must be decreasing as $\tau \uparrow \bar{\tau}$. Formally, a necessary and sufficient condition for these two results is that $\tau/p$ is decreasing in $\tau$ $(\partial p/\partial \tau > p/\tau)$ as $\tau \uparrow \bar{\tau}$. The proof proceeds by contradiction to show that if $\partial p/\partial \tau = p/\tau$ as $\tau \uparrow \bar{\tau}$, then the respective first order conditions for the taxpayer and the tax authority are not simultaneously satisfied.

The comparative static results for $p$ and $q$ are proved only local to $\tau = \bar{\tau}$, for model complexity frustrated all attempts at a more general result. However, Figure 2 depicts the optimal audit regime for a simulation of the model with logarithmic utility, $U\,[y] = \ln y$, (which implies constant relative risk aversion) and exponential audit effectiveness, $h\,[L] = 1 - e^{-2L}$. For this simple specification of the model, and choosing reasonable values for the fine and tax rates ($f = 1.5$, $\theta = 0.3$), $p$ and $q$ respond monotonically to $\tau$ over the whole interval $\tau \in [\underline{\tau}, \bar{\tau}]$.[4] In these cases audit effectiveness is an inferior input in the 'production' of expected revenue.

The final result in Proposition 5 is that optimal compliance is non-monotonic in $\tau$ near $\tau = \bar{\tau}$ (Figure 3). Although optimal compliance is seen to fall in this region, nevertheless expected revenue continues to increase: the tax authority chooses to allow non-compliance to increase in response to an increase in $\tau$, even though it could choose to allow it to decrease. Some intuition from the result is seen by rewriting expected revenue in (9) as:

$$E\,[R] = n\,\{\theta\,(1 - pqf)\,X\,[p,f] + pqf\theta y\}. \qquad (13)$$

The first term in (13) is dependent on the level of compliance, while the second is independent of the level of compliance. Near $\tau = \bar{\tau}$ I have that

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[4]The level of income, $y$, can be chosen arbitrarily under constant relative risk aversion, as the taxpayer's optimal compliance ($x^*\,[p,qf]$) is linear in income, so $y$ acts only as a scale parameter.

$pqf \rightarrow 1$, so, from (13), the compliance-independent component accounts for an increasing proportion of total expected revenue. In the limit, the costs of lowering $X^* [p, f]$ become dominated by the gains from increasing the compliance-independent component of expected revenue.

# 5  Extensions

## 5.1  Probability Weighting

A prominent feature of descriptive accounts of decision-making under risk is that individuals tend to overweight unlikely outcomes and underweight likely outcomes, relative to their objective probabilities (Kahneman and Tversky, 1979; Neilson, 2003). Consistent with this idea, empirical studies of tax-payers' subjective beliefs about their audit probability suggest that many subjects overestimate this (low) probability (e.g. Alm et al., 1992; Scholz and Pinney, 1995). It is therefore of interest to examine how this considera-tion alters the analysis of the previous section.

Following the insights of Quiggin (1982), probability weighting is modelled by a transformation of the cumulative probability distribution according to a probability weighting function, $w [p]$, on which I make the following assump-tions. First, $w [p]$ is continuous, differentiable on $p \in (0, 1)$, strictly increas-ing, and satisfies $w [0] = 0$ and $w [1] = 1$. Second, there exists a $p_f \in (0, 1)$ at which $w [p]$ intersects the diagonal from above. Third, it is concave on an initial interval and convex beyond that (s-shaped). The various functional forms for $w [p]$ so far proposed in the literature (e.g. Rieger and Wang, 2006; Prelec, 1998; Tversky and Fox, 1994; Tversky and Kahneman, 1992) satisfy these assumptions.

Denoting $(x^*, p^*, q^*)$ as the equilibrium of the model of Section 3 (without probability weighting) and $(x^w, p^w, q^w)$ as the equilibrium of the model with probability weighting, I then have the following Proposition:

**Proposition 6** *If taxpayers transform the objective audit probability according to $w[p]$ then for $p^* \in (0,1)$:*

   *i) As $p^* \downarrow 0$ it holds that $p^w > p^*$;*
  *ii) If $p^* = p_f$ then $p^w < p^*$;*
 *iii) As $p^* \uparrow 1$ it holds that $p^w > p^*$.*

Proposition 6 makes clear that probability weighting can either increase or decrease the optimal audit probability depending on the level of $p^*$. At extreme audit probabilities - including the most realistic case of $p$ close to zero - the tax authority chooses a higher audit probability under probability weighting. However, in an interval around the fixed point at $p_f$, probability weighting lowers the tax authority's optimal choice of $p$. The explanation is that the optimal $p$ depends both on the *level* of $w[p]$ and its *slope*, $w'[p]$. When $w[\cdot]$ is overweighting there is an incentive to reduce $p$, as the bias in taxpayers' judgments is a substitute for the objective audit probability. However, when $w'[p] > 1$ there is an incentive to raise $p$, since $w[p]$ increases faster than $p$. Close to $p^* = 0$ and $p^* = 1$, I have $w[p^*] \approx p^*$ and $w'[p^*] > 1$, so the slope effect dominates, and is positive. At the fixed point, however, I have $w'[p_f] < 1$, so the slope effect is negative.

## 5.2   Uncertainty

The previous section assumes that taxpayers know the tax authority's choice of the audit probability and effectiveness. In practice, however, the tax authority does not normally announce its choice, and there may be sound theoretical grounds for maintaining secrecy (Alm, 1988; Snow and Warren Jr., 2005a). Therefore, taxpayers typically face uncertainty over both of these parameters. Let $(\widetilde{p}, \widetilde{q})$ be random variables describing taxpayers' uncertainty about $(p, q)$, where I assume that taxpayers' expectations about $(p, q)$ are rational in the sense that $E[\widetilde{p}] = p$ and $E[\widetilde{q}] = q$. Let $(x^u, p^u, q^u)$ denote the equilibrium under uncertainty, then I have the following Proposition:

**Proposition 7** *Under p-uncertainty it holds that $p^u = p^*$ and $q^u = q^*$.*

Proposition 7 demonstrates that the analysis of Section 4 is robust to tax-payer uncertainty over $p$. The result arises as a straightforward consequence of the linearity of taxpayers' expected utility in audit probability. Formally, suppose $\widetilde{p}$ is distributed according to $P[\varepsilon]$, then taxpayers' expected utility is

$$E[U] = U[W_g]\left(1 - \int \varepsilon \, dP[\varepsilon]\right) + U[W_b]\int \varepsilon \, dP[\varepsilon]. \qquad (14)$$

However, as rational expectations imply that $\int \varepsilon \, dG[\varepsilon] = p$, equation (14) is equivalent to (1). The tax authority's optimization problem is therefore unchanged.

Turning to $q$-uncertainty, suppose $\widetilde{q}$ is distributed according to $Q[\varepsilon]$, then the taxpayers' first order condition in (3) becomes

$$\theta \int \left\{p\left(\varepsilon f - 1\right)U'[W_b[\varepsilon]] - (1-p)U'[W_g[\varepsilon]]\right\} \, dQ[\varepsilon] = 0,$$

and (11) becomes

$$\frac{\partial X^u[p,f]}{\partial p} = -\frac{\left\{\begin{array}{c} U'[W_g] + \int (\varepsilon f - 1 - qf e_q) U'[W_b[\varepsilon]] \, dQ[\varepsilon] \\ +e_q (W_g - W_b)\int (\varepsilon f - 1) U''[W_b[\varepsilon]] \, dQ[\varepsilon] \end{array}\right\}}{\theta\left\{(1-p)U''[W_g] + p\int (\varepsilon f - 1)^2 U''[W_b[\varepsilon]] \, dQ[\varepsilon]\right\}}. \qquad (15)$$

Comparing (11) and (15), how the tax authority's problem is affected by $q$-uncertainty is determined by whether the integrals in (15) are increasing or decreasing under a mean-preserving spread of $Q[\varepsilon]$. From the results of Rothschild and Stiglitz (1971), an integrand increases (decreases) with a mean-preserving spread if it is convex (concave). It follows immediately from (15), therefore, that the effects of $q$-uncertainty for $(p,q)$ depend on both the third and fourth derivatives of the utility function. Kimball (1990) shows that assumption A3 (DARA) implies that $U''' > 0$ - a property which

Menezes et al. (1980) term downside risk aversion.[5] Together, assumptions A2 and A3 therefore imply that $-U'''/U'' > 0$, a property Kimball (1990) terms prudence.

However, in order to sign the fourth derivative of utility, I introduce the stronger concept of standard risk aversion (Kimball, 1993). Taxpayers are standard risk averse if their preferences satisfy DARA (A3) *and* decreasing absolute prudence (DAP). The latter property is that $-U'''[x]/U''[x]$ is decreasing in $x$, which Kimball (1993) shows to imply $U'''' < 0$.[6] Because DARA is still assumed, a standard risk averse taxpayer is necessarily downside risk averse and prudent. A standard risk averse taxpayer is also 'proper risk averse' in the sense of Pratt and Zeckhauser (1987). I then have a final Proposition.

**Proposition 8** *If*

*i) Taxpayers are standard risk averse;*
*ii) Taxpayer beliefs satisfy*

$$e_q \max\left[qf, \frac{1-p}{2p}\right] < qf - 1 < e_q\left(\frac{1-p}{p}\right);$$

*then, under q-uncertainty, $p^u < p^*$ and $q^u > q^*$.*

The proof of Proposition 8 proceeds by analyzing the second derivatives of the integrands in (15) at the equilibrium of the model. Under the restrictions of the Proposition, I am able to prove that $\partial X[p, f]/\partial p > \partial X^u[p, f]/\partial p$. As the tax authority operates on the downward sloping interval of $X[p, f]$

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[5]Menezes et al. (1980) define an increase in downside risk to be a mean-and-variance preserving shift of probability to the lower tail of the distribution. They show that aversion to downside risk is equivalent to $U''' > 0$.

[6]Kimball (1990) shows that absolute prudence $-U'''/U''$ measures the strength of the precautionary saving motive, so that DAP can be interpreted as a precautionary saving motive that decreases in intensity with wealth.

(Figure 1), to restore equilibrium it must raise $p$, from which the result follows. The restrictions in $(ii)$ place limits on the dispersion of taxpayer beliefs around the true value of $q$. In particular, they require that taxpayers believe that the effective fine rate satisfies $qf > 1$. If taxpayers place sufficient probability weight on the possibility that $qf < 1$, then the relative magnitudes of $p^u$ and $p^*$ can be reversed.

# 6   Conclusion

The economics of tax compliance has developed as a special case of Becker's (1968) model of crime and punishment. However, tax evasion is in some ways a unique type of crime, making it worthwhile exploring the implications of alternative assumptions. In particular, the political economy considerations inherent in the enforcement of compliance imply that the tax authority is not a simple law enforcer, but also plays an economic role in raising government revenue. I therefore consider the private objective function of the tax authority to maximize expected revenue, rather than assuming the maximization of social welfare. Second, with fine rates severely constrained in practice, I instead analyze the trade-off between audit probability and effectiveness.

Characterizing the tax authority in this way leads to some descriptively important changes to the predictions of the standard portfolio model. In particular, I have shown that at any interior equilibrium - the type that we observe empirically - the expected-revenue maximizing audit strategy does not maximize voluntary compliance, and that increases in the tax authority's budget can lead to falls in voluntary compliance, while still increasing expected revenue. While not contradicting the intuition of Becker's 'hang 'em with probability zero' equilibrium, the model nevertheless leads to the conclusion that the tax authority will choose to set an effective fine rate that does not exceed two - a prediction closely in line with observed practice.

There are further extensions of the model that future research might prof-

itably explore. For instance, a key assumption one would like to relax is that of homogeneous taxpayers, which in turn might allow for an integration of the present approach with the literature on the design of audit selection rules. The model can also be used to derive policy implications for tax authorities considering changes to their audit portfolio through, for instance, the introduction of 'light-touch' audits - audit types that can be performed quickly and cheaply - as a partial replacement for (longer and more expensive) traditional audit types.

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# Appendix

## Proof of Proposition 1

*Existence*: I begin by showing that $\lim_{p\downarrow 0} G[X,p] > 0$. As $p \downarrow 0$ I have that $h[\tau/p] \uparrow 1$ and $e_q \downarrow 0$. Therefore, (11) gives
$\lim_{p\downarrow 0} \partial X[p,f]/\partial p = -\lim_{p\downarrow 0}(\theta/D)(U'[W_g] + (f-1)U'[W_b]) > 0$,
which, in turn, implies that $\lim_{p\downarrow 0} G[X,p] = n \lim_{p\downarrow 0}\left(W_g - W_b + \theta\frac{\partial X[p,f]}{\partial p}\right) > 0$. I now show that $G[X,p] < 0$ where $p = (h[\tau/p]f - 1)/(h[\tau/p]f - 1 + e_q) < 1$. Setting $G[X,p] = 0$ in (10), and substituting for $\frac{\partial X[p,f]}{\partial p}$ from (11) I obtain:

$$(W_g - W_b)\left\{ \begin{array}{c} (1-p)(1-e_q)U''[W_g] \\ -(qf-1)\{e_q(1-p) - p(qf-1)\}U''[W_b] \end{array} \right\}$$
$$= (1-pqf)\{U'[W_g] - \{1 - qf(1-e_q)\}U'[W_b]\} \quad \text{(A.1)}$$

Suppose, by contradiction, that $e_q = p(qf-1)/(1-p)$, then substituting in (A.1) obtains $(W_g - W_b)U''[W_g] = (qf-1)(U'[W_b] - U'[W_g])$, which is a contradiction since the l.h.s. is negative and the r.h.s. is positive, implying $G[X,p] < 0$. It follows, by continuity, that there exists a $p$ satisfying $p > 0$ and $p < (h[\tau/p]f - 1)/(h[\tau/p]f - 1 + e_q)$ such that $G[X,p] = 0$.

*Uniqueness*: I first show that $E[R]$ is a convex function of $(x,p)$: the determinant of the Hessian matrix is $|H| = (fn\theta\partial(ph[\tau/p])/\partial p)^2 > 0$. The iso-expected revenue curves in Figure 1 are therefore concave to the origin. The constraint $X[p,f]$ is not globally concave because, taking $q$ as constant, compliance is an increasing and convex function of $p$. Since $q$ is approximately constant close to unity, $X[p,f]$ is increasing and convex for $p$ sufficiently close to zero. However, to generate multiple equilibria would require $X[p,f]$ to be downward sloping on the convex interval, and for the convex interval to be sandwiched between two concave intervals, neither of which is the case.

It remains to check whether the constraint and objective functions coincide at more than a single point on the interval where both are concave. To

see this is not the case, note that iso-expected revenue intersects the line $x = 0$ for $p = p_R$, where $p_R = 1/h\left[\tau/p_R\right]f$. The constraint $X\left[p, f\right]$ intersects $x = 0$ for $p = p_x$ (which may not be unique), where $(1 - p_x)U'\left[y\right] - p_x\left(h\left[\tau/p_x\right]f - 1\right)U'\left[y\left(1 - h\left[\tau/p_x\right]f\theta\right)\right] = 0$. Substituting $p_R$ into the definition of $p_x$ yields $\left(\left(h\left[\tau/p_R\right]f - 1\right)/h\left[\tau/p_R\right]f\right)\left(U'\left[y\right] - U'\left[y\left(1 - h\left[\tau/p_R\right]f\theta\right)\right]\right) < 0$, from which it follows that that $p_x < p_R$.

## Proof of Proposition 2

Part $(i)$: If $x = 0$ then $E\left[R\right] = pqf\theta y$. Since $\partial\left(pq\right)/\partial p = q + p\left(\partial q/\partial p\right) = q\left(1 - e_q\right) > 0$ it follows that $\partial E\left[R\right]/\partial p > 0$, implying a corner solution at $p = 1$.

Part $(ii)$: If $pqf = 1$ is feasible $(\tau \geq \bar{\tau})$ then there is always a solution to $G\left[X, p\right] = 0$ in (10), since it implies that $x = y$, so also $W_g = W_b$.

## Proof of Proposition 3

From (10) it is immediate that $G\left[X, p\right] = 0$ implies
$\partial X\left[p, f\right]/\partial p = -\left(W_g - W_b\right)\left(1 - e_q\right)/\left\{\theta\left(1 - pqf\right)\right\} < 0$.

## Proof of Proposition 4

From (5) an interior equilibrium for compliance must satisfy $qf < p^{-1}$. I now show that all interior equilibria also satisfy the inequality $qf < (1 - p)^{-1}$. Suppose, by contradiction, that $qf = (1 - p)^{-1}$, so $p = (qf - 1)/qf$ and $pqf = qf - 1$. Substituting $p = (qf - 1)/qf$ in (3) gives $U'\left[W_g\right] - (qf - 1)^2 U'\left[W_b\right] = 0$. Now also suppose $\tau = \underline{\tau}$ which implies $e_q = pqf$. Substituting for $e_q$ in (A.1) I obtain

$$G\left[X, p\right] = 0 \Leftrightarrow \left(W_g - W_b\right)\left\{(1 - p)U''\left[W_g\right] - p\left(qf - 1\right)U''\left[W_b\right]\right\}$$
$$= U'\left[W_g\right] - \left\{1 - qf\left(1 - pqf\right)\right\}U'\left[W_b\right]. \quad \text{(A.2)}$$

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Substituting from (3) in both sides gives:

$$G\left[X,p\right]=0 \Leftrightarrow (W_g - W_b)(1-p)\,U'\left[W_g\right]\{A\left[W_b\right]-A\left[W_g\right]\}$$
$$= p^{-1}\left\{U'\left[W_g\right]-(qf-1)^2 U'\left[W_b\right]\right\}=0,$$

But this is a contradiction since the l.h.s. is strictly positive by assumption A3 (DARA), while the r.h.s. is zero. It follows that $(U'\left[W_g\right]-\{1-qf\left(1-pqf\right)\}U'\left[W_b\right])$ cannot be zero at an interior equilibrium. Instead, for $\tau\in(\underline{\tau},\overline{\tau})$, it must hold that $(U'\left[W_g\right]-\{1-qf\left(1-pqf\right)\}U'\left[W_b\right])<0$. This implies that $U'\left[W_g\right]/U'\left[W_b\right]<1-qf\left(1-pqf\right)$. Using (3) I have that $U'\left[W_g\right]/U'\left[W_b\right]=p\left(qf-1\right)/\left(1-p\right)$, so, solving the resulting quadratic in $(qf)$, this implies that $qf\in\left(1,\min\left[p^{-1},(1-p)^{-1}\right]\right)$. Then $\max_p\min\left[p^{-1},(1-p)^{-1}\right]=2$ (at $p=1/2$), implying $qf<2$.

## Proof of Proposition 5

Suppose, by contradiction, that $\partial p/\partial\tau=p/\tau$, such that $\partial q/\partial\tau=\partial h\left[\tau/p\right]/\partial\tau=0$. Then an increase in $\tau$ in (3) leaves $q$ unchanged and increases $p$. To restore the first order condition it follows that $\left.\frac{\partial X^*[p,f]}{\partial p}\right|_{\frac{\partial p}{\partial\tau}=\frac{p}{\tau}}=-\left(\theta/D\left[X,p\right]\right)\left(U'\left[W_g\right]+(qf-1)U'\left[W_b\right]\right)>0$. In the limit as $\tau\uparrow\overline{\tau}$ I have that $W_g-W_b\to 0$ and $qf\to 1$, in which case $\left.\frac{\partial X^*[p,f]}{\partial p}\right|_{\frac{\partial p}{\partial\tau}=\frac{p}{\tau}}$ collapses to $\lim_{\tau\uparrow\overline{\tau}}\left.\frac{\partial X[p,f]}{\partial p}\right|_{\frac{\partial p}{\partial\tau}=\frac{p}{\tau}}=-U'\left[W_g\right]/\{\theta\left(1-p\right)U''\left[W_g\right]\}>0$. A further expression for $\lim_{\tau\uparrow\overline{\tau}}\left.\frac{\partial X[p,f]}{\partial p}\right|_{\frac{\partial p}{\partial\tau}=\frac{p}{\tau}}$ is derived by total differentiation of the equality in (A.1), giving $\lim_{\tau\uparrow\overline{\tau}}\left.\frac{\partial X[p,f]}{\partial p}\right|_{\frac{\partial p}{\partial\tau}=\frac{p}{\tau}}=-\{(1-e_q)/e_q\}\{U'\left[W_g\right]/\{\theta\left(1-p\right)U''\left[W_g\right]\}\}$.
The two expressions are equal iff $\lim_{\tau\uparrow\overline{\tau}}(1-e_q)/e_q=1$, which establishes a contradiction since $\lim_{\tau\uparrow\overline{\tau}}e_q=1$. From analysis of derivatives it follows that $\lim_{\tau\uparrow\overline{\tau}}\partial p/\partial\tau>p/\tau>0$, so also $\lim_{\tau\uparrow\overline{\tau}}\partial q/\partial\tau<0$.

To establish the sign of $\lim_{\tau\uparrow\overline{\tau}}\partial X^*\left[p,f\right]/\partial\tau$ I can now denote $\partial p/\partial\tau=\beta p/\tau$, where $\beta>1$ is a scalar. It follows that $\left.\frac{\partial q}{\partial\tau}\right|_{\frac{\partial p}{\partial\tau}=\frac{\beta p}{\tau}}=\frac{qe_q}{\tau}(1-\beta)<0$. Differentiating $T\left[X^*,p\right]=0$ in (3) I have that:

$$\frac{\partial X^*[p,f]}{\partial \tau}\bigg|_{\frac{\partial p}{\partial \tau}=\frac{\beta p}{\tau}} \gtrless 0 \Leftrightarrow \beta \lesseqgtr -\frac{e_q\{qfU'[W_b]-(qf-1)(W_g-W_b)U''[W_b]\}}{\left(\begin{array}{c} U'[W_g]-U'[W_b]\{1-fq(1-e_q)\} \\ +e_q(qf-1)(W_g-W_b)U''[W_b] \end{array}\right)}.$$

$$(\text{A.3})$$

In the limit as $\tau \uparrow \overline{\tau}$, (A.3) implies that $\beta > -\lim_{\tau\uparrow\overline{\tau}} e_q/(1-e_q) < 0$, so it must be that $\lim_{\tau\uparrow\overline{\tau}} \partial X^*[p,f]/\partial\tau < 0$.

**Proof of Proposition 6**

Part $(i)$: Under probability weighting (11) becomes:

$$\frac{\partial x^w}{\partial p^w} = -\left(\frac{\theta}{E^w}\right)\left\{\begin{array}{c} w'[p^w]\left(U'[W_g^w]+(q^w f-1)U'[W_b^w]\right) \\ +e_q^w\left(\frac{w[p^w]}{p^w}\right)\{(q^w f-1)(W_g^w-W_b^w)U''[W_b^w]-q^w fU'[W_b^w]\} \end{array}\right\},$$

where $E^w = \theta^2\{w[p^w](q^w f-1)^2 U''[W_b^w]+(1-w[p^w])U''[W_g^w]\}$. Suppose, by contradiction, that $(p^w, x^w) = (p^*, x^*)$ then I have that:

$$\frac{\partial x^w}{\partial p^w} - \frac{\partial x^*}{\partial p^*} =$$
$$\left(\frac{\theta}{D^*E^*}\right)\left\{\begin{array}{c} p^*\{U'[W_g^*]+(q^*f-1)U'[W_b^*]\}w[p^*](1-e_w^*)(q^*f-1)^2 U''[W_b^*] \\ +p^*\{1-w'[p^*]-w[p^*](1-e_w^*)\}U''[W_g^*] \\ -e_q(w[p^*]-p^*)U''[W_g^*]\{(q^*f-1)(W_g^*-W_b^*)-q^*fU'[W_b^*]\} \end{array}\right\},$$

$$(\text{A.4})$$

where $e_w$ is the elasticity of $w[p]$. As $p^* \downarrow 0$ I have that $w[p^*] = p^*$, so $e_w^*[0] = w'[0] > 1$. This implies that $1-w'[0]-w[0](1-e_w^*[0]) = 1-w'[0] < 0$. Using these observations in (A.4) yields that $\frac{\partial x^w}{\partial p^w} - \frac{\partial x^*}{\partial p^*} > 0$, contradicting the supposed solution at $(p^w, x^w) = (p^*, x^*)$. Since $\partial G[x,p]/\partial p < 0$ it follows that $p^w > p^*$, and therefore $q^w < q^*$.

Part $(ii)$: At $p^* = p_f$ I have $e_w^* = w'[p_f] < 1$ and $1-w'[p^*]-w[p^*](1-e_w^*) = (1-w[p_f])(1-w'[p_f]) > 0$. Hence, $\frac{\partial x^w}{\partial p^w} - \frac{\partial x^*}{\partial p^*} < 0$, contradicting the supposed solution at $(p^w, x^w) = (p^*, x^*)$. Since $\partial G[x,p]/\partial p < 0$ it follows that $p^w < p^*$, and therefore $q^w > q^*$.

Part $(iii)$: As $p^* \uparrow 1$ I have $e_w^*[1] = w'[1] > 1$ and $1 - w'[1] - w[1](1 - e_w^*[1]) = 0$. An analogous argument to Part $(i)$ therefore applies.

**Proof of Proposition 8**

Substituting (15) into (10) gives

$$(W_g - W_b) \left\{ \begin{array}{c} (1-p)(1-e_q) U''[W_g] \\ -\int (\varepsilon f - 1) \{e_q(1-p) - p(\varepsilon f - 1)\} U''[W_b[\varepsilon]] \, dQ[\varepsilon] \end{array} \right\}$$
$$= (1 - pqf) \left( U'[W_g] + \int (\varepsilon f - 1 - qf e_q) U'[W_b[\varepsilon]] \, dQ[\varepsilon] \right). \quad (A.5)$$

Suppose, en route to a contradiction, that $(p^*, x^*) = (p^u, x^u)$ then both (A.5) and the equivalent relation under certainty (A.1) must hold. Taking the second derivative of the integrand in the r.h.s. of (A.5) gives

$$\frac{\partial^2 (\varepsilon f - 1 - qf e_q)}{(\partial \varepsilon)^2} = -\left( \frac{W_g - W_b}{q} \right) \left( 2U''[W_b] - (\varepsilon f - 1 - qf e_q) \frac{W_g - W_b}{q} U'''[W_b] \right).$$
$$(A.6)$$

Within the second bracket, the first term is negative under risk aversion and the second is negative under downside risk aversion (as $\varepsilon > (1 + qf e_q)/f$ by assumption). According to Rothschild and Stiglitz (1971), an integrand increases (decreases) with a mean-preserving spread if it is convex (concave). Therefore (A.6) implies

$$\int (\varepsilon f - 1 - qf e_q) U'[W_b[\varepsilon]] \, dQ[\varepsilon] > (qf - 1 - qf e_q) U'[W_b].$$

Using the assumption of decreasing absolute prudence, which implies $U'''' < 0$, similar reasoning can be used to show that, if beliefs satisfy $(e_q(1-p) + 2p)/2pf < \varepsilon < (e_q(1-p) + p)/pf$, then

$$\int \left(\varepsilon f - 1\right) \left\{e_q \left(1 - p\right) - p \left(\varepsilon f - 1\right)\right\} U'' \left[W_b \left[\varepsilon\right]\right] dQ \left[\varepsilon\right]$$
$$> \left(qf - 1\right) \left\{e_q \left(1 - p\right) - p \left(qf - 1\right)\right\} U'' \left[W_b\right].$$

But then (A.1) and (A.5) cannot hold for $(p^*, x^*) = (p^u, x^u)$ as the l.h.s. of (A.5) is smaller than the l.h.s. of (A.1), while the r.h.s. of (A.5) exceeds the r.h.s. of (A.1). Instead, it must hold that $\partial X \left[p, f\right] / \partial p > \partial X^u \left[p, f\right] / \partial p$. In order to restore (10) it must hold that $p^u < p^*$, which implies $q^u > q^*$ and, as $\partial X \left[p, f\right] / \partial p < 0$, $x^u > x^*$.

# List of Figures


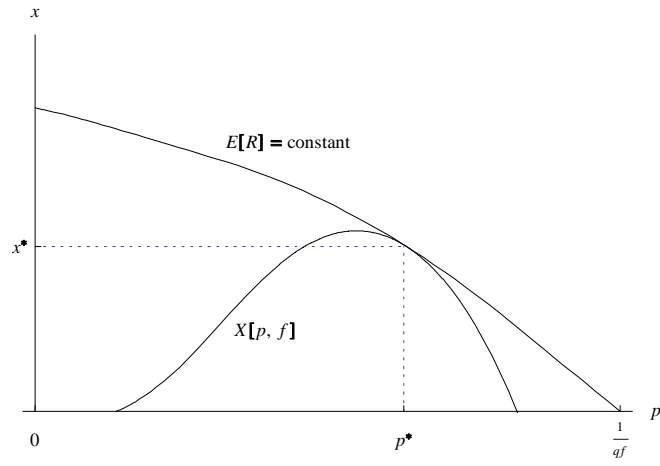
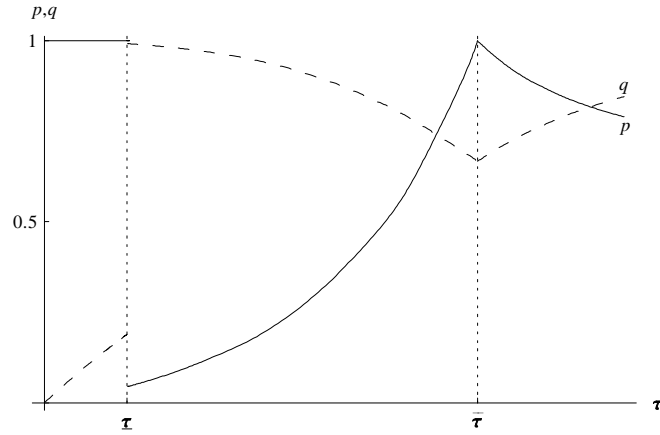Figure 1: Equilibrium between taxpayers and the tax authority.

Figure 2: Optimal audit probability and effectiveness (for CRRA utilty and $h[L]$ as the exponential distribution function).
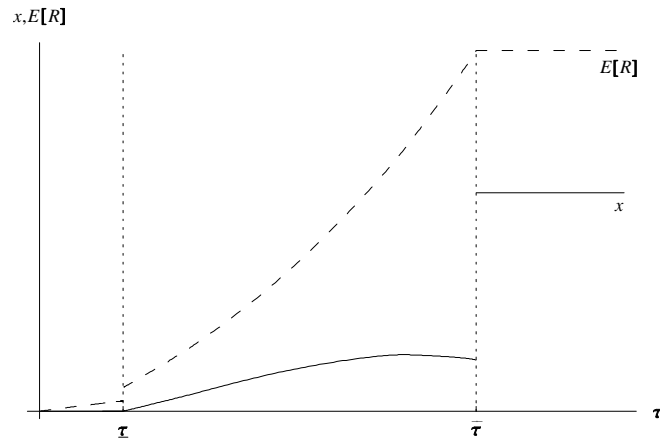


Figure 3: Optimal compliance and expected revenue (for CRRA utilty and $h[L]$ as the exponential distribution function).