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TESTING UNEMPLOYMENT THEORIES:
A MULTIVARIATE LONG MEMORY APPROACH

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Abstract

This paper investigates the empirical relevance of different unemployment theories in three major economies, namely the UK, the US and Japan, by estimating the degree of dependence in the unemployment series. Both univariate and multivariate long memory methods are used. The results vary depending on whether the former or the latter approach is followed. Specifically, when taking a univariate approach, the unit root null cannot be rejected in case of the UK and Japanese unemployment series, and some degree of mean reversion ($d < 1$) is found in the case of the US unemployment rate. When applying multivariate methods instead, higher orders of integration are still found for the UK and Japanese series, but the NAIRU hypothesis cannot be rejected in the case of the US.

Keywords: Unemployment rate; Multivariate long memory; Fractional integration

JEL Classification: C22, C32, E24

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1. Introduction

This paper investigates the empirical relevance of different unemployment theories in three major economies, namely the UK, the US and Japan, by estimating the degree of dependence in the unemployment series. For this purpose, it applies long memory methods and in particular, fractional integration techniques, which are more general than the standard approaches based on integer degrees of differentiation. The existing empirical literature has either used univariate fractional integration or multivariate fractional cointegration methods for estimating the differencing parameter with parametric, semiparametric or nonparametric techniques; little attention has instead been paid to multivariate I(d) processes, which allow for potential correlation among the variables of interest. The present is a thorough study using both univariate and multivariate techniques. It shows that the results vary substantially depending on whether the former or the latter approach is followed, and that taking into account the correlations between the variables is crucial to estimate the degree of integration of the series accurately, and therefore to obtain reliable evidence to discriminate between different unemployment theories. In brief, we find that unemployment in the UK and Japan is highly persistent, which supports the hysteresis hypothesis in these two countries, whilst for the US the results are consistent with the NAIRU hypothesis.

The structure of the paper is as follows. Section 2 briefly reviews the main unemployment theories and what they imply for the degree of dependence of the data. Section 3 outlines the methodology. Section 4 describes the data and discusses the empirical results, while Section 5 concludes the paper.

2. Unemployment theories

There are two main theoretical approaches to understanding the behaviour of the unemployment rate. The natural rate theory (see Friedman, 1968, and Phelps, 1967, 1968) implies that it should fluctuate around a stationary equilibrium level, known as the natural rate or NAIRU, which is determined by economic fundamentals. In "structuralist" models (see Phelps, 1994) this is "endogenised", i.e. the equilibrium level to which unemployment reverts when hit by shocks can shift over time as a result of infrequent structural breaks reflecting changes in economic fundamentals. Hence mean-reversion occurs provided the breaks are taken into account. Various theoretical models have been put forward to endogenise the natural rate of unemployment. They rely alternatively on productivity growth (Pissarides, 1990), real interest rates (Blanchard, 1999), stock prices (Phelps, 1999), institutional variables (Nickell, 1998 and Nickell and Van Ours, 2000), or the interaction between institutional and macroeconomic variables (Blanchard and Wolfers, 2000).

The observed high persistence of unemployment in Europe led to the development of an alternative class of models, i.e. hysteresis models (see Blanchard and Summers, 1986, 1987, and Barro, 1988) where unemployment is a path-dependent variable, with temporary shocks having permanent or very highly persistent effects. In this framework, unemployment may exhibit long memory, with a (near) unit root.

Unemployment theories have been tested in a huge number of papers. Initially, standard unit root tests (such as ADF or Phillips-Perron) were carried out (see, e.g., Blanchard and Summers, 1986, and Alogoskoufis and Manning, 1988), often supporting the hysteresis hypothesis (see, e.g., the studies of Gordon (1989) for France, Germany, the US, Japan and the UK, Graafland (1991) for the Netherlands, Lopez et al. (1996) for Spain and Wilkinson (1997) for Canada). Studies allowing for structural breaks (see, e.g., Mitchell, 1993, Bianchi and Zoega, 1998, and Papell et al, 2000) have tended instead to support structuralist theories.

Panel approaches have subsequently been used to deal with the well-known problem of the low power of standard unit root tests (see, e.g., Song and Wu (1998) and Leon-Ledesma (2002)), generally finding that hysteresis models work better in Europe, and NAIRU models in the US. Panel analyses allowing for breaks as well (see Murray and Papell (2000) and Strazicich, Tieslau and Lee (2009)) are more supportive of structuralist theories.

Another recent strand of the literature estimates fractionally integrated (ARFIMA) models to test for long memory in the unemployment rate (see, for instance, Tschernig and Zimmermann, 1992; Crato and Rothman, 1996; Gil-Alana, 2001, 2002; etc.). By allowing for fractional orders of integration, such a modelling approach is suitable for both stationary processes (NAIRU models), and highly persistent/nonstationary ones (hysteresis hypothesis), and by incorporating structural breaks it can also be used to model processes exhibiting regime change (structuralist theories). Recent studies of this type include Caporale and Gil-Alana (2007, 2008). The former paper proposes a model of the US unemployment as a fractionally integrated process interacting with some nonlinear functions of labour-demand variables such as real oil prices and real interest rates, and also finds evidence of a long memory component. The results are consistent with a hysteresis model with path dependence rather than a non-accelerating inflation rate of unemployment (NAIRU) model. The latter paper uses a general procedure for fractional integration and structural breaks at unknown points in time, allowing for different orders of integration and deterministic components in each subsample as well as for nonlinearities. This study suggests that a structuralist interpretation is more appropriate for the US and Japan, whilst a hysteresis model accounts better for the UK experience. However, these two papers are based on univariate models that do not take into account the possible correlation between the unemployment series.

3. Methodology

For the purposes of the present study, we define an *I(0)* process as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. In this context, x_t is said to be *I(d)* if it can be represented in the form:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \ldots, \tag{1}$$

with $x_t = 0$ for $t \leq 0$, where d can be any real value, L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is $I(0)$. The polynomial $(1-L)^d$ in equation (1) can be expressed in terms of its binomial expansion, such that, for all real d,

$$(1-L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \ldots,$$

and thus

$$(1-L)^d x_t = x_t - dx_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \ldots$$

In this context, d plays a crucial role since it indicates the degree of dependence of the time series: the higher the value of d is, the higher the level of association will be between the observations. The above process also admits an infinite Moving Average (MA) representation such that

$$x_t = \sum_{k=0}^{\infty} a_k u_{t-k},$$

where

$$a_k = \frac{\Gamma(k+d)}{\Gamma(k+1)\Gamma(d)},$$

and $\Gamma(x)$ represents the Gamma function. Thus, the impulse responses are also clearly affected by the magnitude of d, and the higher the value of d is, the higher the responses will be.

Given the parameterisation in (1), several cases can be distinguished depending on the value of d. Specifically, if $d = 0$, $x_t = u_t$, x_t is said to be "short memory" or $I(0)$, and if the observations are autocorrelated they are of a "*weakly*" form (e.g., autoregressive), in the sense that they decay at an exponential rate; if $d > 0$, x_t is said to be "long memory", or "*strongly autocorrelated*", because of the strong association between observations far away in time. If d belongs to the interval (0, 0.5) x_t is still covariance stationary, while $d \geq 0.5$ implies nonstationarity. Finally, if $d < 1$, the series is mean reverting in the sense that the effects of shocks disappear in the long run, contrary to what happens if $d \geq 1$ when they persist forever.

As mentioned before, estimating d is crucial to be able discriminate between different unemployment theories. Specifically, $d = 0$ can be thought of as being consistent with the NAIRU hypothesis, while long memory ($d > 0$) and unit roots ($d = 1$) support the "hysteresis" approach to unemployment.

In a univariate context, there exist several methods for estimating and testing the fractional differencing parameter d. Some of them are parametric while others are semiparametric and can be specified in the time or in the frequency domain. In this paper, we use a Whittle estimate of d in the frequency domain (Dahlhaus, 1989) along with a testing procedure, which is based on the Lagrange Multiplier (LM) principle and that also uses the Whittle function in the frequency domain. It tests the null hypothesis:

$$H_o: d = d_o, \tag{2}$$

for any real value d_o, in a model given by the equation (1), where x_t can be the errors in a regression model of the form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, ..., \tag{3}$$

where y_t is the observed time series, β is a (kx1) vector of unknown coefficients and z_t is a set of deterministic terms that might include an intercept (i.e., $z_t = 1$), an intercept with a linear time trend ($z_t = (1, t)^T$), or any other type of deterministic processes. Robinson (1994) showed that, under certain very mild regularity conditions, the LM-based statistic $(\hat{r})$:

$$\hat{r} \rightarrow_d N(0,1) \quad as \quad T \rightarrow \infty, \tag{4}$$

where " $\rightarrow_d$ " stands for convergence in distribution, and this limit behaviour holds independently of the regressors z_t used in (3) and the specific model for the *I(0)* disturbances u_t in (1).

As in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives have the same null and limit theory as the LM test of Robinson (1994). Lobato and Velasco (2007) essentially employed such a Wald testing procedure, even though it requires a consistent estimate of d; therefore the LM test of Robinson (1994) seems computationally more attractive. A semiparametric Whittle approach (Robinson, 1995) will also be implemented in the paper.

A disadvantage of the univariate methods is that they do not take into account the potential cross-dependence of the series. Thus, in this paper, we also consider multivariate models of the form:

$$D x_t = u_t, \quad t = 1, 2, ..., \tag{5}$$

where D is a (nxn) diagonal matrix of the form $(1-L)^{d_i}$, where d_i indicates the degree of integration of the i^{th} variable, and u_t is now an (nx1) vector of *I(0)* variables that may have a finite Vector Autoregressive (VAR) representation. To estimate the process given by (5) we use the approximate frequency domain maximum likelihood approach proposed by Boes et al. (1989). The discussion of the multivariate version of the procedure can be found in Hosoya (1996).

4. Data and empirical results

The data source is the St. Louis Federal Reserve Bank database. We use the following three series:

1. Harmonized Unemployment Rate: All Persons for United Kingdom, quarterly, seasonally adjusted, 1971-01-01 to 2011-10-01, series ID: GBRURHARMQDSMEI
2. Harmonized Unemployment Rate: All Persons for the United States, quarterly, seasonally adjusted, 1971-01-01 to 2011-10-01, series ID: USAURHARMQDSMEI
3. Harmonized Unemployment Rate: All Persons for Japan, monthly, seasonally adjusted, 1971-01-01 to 2011-10-01, series ID: JPNURHARMMDSMEI, transformed to quarterly by taken average of months inside a quarter.

[Insert Figure 1 about here]

Prior to the estimation we take logs of the series, and for the multivariate approach we standardise them by substracting the mean.

We start with the univariate approach and estimate d in the model given by the equations (1) and (3) with $z_t = (1, t)^T, t > 0, 0$ otherwise, i.e.

$$y_t \;=\; \alpha \;+\; \beta t \;+\; x_t\,, \quad (1-L)^d x_t \;=\; u_t\,, \quad t \;=\; 1, 2, \ldots, \tag{6}$$

where y_t is the log-transformed series, with different assumptions for the $I(0)$ error term u_t. In particular, we assume in turn that u_t is white noise and autocorrelated, in the latter case using a nonparametric specification due to Bloomfield (1973). In this model, u_t is specified exclusively in terms of its spectral density function, which is given by

$$f(\lambda;\tau) \;=\; \frac{\sigma^2}{2\pi}\; exp\left(2\sum_{r=1}^{m}\tau_r \cos(\lambda r)\right), \tag{7}$$

where σ^2 is the variance of the error term, and m is the number of parameters required to describe the short run dynamics of the series. Its main advantage is that it mimics the behaviour of ARMA (AutoRegressive Moving Average) structures with a small number of parameters. Moreover, it works extremely well in the context of the LM tests of Robinson (1994) (Gil-Alana, 2004).

Given the above model, we consider the three standard cases examined in the literature, i.e., the case of no regressors, i.e. $\alpha = \beta = 0$ in (6), an intercept (α unknown and $\beta = 0$) and an intercept with a linear time trend (α and β unknown in (6)). The t-values (not reported) indicate that a time trend is not required, an intercept being sufficient to describe the deterministic part of the process in all cases.

[Insert Table 1 about here]

We report the estimates of d along with the 95% confidence band of the non-rejection values of d using Robinson' s (1994) parametric approach. Starting with the case of white noise disturbances. It can be seen that the three estimates of d are above 1 and the unit root null hypothesis is rejected in favour of higher degrees of integration for the UK and Japan but not for the US. However, when using the semiparametric method of Robinson (1995) the three estimates are below 1 and the unit root cannot be rejected for any of the three series. Finally, when using the nonparametric approach of Bloomfield (1973) the unit root null cannot be rejected for the UK and Japan, but it is rejected in favour of mean reversion ($d < 1$) in the US case. The results for the three specifications are consistent in the sense the highest degrees of integration (and thus of dependence) are found for the UK, followed by Japan, with the US exhibiting the lowest degrees of integration.

The multivariate model we consider is the following:

$$\begin{pmatrix} (1-L)^{d_{UK}} & 0 & 0 \\ 0 & (1-L)^{d_{US}} & 0 \\ 0 & 0 & (1-L)^{d_{JP}} \end{pmatrix} \begin{pmatrix} y_{UK,t} \\ y_{US,t} \\ y_{JAP,t} \end{pmatrix} = \begin{pmatrix} u_{UK,t} \\ u_{US,t} \\ u_{JAP,t} \end{pmatrix}, \quad t = 1, 2, \ldots$$

where y_t is now a (3x1) vector with the unemployment series, and u_t is a VAR process of the form:

$$\left(I - F_p(L)\right) u_t = \varepsilon_t , \qquad t = 1, 2, \ldots$$

with ε_t as a white noise vector process, with variance covariance matrix

$$V(\varepsilon_t) = \Lambda \Lambda^T , \quad \Lambda = \begin{pmatrix} \omega_{11} & 0 & 0 \\ \omega_{21} & \omega_{22} & 0 \\ \omega_{21} & \omega_{22} & \omega_{23} \end{pmatrix}$$

and p = 4 according to the Akaike Information Criterion.

[Insert Table 2 about here]

The estimated fractional differencing parameter is now very different in the UK and Japan compared to the US case. Specifically, it is equal to 0.615 for the UK, 0.568 for Japan, and 0.086 for the US (see Table 2). The unit root null is rejected in favour of mean reversion (d < 1) in the UK and the US at the 5% significance level and in Japan at the 10% level. On the other hand, the I(0) null (i.e., d = 0) cannot be rejected for the UK and Japan at the 5% significance level and for the US at any level.

It is noteworthy that the results differ substantially from those obtained with the univariate methods. Whilst in the univariate case the unit root null hypothesis could not be rejected for any of the three series, it is decisively rejected in the multivariate contect in favour of mean reversion. Moreover, the NAIRU hypothesis (d = 0), which was clearly rejected in the univariate models, cannot be rejected in the case of the US in the multivariate context. Since the multivariate model allows for correlations between the unemployment series, which are neglected in the univariate approach, more weight should be given to the multivariate results supporting the NAIRU hypothesis for the US and the "hysteresis" view for the UK and Japan, with a higher degree of persistence in the unemployment rate in the UK than in Japan.[1]

5. Conclusions

This study revisits the issue of the degree of dependence in the unemployment series with the aim of discriminating between alternative unemployment theories. Specifically, it carries out both a univariate and multivariate analysis of the long memory properties of the unemployment series in the UK, the US and Japan. The latter type of framework has the advantage of allowing for possible cross-country correlations overlooked in previous empirical studies. The results are indeed very different depending on whether a univariate or multivariate approach is taken, showing the importance of modelling cross-country correlations to draw valid inference.

The main findings can be summarised as follows. When taking a univariate approach, the unit root null cannot be rejected in case of the UK and Japanese unemployment series, and some degree of mean reversion ($d < 1$) is found in the case of the US unemployment rate. When applying multivariate methods instead, higher orders of integration are still found for the UK and Japanese series, but the NAIRU hypothesis cannot be rejected in the case of the US.

[1] This ranking of persistence is consistent with the univariate results: the UK displays the highest degree of dependence, followed by Japan and the US.

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Figure 1. Harmonized Unemployment Rate: UK, US, Japan, %

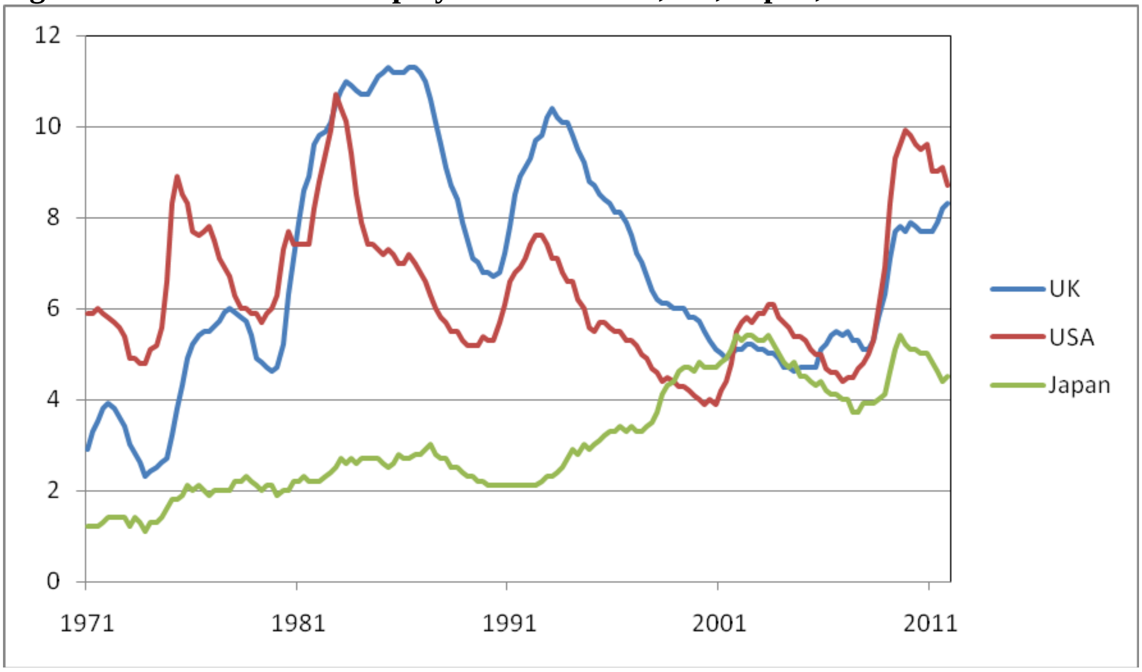

Table 1: Estimates of d (and 95% confidence intervals) from the univariate approach

	Parametric Robinson (1994)	Semiparametric Robinson (1995)	Nonparametric Bloomfield (1973)
UNITED KINGDOM	1.457 (1.354, 1.588)	0.954 (0.771, 1.228)	1.081 (0.667, 1.433)
JAPAN	1.126 (1.019, 1.266)	0.894 (0.771, 1.228)	0.862 (0.534, 1.346)
UNITED STATES	1.052 (0.944, 1.299)	0.780 (0.771, 1.228)	0.683 (0.298, 0.993)

Table 2: Estimated coefficients in the multivariate model

			UNITED KINGDOM			JAPAN			UNITED STATES		
Estimates of d			d_{UK} = 0.615 (0.171)			d_{JAP} = 0.568 (0.307)			d_{US} = 0.086 (0.221)		
F_1			F_2			F_3			F_4		
0.836 (0.168)	0.311 (0.068)	-0.123 (0.050)	0.206 (0.135)	-0.251 (0.132)	-0.048 (0.066)	-0.033 (0.168)	-0.092 (0.126)	0.068 (0.064)	-0.282 (0.088)	0.109 (0.071)	0.020 (0.052)
-0.177 (0.110)	1.608 (0.232)	-0.125 (0.065)	0.139 (0.148)	-0.605 (0.279)	0.114 (0.082)	0.170 (0.146)	-0.020 (0.157)	-0.081 (0.068)	-0.168 (0.108)	0.004 (0.094)	-0.061 (0.085)
0.056 (0.141)	0.021 (0.112)	0.352 (0.300)	0.065 (0.193)	0.117 (0.200)	0.205 (0.095)	0.090 (0.190)	-0.143 (0.213)	0.284 (0.089)	-0.248 (0.152)	0.010 (0.139)	-0.083 (0.095)
Variance – Covariance matrix of the estimated residuals: $V(\varepsilon_t) = \Lambda \Lambda^T$											
Λ			ω_{11} = 0.553 (0.031)								
			ω_{21} = 0.257 (0.053)			ω_{22} = 0.649 (0.036)					
			ω_{31} = 0.260 (0.071)			ω_{32} = 0.157 (0.068)			Ω_{33} = 0.857 (0.048)		

Fig. 2 Fitting periodogram of the three series and the estimated spectra (UK,US and Japan)