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and Strong Dependence:
An Empirical Study**

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**AFRICAN GROWTH, NON-LINEARITIES AND STRONG DEPENDENCE:
AN EMPIRICAL STUDY**

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Abstract

This paper applies a fractional integration approach incorporating Chebyshev polynomials in time to allow for possible non-linearities in GDP growth. This is particularly appropriate in the case of African countries, where growth has been affected by various conflicts. The results for a sample of 28 countries confirm the existence of non-linearities in most cases, the only exceptions being the Central African Republic, Niger, Sierra Leone and Somalia. Further, there is heterogeneity across countries in terms of the degree of persistence, the GDP series being characterised in different cases by mean reversion, unit root behaviour, and orders of integration significantly higher than 1 respectively.

Keywords: GDP growth, African countries, non-linearities, fractional integration, Chebyshev polynomials

JEL Classification: C22, C50

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1. Introduction

This paper examines the statistical properties of the growth rates of several African countries using statistical techniques based on the concepts of fractional integration and long-range dependence. It is normally assumed that GDP (and/or its log transformation) is a non-stationary, integrated of order 1 (or I(1)), series and its first difference, i.e. the growth rate, a stationary I(0) one (Nelson and Plosser, 1982). However, this is a rather restrictive assumption: the possibility of fractional degrees of integration has more recently been taken into account in several studies on GDP growth (Michelacci and Zaffaroni, 2000; Silverberg and Verspagen, 2000; Gil-Alana, 2003, 2008; Mayoral, 2006; Caporale and Gil-Alana, 2013; etc.). For instance, Michelacci and Zaffaroni (2000) provided evidence of long memory and mean-reverting behaviour in US per capita output. Their paper, however, was criticised by Silverberg and Verspagen (2000), who questioned its methodology and reported I(1) nonstationary behaviour in US output. Mayoral (2006) examined annual real GNP and GNP per capita in the US for the time period 1869-2001, using several parametric and semiparametric fractional integration methods. Her results, though slightly different depending on the technique used, suggested that the orders of integration lie in the interval $[0.5, 1)$, which implies nonstationarity, high persistence and mean-reverting behaviour. Caporale and Gil-Alana (2013) showed that the behaviour of US per capita real output is captured well by a linear trend model with stationary long-memory behaviour and breaks, and that mean reversion occurs.

It is well known that fractional integration, non-linearities and structural breaks are intimately related issues (Cheung, 1993; Diebold and Inoue, 2001; Giraitis et al., 2001; Kapetanios and Shin, 2003; Mikosch and Starica, 2004; Granger and Hyung, 2004; etc.). In particular, fractional integration can be an artifact generated by the

presence of breaks that are not taken into account. Further, changes can occur smoothly rather than suddenly as implied by structural breaks; Ouliaris et al. (1989) therefore proposed regular polynomials to approximate deterministic components in the data generation process (DGP). However, as later pointed out by Bierens (1997), Chebyshev polynomials might be a better mathematical approximation of the time functions, since they are bounded and orthogonal; being cosine functions of time, they are a very flexible tool to approximate deterministic trends.

In the specific case of the African countries, growth rates might be not only persistent, but also subject to non-linearities resulting from civil wars, ethnic conflicts etc. Therefore the present study adopts a GDP growth model incorporating both features (non-linearities and persistence) in a single framework.

The remainder of the paper is structured as follows. Section 2 briefly reviews the previous literature on economic growth in Africa. Section 3 outlines the methodology. Section 4 describes the data and discusses the empirical results. Section 5 concludes the paper.

2. Literature review

Relatively few studies have focused on economic growth in Africa. In the paper by Fosu (1992a), who used data from 1956 to 1985 for 31 sub-Saharan countries, the country-specific analysis is complemented by an investigation of the extent to which growth differentials between countries can be explained by differences in production output. Political instability and corruption are found to have adverse effects on growth and to have played a major role in the economic stagnation of sub-Saharan Africa, accounting for a substantial reduction in the region's overall GDP growth over the period 1956-1985.

Fosu (1992b) investigated the effect of export instability on GDP growth in Africa, and found that these are particularly significant in the case of sub-Saharan Africa. Karikari (1995) examined the role of the government in the growth of a developing nation, using data for Ghana from 1963 to 1984. He concluded that the impact of government on economic growth was negative. Savvides (1995) investigated the factors that explain the differences in per capita growth across Africa, and concluded that these are: initial conditions, investment, economic growth, trade orientation, inflation, financial development and the growth of the government sector. Easterly and Levice (1997) showed that ethnic diversity helps explain cross-country differences in public policies and other economic indicators. Sub-Saharan economic growth is associated with low schooling, political instability, an underdeveloped financial system, distorted foreign exchange markets, high government deficits and insufficient infrastructures.

Guillaumont et al. (1999) showed, using a cross-section including a sample of African and non-African countries, that instability lowered African growth in the seventies and eighties. They concluded that Africa has a higher level of primary instability (climatic, terms of trade and political instability) which lowers growth. Brempong and Traynor (1999) also found an inverse relationship between political instability and economic growth (as well as joint endogeneity of these two variables), and an indirect effect of political instability on economic growth through lower long-run capital accumulation. Gomanee et al. (2005) found a positive relationship between foreign aid and growth in a sample of 25 sub-Saharan countries: on average, a percentage point increase in the foreign aid/GNP ratio contributes one-quarter of a percentage point to the growth rate. Aghion et al. (2008) showed that mark-ups are higher in South Africa manufacturing industries than in corresponding industries

worldwide, which has a large negative effect on productivity growth in the South African manufacturing industry.

3. Methodology

As a first step, we carry out standard unit root tests, specifically the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979), as well as its generalization, i.e. the GLS specification (Elliot et al., ERS, 1996), and the Kwiatkowski et al. (KPSS, 1992) test for the null of stationarity against the alternative of a unit root.

We then consider the following non-linear model:

$$y_t = \sum_{i=0}^m \theta_i P_{iT}(t) + x_t, \quad t = 1, 2, \dots, \quad (1)$$

with m indicating the order of the Chebyshev polynomial, and x_t following an I(d) process of the form

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \dots, \quad (2)$$

with $x_t = 0$ for $t \leq 0$, and $d > 0$, where L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is I(0).

The Chebyshev polynomials $P_{iT}(t)$ in equation (1) are defined as:

$$P_{0,T}(t) = 1, \\ P_{i,T}(t) = \sqrt{2} \cos(i\pi(t-0.5)/T), \quad t = 1, 2, \dots, T; \quad i = 1, 2, \dots \quad (3)$$

(see Hamming (1973) and Smyth (1998) for a detailed description of these polynomials). Bierens (1997) uses them in the context of unit root testing. According to Bierens (1997) and Tomasevic and Stanivuk (2009), it is possible to approximate highly non-linear trends with rather low degree polynomials. If $m = 0$ the model contains an intercept, if $m = 1$ it also includes a linear trend, and if $m > 1$ it becomes non-linear - the higher m is the less linear the approximated deterministic component becomes.

An issue that immediately arises here is how to determine the optimal value of m . As argued in Cuestas and Gil-Alana (2012), if one combines (1) and (2) in a single equation, standard t -statistics will remain valid with the error term being $I(0)$ by definition. The choice of m will then depend on the significance of the Chebyshev coefficients. Note that the model combining (1) and (2) becomes linear and d can be estimated parametrically or tested as in Robinson (1994), Demetrescu, Kuzin and Hassler (2008) and others (see Cuestas and Gil-Alana, 2012).

The method proposed here is a slight modification of Robinson's (1994). He considers the same set-up as in (1) and (2) with the first component in the right hand side in (1) replaced by $\theta' z_t$, and testing the null hypothesis:

$$H_o : d = d_o, \quad (4)$$

for any real vector value d_o . Under H_o (4), the model in Robinson (1994) becomes:

$$y_t^* = \theta' z_t^* + u_t, \quad t = 1, 2, \dots, \quad (5)$$

where $y_t^* = (1-L)^{d_o} y_t$, $z_t^* = (1-L)^{d_o} z_t$, and the symbol ' indicating transposition.

Then, given the linear structure of the above relationship and the $I(0)$ nature of the error term u_t , the coefficients in (5) can be estimated by standard ordinary least square/generalized least square (OLS/GLS) methods.¹ The same applies in our case, with (1) containing the Chebyshev polynomials: despite the non-linear structure, the relationship is linear in the parameters. Thus, combining equations (1) and (2) we obtain

$$y_t^* = \theta' P_T^*(t) = \sum_{i=0}^m \theta_i P_{iT}^*(t) + u_t, \quad t = 1, 2, \dots, \quad (6)$$

where

$$P_{iT}^*(t) = \rho(L; d_o) P_{iT}(t),$$

¹ Although Robinson (1994) focuses exclusively on the linear case, he argues (p. 1421) that “ (...) undoubtedly a non-linear regression will also leave our limit distributions unchanged, under standard regularity conditions.”. These conditions can be found in Robinson (1994).

which can also be expressed as in Robinson (1994) ($P_T^*(t) = z_t^*$), and then, using OLS/GLS methods, under the null hypothesis (4), the residuals are

$$\hat{u}_t = y_t^* - \sum_{i=0}^m \hat{\theta}_i P_{iT}^*(t);$$

with

$$\hat{\theta} = \left(\sum_{t=1}^T P_T^*(t) P_T^*(t)' \right)^{-1} \left(\sum_{t=1}^T P_T^*(t) y_t^* \right),$$

and $P_T^*(t)$ as the $(m \times 1)$ vector of transformed Chebyshev polynomials. Using the above residuals \hat{u}_t , we estimate the variance,

$$\hat{\sigma}^2(\tau) = \frac{2\pi}{T} \sum_{j=1}^T g(\lambda_j; \hat{\tau})^{-1} I_{\hat{u}}(\lambda_j); \quad \lambda_j = 2\pi j/T, \quad (7)$$

where $I_{\hat{u}}(\lambda_j)$ is the periodogram of \hat{u}_t ; g is a function related to the spectral density of u_t (i.e., $s.d.f.(u_t) = (\sigma^2/2\pi)g(\lambda_j; \tau)$); and the nuisance parameter τ is estimated, for example, by $\hat{\tau} = \arg \min_{\tau \in T^*} \sigma^2(\tau)$, where T^* is a suitable subset of the R^q Euclidean space.²

The test statistic (based on Robinson (1994)) for testing H_o (4) in (1) and (2) uses the Lagrange Multiplier (LM) principle, and is given by

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a}, \quad (8)$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I_{\hat{u}}(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left(\sum_j^* \psi(\lambda_j) \psi(\lambda_j)' - \sum_j^* \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \left(\sum_j^* \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right) \sum_j^* \hat{\varepsilon}(\lambda_j) \psi(\lambda_j)' \right)$$

² Alternative methods for estimating the variance, e.g., non-parametric ones, could also be used. Here we take the same approach as in Robinson (1994).

with

$$\psi(\lambda_j) = \operatorname{Re} \left(\frac{\partial}{\partial d} \log \rho(e^{i\lambda_j}; d) \right), \quad \text{and} \quad \hat{\varepsilon}(\lambda_j) = \left. \frac{\partial}{\partial \tau} \log g(\lambda_j; \tau) \right|_{\tau = \hat{\tau}},$$

and the sum over * above refers to all the bounded discrete frequencies in the spectrum.

Under very mild regularity conditions,³ it can be shown that, as in Robinson (1994)

$$\hat{R} \rightarrow_d \chi_1^2 \quad \text{as} \quad T \rightarrow \infty, \quad (9)$$

and, based on Gaussianity of u_t , one can also show the Pitman efficiency of the test against local departures from the null. In other words, if one considers local alternatives of the form:

$$H_a: d = d_o + \delta T^{-1/2},$$

where δ is a non-null parameter vector, $\hat{R} \rightarrow_d \chi_1^2(\Lambda)$ as $T \rightarrow \infty$, indicating a non-central chi-squared distribution with a non-centrality parameter which is optimal under Gaussianity of u_t . Note that this method is a testing procedure and therefore we do not directly estimate the fractional differencing parameter vector but simply present confidence intervals based on the non-rejections for a given set of values. However, we display estimates of d , based on the values minimizing the absolute value of the test statistic. Monte Carlo evidence suggests that this approach performs well (see Cuestas and Gil-Alana, 2012).

4. Empirical results

We use data on real GDP per capita in 28 African countries at 2005 constant prices. The source is the Penn World Table.

[Insert Table 1 about here]

³ These conditions only include moments up to a second order.

Table 1 provides a list of the countries with the corresponding sample periods, the longest being those starting in 1950 for the Congo Democratic Republic, Ethiopia, Morocco, Nigeria, South Africa and Uganda. The start date is 1954 for Zimbabwe, 1955 for Zambia and Ghana, 1960 for Algeria, Botswana, Burundi, Central African Rep., Chad, Congo Republic, Cape Verde, Equatorial Guinea, Gambia, Guinea Bissau, Mali, Mauritania, Mozambique, Namibia and Niger, 1961 for Sierra Leone and Tunisia, 1970 for Angola and Somalia. The end date is 2010 in all cases.

[Insert Tables 2 and 3 about here]

The unit root test results (ADF, KPSS and ERS) reported in Tables 2 (in levels) and 3 (in first differences) suggest that the level series are I(1), whilst the GDP growth rates are I(0) in all cases. However, it is well known that such tests have very low power if the DGP is characterised by fractionally integration (see, Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996; and more recently Ben Nasr *et al.*, 2014); on the other hand, fractional integration may be a spurious phenomenon caused by the presence of non-linearities and structural breaks in the data that have not been taken into account.⁴ For these reasons, next we allow for non-linear trends in the context of fractional integration, and consider the following model,

$$y_t = \sum_{i=0}^m \theta_i P_{iT}(t) + x_t, \quad (1 - L)^d x_t = u_t, \quad (10)$$

assuming that u_t is a white noise process. Allowing for autoregressive behaviour in the error term u_t in (10) produced coefficients close to 0 in all cases. We also performed a LR test that strongly supports the white noise specification for all the series examined.

First we assume that $m = 3$ to allow for a high degree of non-linear behaviour. Table 4 displays in the second column the estimates of d along with their corresponding

⁴ This point has been made by several authors including Bhattacharya *et al.* (1983), Teverovsky and Taquu (1997), Smith (2005), Ohanissian *et al.* (2008), Perron and Qu (2010), etc.

95% confidence bands showing the values of d for which the null hypothesis (4) cannot be rejected. The remaining columns display the estimated coefficients along with their corresponding t -values.

[Insert Table 4 about here]

For the Central African Republic, Niger, Sierra Leone and Somalia there is no evidence of non-linearities, since the two coefficients on the non-linear terms (i.e., θ_2 and θ_3) are statistically insignificant. Further, the order of integration varies considerably across these countries: for the Central African Republic and Somalia, the estimated value of d is significantly smaller than 1 (0.37 and 0.49 respectively), which implies in both cases mean-reverting behaviour; for Niger the estimate of d is below 1, but the unit root null hypothesis cannot be rejected; and for Sierra Leone the estimated value of d is 1.32 and the hypothesis of $d = 1$ is decisively rejected in favour of $d > 1$.

The countries exhibiting a large degree of non-linearity are those for which all four coefficients are statistically significant, namely Cabo Verde, Equatorial Guinea, Gambia, Mauritania, Mozambique and Uganda. In four of them (Cabo Verde, Equatorial Guinea, Mozambique and Uganda) the unit root null (i.e., $d = 1$) cannot be rejected, while for the remaining two (Gambia and Mauritania) the null of mean reversion (i.e., $d < 1$) cannot be rejected.

In between, there are some cases with at least one of the two non-linear coefficients being statistically significant. Specifically, a significant θ_3 is found for Algeria, Ethiopia, Gambia, Morocco, Nigeria, Namibia, South Africa, Tunisia and Zambia, and a significant θ_2 -coefficient for Botswana, Burundi, Chad, Congo Democratic Republic, Congo Republic, Guinea Bissau and Mali. For this group of countries, mean reversion ($d < 1$) is found in Algeria, Botswana, Guinea Bissau, Malia, Namibia and Tunisia, whilst the unit root null cannot be rejected in Angola, Burundi,

Chad, Congo Democratic Republic, Congo, Ethiopia, Ghana, Morocco, Nigeria, South Africa, Zambia and Zimbabwe. Therefore, we can conclude by saying that there is some evidence of non-linearity in all except the above mentioned four countries (Central African Republic, Niger, Sierra Leone and Somalia).

[Insert Tables 5 and 6 about here]

Tables 5 and 6 display the results for $m = 2$ and $m = 1$ respectively. They are completely in line with those reported above for the case of $m = 3$. Table 7 reports the selected model for each country. In fourteen countries the specification with $m = 3$ is found to be the most appropriate - these are Algeria, Cabo Verde, Equatorial Guinea, Ethiopia, Gambia, Ghana, Mauritania, Mozambique, Namibia, Nigeria, South Africa, Tunisia, Uganda and Zambia. For another group of countries, including Angola, Botswana, Burundi, Chad, Congo Democratic Republic, Congo Republic, Guinea Bissau, Mali, Morocco and Zimbabwe, the best model is the one with $m = 2$; for Somalia, the specification includes a linear time trend, and finally, for Central African, Niger and Sierra Leone it only includes an intercept. Mean reversion is only found for the following countries: Central African Republic, Gambia, Mali, Mauritania and Somalia, and orders of integration significantly above 1 are estimated only for Angola and Sierra Leone. For the remaining countries, the unit root null cannot be rejected.

5. Conclusions

This paper applies a fractional integration approach incorporating Chebyshev polynomials to allow for possible non-linearities in GDP per capita. This is particularly appropriate in the case of African countries, where growth has been affected by various conflicts. The results for a sample of 28 countries confirm the existence of non-linearities in most cases, the only exceptions being the Central African Republic, Niger,

Sierra Leone and Somalia. For the remaining countries strong evidence of non-linearities is obtained for Cabo Verde, Equatorial Guinea, Gambia, Mauritania, Mozambique and Uganda, followed by Algeria, Ethiopia, Gambia, Morocco, Nigeria, Namibia, South Africa, Tunisia and Zambia (where θ_3 is statistically significant), and for Botswana, Burundi, Chad, Congo Democratic Republic, Congo Republic, Guinea Bissau and Mali (with a significant θ_2 -coefficient).

Heterogeneity across countries is another feature of our results, mean-reversion, unit root behaviour and orders of integration significantly higher than 1 being found in different cases. Overall, the evidence presented in this study confirms the importance of taking into account non-linearities when modelling GDP per capita in countries such as the African ones where various types of conflicts have disrupted economic growth at different stages.

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Table 1: List of countries and sample size

Country	Starting date	Ending date	No. of observations
Algeria	1960	2010	51
Angola	1970	2010	41
Botswana	1960	2010	51
Burundi	1960	2010	51
Cape Verde	1960	2010	51
Central African Rep.	1960	2010	51
Chad	1960	2010	51
Congo Dem. Rep.	1950	2010	61
Congo Rep.	1960	2010	51
Equatorial Guinea	1960	2010	51
Ethiopia	1950	2010	61
Gambia	1960	2010	51
Ghana	1955	2010	56
Guinea Bissau	1960	2010	51
Mali	1960	2010	51
Mauritania	1960	2010	51
Morocco	1950	2010	61
Mozambique	1960	2010	51
Namibia	1960	2010	51
Niger	1960	2010	51
Nigeria	1950	2010	61
South Africa	1950	2010	61
Sierra Leone	1961	2010	50
Somalia	1970	2010	41
Tunisia	1961	2010	50
Uganda	1950	2010	61
Zambia	1955	2010	56
Zimbabwe	1954	2010	57

Table 2: Unit root test results (levels)

Country	ADF		KPSS		ERS	
	Intercept	T. trend	Intercept	T. trend	Intercept	T. trend
Algeria	-0.629196	-2.961852	0.783198***	0.102005	14.24983	11.32340
Angola	1.836251	0.518612	0.399583*	0.187202**	32.79653	66.76069
Botswana	-0.107168	-3.055937	0.933389***	0.179059**	153.4363	12.79852
Burundi	-1.993673	-1.678875	0.240580	0.220282***	13.50598	41.10326
Cape Verde	4.246626	0.823215	0.855589***	0.227643***	250.1908	105.5559
Central African Rep.	-0.949002	-3.270792*	0.913144***	0.089585	33.80204	8.450531
Chad	-0.096397	-0.813696	0.359030*	0.193348**	11.78548	25.67639
Congo Dem. Rep.	-0.272646	-3.218803*	0.783635***	0.202449**	20.92074	47.22060
Congo Rep.	-1.653857	-1.378591	0.650773**	0.201905**	37.91988	25.24673
Equatorial Guinea	2.389068	-3.028438	0.609258**	0.202397**	1.184983***	0.172974***
Ethiopia	0.341809	-0.208754	0.422679*	0.134162*	35.60427	27.64280
Gambia	-2.428238	-2.430300	0.191464	0.185197**	4.997211	12.26448
Ghana	0.403107	-0.296392	0.414326*	0.174116**	20.58400	23.93080
Guinea Bissau	-2.080610	-2.322633	0.221729	0.173441**	5.749240	16.62405
Mali	1.001084	-2.837289	0.892943***	0.213381**	46.06339	30.07915
Mauritania	-2.383145	-2.531172	0.655820**	0.171435**	43.97707	21.38731
Morocco	1.120514	-1.826865	0.951615***	0.080592	145.5967	17.23232
Mozambique	3.378613	1.442896	0.562350**	0.195555**	114.5698	116.0452
Namibia	-1.669098	-1.843135	0.300645	0.127344*	22.41283	20.02045
Niger	-0.780237	-2.410178	0.860278***	0.139081*	26.41376	15.52441
Nigeria	-2.166709	-2.142204	0.113443	0.116760	4.180557	9.872862
South Africa	-0.287500	-1.418518	0.828434***	0.122150*	86.17429	16.62109
Sierra Leone	-1.943010	-1.946805	0.155448	0.143902*	10.34364	14.31889
Somalia	-1.055053	-3.359393*	0.715713**	0.067251	14.63175	9.032007
Tunisia	-0.370069	-2.266093	0.917923***	0.080777	276.6127	9.951711
Uganda	0.149478	-0.548710	0.308285	0.181477**	17.70829	39.34480
Zambia	-1.027385	-0.834667	0.399372*	0.137932*	9.303952	30.47720
Zimbabwe	-1.933025	-1.447110	0.381560*	0.381560***	13.87958	22.59612

*Rejection at the 10%; **Rejection at the 5%; ***Rejection at the 1%

Table 3: Unit root test results (first differences)

Country	ADF		KPSS		ERS	
	Intercept	T. trend	Intercept	T. trend	Intercept	T. trend
Algeria	-8.031377***	-8.035174***	0.136037	0.086206	1.261192***	3.817498***
Angola	-4.265845***	-5.075182***	0.485975**	0.110410	1.388506***	5.313196**
Botswana	-7.102229***	-7.040803***	0.144804	0.128882*	1.075163***	3.638831***
Burundi	-6.098586***	-6.566598***	0.277927	0.088027	3.828045*	7.697453
Cape Verde	-4.522635***	-6.154891***	0.678958**	0.112596	1.468414***	4.822752**
Central African Rep.	-7.091284***	-7.016827***	0.083676	0.081268	1.295454***	3.169795***
Chad	-5.484588***	-5.673970***	0.271841	0.062483	1.044450***	3.716404***
Congo Dem. Rep.	-7.543291***	-7.609963***	0.278939	0.112461	2.999226*	4.710365**
Congo Rep.	-5.376108***	-5.349663***	0.146577	0.058986	1.315274***	4.093757***
Equatorial Guinea	-1.562729	-3.101588	0.293199	0.076178	6.292617	5.687500**
Ethiopia	-7.915775***	-8.073409***	0.269474	0.224027***	1.423644***	4.164478***
Gambia	-7.251590***	-7.230887***	0.095739	0.075360	1.912782**	4.597054**
Ghana	-6.770594***	-7.040604***	0.314692	0.149621**	1.416303***	4.426892**
Guinea Bissau	-8.051499***	-8.239527***	0.151576	0.050941	1.280371***	4.316129**
Mali	-6.051802***	-6.997345***	0.455721*	0.103298	0.984308***	2.057099***
Mauritania	-8.772692***	-9.003160***	0.248982	0.139610*	1.586479***	4.669348**
Morocco	-8.754953***	-9.080136***	0.300751	0.097564	0.995380***	3.117883***
Mozambique	-4.392688***	-5.133809***	0.536365**	0.196035**	1.450947***	3.902841***
Namibia	-7.529205***	-7.451359***	0.154072	0.154333**	1.084579***	3.706657***
Niger	-7.097787***	-7.016457***	0.115738	0.117548	1.559816***	4.227351**
Nigeria	-5.467455***	-5.421933***	0.103756	0.103436	0.974573***	3.401545***
South Africa	-5.349220***	-5.319508***	0.168457	0.166160**	1.016475***	3.446017***
Sierra Leone	-4.252713***	-4.137187***	0.173099	0.142885*	2.780876**	6.506296*
Somalia	-7.193110***	-7.089378***	0.087792	0.086363	1.669479***	5.014989**
Tunisia	-7.846147***	-7.759871***	0.051523	0.051546	1.033970***	3.767401***
Uganda	-5.542541***	-6.082612***	0.377880*	0.167617**	3.152519*	6.618272*
Zambia	-6.098201***	-6.105915***	0.205915	0.182011**	1.398511***	4.191256***
Zimbabwe	-8.286360***	-8.571890***	0.286539	0.045342	0.908183***	3.453683***

*Rejection at the 10%; **Rejection at the 5%; ***Rejection at the 1%

Table 4: Estimated coefficients in a model with $m = 3$

Country	d (95 interval)	θ_0	θ_1	θ_2	θ_3
Angola	1.16 (0.93, 1.45)	2355.81 (1.87)	-315.19 (-0.41)	555.24 (1.73)	-240.75 (-1.20)
Algeria	0.60 (0.33, 0.93)	4991.13 (16.39)	-663.64 (-3.82)	-53.06 (0.41)	-265.79 (-2.57)
Botswana	0.56 (0.21, 0.98)	4882.13 (14.67)	-3103.37 (-16.13)	306.28 (2.07)	-192.36 (-1.60)
Burundi	0.88 (0.61, 1.25)	468.54 (7.56)	-13.64 (-0.38)	-75.24 (-3.72)	-0.67 (-0.04)
Central African Rep.	0.37 (0.11, 0.72)	760.41 (43.43)	-163.42 (-14.19)	-1.65 (-0.16)	-7.19 (-0.83)
Chad	0.97 (0.65, 1.40)	838.6281 (3.41)	-78.58 (-0.55)	122.42 (1.65)	-62.39 (-1.24)
Congo Dem. Rep.	0.93 (0.67, 1.19)	421.55 (2.74)	185.01 (-2.08)	-90.63 (-1.88)	-16.68 (-0.50)
Congo Rep.	1.03 (0.68, 1.44)	1963.07 (3.48)	-376.50 (-1.13)	-281.73 (-1.75)	-24.47 (-0.23)
Cabo Verde	1.16 (0.96, 1.39)	1691.49 (2.95)	-669.56 (-1.90)	257.05 (1.75)	-179.91 (-1.96)
Equatorial Guinea	1.06 (0.81, 1.34)	4750.37 (1.67)	-3744.48 (-2.16)	2790.42 (3.45)	-1951.67 (-3.71)
Ethiopia	0.98 (0.78, 1.17)	409.13 (4.23)	-33.71 (-0.59)	-2.22 (-0.07)	-50.02 (-2.56)
Gambia	0.51 (0.11, 0.98)	1229.70 (28.12)	4.27 (0.16)	-58.52 (-2.84)	-42.83 (-2.50)
Ghana	0.88 (0.54, 1.21)	1365.27 (6.37)	-114.28 (-0.93)	100.80 (1.43)	138.54 (-2.81)
Guinea Bissau	0.70 (0.49, 0.97)	915.37 (8.05)	21.18 (0.33)	-100.56 (-2.30)	-12.68 (-0.38)
Mali	0.69 (0.47, 0.97)	670.18 (14.78)	-150.70 (-5.94)	46.81 (2.66)	-10.94 (-0.81)
Mauritania	0.53 (0.26, 0.82)	1454.17 (16.36)	-252.74 (-4.74)	-128.31 (-3.14)	-178.55 (-5.30)
Morocco	0.92 (0.77, 1.11)	2085.21 (6.36)	-774.31 (-4.09)	15.95 (0.15)	-136.88 (-1.91)
Mozambique	1.01 (0.77, 1.28)	422.38 (6.40)	-80.86 (-2.06)	64.80 (3.38)	-65.28 (-5.12)
Namibia	0.52 (0.15, 0.90)	3737.79 (23.97)	-223.94 (-2.43)	51.13 (0.70)	-400.10 (-6.67)
Niger	0.70 (0.28, 1.10)	660.64 (10.24)	171.06 (4.74)	26.69 (1.07)	-26.46 (-1.39)
Nigeria	1.09 (0.81, 1.50)	1408.27 (2.38)	21.07 (0.05)	-22.01 (-0.13)	-201.32 (-1.94)
South Africa	1.12 (0.93, 1.40)	5329.23 (6.36)	-856.77 (-1.68)	-161.35 (-0.72)	-432.70 (-3.06)
Sierra Leone	1.32 (1.07, 1.67)	352.28 (0.66)	168.06 (0.49)	-109.71 (-0.92)	-21.50 (-0.30)
Somalia	0.49 (0.17, 0.90)	606.98 (17.37)	125.96 (6.01)	1.22 (0.07)	-5.35 (-0.38)

Tunisia	0.58 (0.29, 0.95)	3940.28 (32.34)	-1229.80 (-17.69)	24.94 (0.47)	-259.58 (-6.08)
Uganda	0.94 (0.69, 1.31)	746.92 (7.54)	-55.40 (-0.96)	88.52 (2.87)	-94.87 (-4.50)
Zambia	0.87 (0.59, 1.23)	1167.55 (4.82)	162.23 (1.17)	74.11 (0.92)	-201.23 (-3.57)
Zimbabwe	0.81 (0.47, 1.19)	1.380 (5.45)	0.271 (1.90)	-0.138 (-1.67)	0.075 (1.17)

In bold, significant coefficients according to the t-values at the 5% level.

Table 5: Estimated coefficients in a model with $m = 2$

Country	d (95% interval)	θ_0	θ_1	θ_2
Angola	1.19 (1.02, 1.44)	1819.25 (1.87)	-174.43 (-0.20)	544.00 (1.77)
Algeria	0.77 (0.58, 1.02)	5032.94 (9.43)	-740.27 (-2.44)	-68.93 (-0.35)
Botswana	0.66 (0.40, 1.02)	4776.53 (10.34)	-3160.05 (-12.17)	299.14 (1.79)
Burundi	0.88 (0.61, 1.26)	467.82 (7.78)	-13.76 (-0.39)	-75.24 (-3.72)
Central African Rep.	0.38 (0.11, 0.73)	759.31 (41.76)	161.71 (13.80)	-1.84 (-0.18)
Chad	1.03 (0.78, 1.40)	738.28 (2.56)	-69.34 (-0.39)	120.92 (1.40)
Congo Dem. Rep.	0.94 (0.71, 1.19)	397.13 (2.58)	184.57 (1.99)	-90.42 (-1.82)
Congo Rep.	1.03 (0.68, 1.43)	1926.36 (3.56)	-375.09 (-1.12)	-281.74 (-1.75)
Cabo Verde	1.24 (1.10, 1.42)	1208.17 (1.71)	-485.39 (-1.04)	228.41 (1.27)
Equatorial Guinea	1.28 (1.11, 1.52)	1315.71 (0.22)	-3428.94 (-0.87)	2915.99 (2.01)
Ethiopia	1.08 (0.97, 1.21)	305.87 (2.28)	-10.34 (-0.12)	-5.60 (-0.14)
Gambia	0.70 (0.42, 1.05)	1205.72 (14.60)	-9.40 (-0.20)	-58.69 (-1.81)
Ghana	1.06 (0.89, 1.26)	1094.52 (2.84)	-64.32 (-0.26)	92.91 (0.82)
Guinea Bissau	0.70 (0.49, 0.97)	906.98 (8.11)	17.16 (0.27)	-100.89 (-2.30)
Mali	0.71 (0.51, 0.99)	665.26 (13.96)	-154.24 (-5.75)	46.36 (2.50)
Mauritania	0.86 (0.72, 1.04)	1211.90 (4.34)	-281.78 (-1.73)	-124.78 (-1.88)
Morocco	0.99 (0.86, 1.15)	1903.88 (4.67)	-768.91 (-3.08)	6.76 (2.05)
Mozambique	1.28 (1.17, 1.43)	236.59 (1.69)	-5.20 (-2.04)	54.12 (13.91)
Namibia	0.96 (0.82, 1.15)	3017.15 (4.30)	-223.58 (-0.53)	62.61 (0.28)
Niger	0.79 (0.42, 1.11)	622.62 (7.31)	164.88 (3.39)	27.15 (0.88)
Nigeria	1.19 (0.99, 1.54)	988.58 (1.23)	111.42 (0.21)	-24.06 (-0.11)
South Africa	1.26 (1.13, 1.48)	4216.21 (3.22)	-465.41 (-0.53)	-209.17 (-0.63)
Sierra Leone	1.33 (1.11, 1.75)	284.97 (0.55)	195.31 (-0.57)	-111.81 (-0.91)
Somalia	0.49 (0.16, 0.91)	605.53 (17.40)	124.46 (6.04)	1.05 (0.06)

Tunisia	0.99 (0.85, 1.19)	3507.53 (7.16)	-1230.76 (-4.14)	28.65 (0.19)
Uganda	1.21 (1.07, 1.43)	518.16 (2.09)	4.23 (0.02)	84.58. (1.31)
Zambia	1.09 (0.94, 1.31)	751.44 (1.49)	239.27 (0.75)	70.77 (0.49)
Zimbabwe	0.86 (0.60, 1.22)	1.45 (4.86)	0.28 (1.68)	-0.13 (-1.67)

In bold, significant coefficients according to the t-values at the 5% level.

Table 6: Estimated coefficients in a model with $m = 1$

Country	d (95% interval)	θ_0	θ_1
Angola	1.25 (1.11, 1.47)	2376.49 (1.88)	-18.71 (-0.02)
Algeria	0.77 (0.59, 1.03)	4949.06 (10.36)	-743.33 (-2.45)
Botswana	0.75 (0.56, 1.06)	4102.00 (9.12)	-3137.99 (-8.88)
Burundi	1.14 (0.99, 1.39)	372.66 (2.81)	-14.54 (-0.16)
Central African Rep.	0.37 (0.12, 0.73)	758.57 (44.51)	161.72 (14.15)
Chad	1.10 (0.91, 1.42)	879.58 (2.64)	-48.72 (-0.21)
Congo Dem. Rep.	1.03 (0.87, 1.23)	231.16 (1.21)	201.54 (1.54)
Congo Rep.	1.15 (0.93, 1.50)	1481.14 (1.99)	-350.40 (-0.68)
Cabo Verde	1.27 (1.15, 1.42)	1431.93 (1.94)	-413.11 (-0.80)
Equatorial Guinea	1.37 (1.23, 1.50)	5259.44 (0.69)	-3262.64 (-0.61)
Ethiopia	1.09 (0.97, 1.22)	293.05 (2.31)	-7.21 (-0.08)
Gambia	0.80 (0.61, 1.09)	1126.75 (10.94)	-8.54 (-0.12)
Ghana	1.08 (0.93, 1.27)	1205.25 (3.20)	-50.48 (-0.19)
Guinea Bissau	0.82 (0.67, 1.04)	764.22 (5.09)	18.35 (0.18)
Mali	0.84 (0.70, 1.05)	763.30 (11.17)	-153.92 (-3.60)
Mauritania	0.90 (0.77, 1.07)	1018.59 (3.56)	-271.53 (-1.44)
Morocco	0.99 (0.86, 1.15)	1913.43 (5.21)	-768.91 (-3.08)
Mozambique	1.30 (1.21, 1.44)	295.49 (1.82)	7.96 (0.07)
Namibia	0.96 (0.82, 1.15)	3104.69 (4.93)	-223.46 (-0.53)
Niger	0.83 (0.58, 1.13)	648.59 (7.50)	167.50 (2.99)
Nigeria	1.19 (0.99, 1.54)	955.76 (1.28)	110.32 (0.21)
South Africa	1.27 (1.14, 1.51)	3911.36 (3.05)	-461.12 (-0.51)
Sierra Leone	1.38 (1.16, 1.81)	75.75 (0.13)	228.14 (0.56)
Somalia	0.49 (0.17, 0.92)	606.23 (18.40)	124.54 (6.05)
Tunisia	0.99 (0.85, 1.19)	3547.91 (8.03)	-1230.76 (-4.13)
Uganda	1.24 (1.12, 1.44)	613.50 (2.38)	21.20 (0.11)
Zambia	1.10 (0.95, 1.31)	838.61 (1.75)	248.11 (0.75)
Zimbabwe	0.93 (0.73, 1.24)	1.268 (3.81)	0.284 (1.68)

In bold, significant coefficients according to the t-values at the 5% level.

Table 7: Order of integration of each series according to the selected models

Country	m = 0	m = 1	m = 2	m = 3
Angola	1.25 (1.09, 1.49)	xxx	1.19 (1.02, 1.44)	xxx
Algeria	xxx	0.77 (0.59, 1.03)	xxx	0.60 (0.33, 0.93)
Botswana	xxx	xxx	0.66 (0.40, 1.02)	xxx
Burundi	1.14 (0.99, 1.40)		0.88 (0.61, 1.25)	xxx
Central African	0.37 (0.12, 0.73)	xxx	xxx	xxx
Chad	1.09 (0.89, 1.42)	xxx	0.97 (0.65, 1.40)	xxx
Congo Dem. Rep.	xxx	xxx	0.94 (0.71, 1.19)	xxx
Congo Rep.	1.15 (0.93, 1.49)	xxx	1.03 (0.68, 1.43)	xxx
Cabo Verde	xxx	xxx	xxx	1.16 (0.96, 1.39)
Equatorial Guinea	xxx	xxx	xxx	1.06 (0.81, 1.34)
Ethiopia	1.08 (0.94, 1.24)	xxx	xxx	0.98 (0.78, 1.17)
Gambia	xxx	xxx	xxx	0.51 (0.11, 0.98)
Ghana	1.06 (0.89, 1.29)	xxx	xxx	0.88 (0.54, 1.21)
Guinea Bissau	0.83 (0.68, 1.04)	xxx	0.70 (0.49, 0.97)	xxx
Mali	xxx	xxx	0.71 (0.51, 0.99)	xxx
Mauritania	xxx	xxx	xxx	0.53 (0.26, 0.82)
Morocco	xxx	xxx	0.99 (0.86, 1.15)	xxx
Mozambique	xxx	xxx	xxx	1.01 (0.77, 1.28)
Namibia	0.93 (0.75, 1.14)	xxx	xxx	0.52 (0.15, 0.90)
Niger	0.83 (0.58, 1.13)	xxx	xxx	Xxx
Nigeria	1.19 (1.02, 1.44)	xxx	xxx	1.09 (0.81, 1.50)
South Africa	1.20 (1.00, 1.54)	xxx	xxx	1.12 (0.93, 1.40)
Sierra Leone	1.24 (1.08, 1.50)	xxx	xxx	xxx
Somalia	xxx	0.49 (0.17, 0.92)	xxx	xxx
Tunisia	xxx	0.99 (0.85, 1.19)	xxx	0.58 (0.29, 0.95)
Uganda	xxx	xxx	Xxx	0.94 (0.69, 1.31)
Zambia	1.15 (0.99, 1.38)	xxx	xxx	0.87 (0.59, 1.23)
Zimbabwe	xxx	xxx	0.86 (0.60, 1.22)	xxx