Department of
Economics and Finance

Working Paper No. 15-27

Economics and Finance Working Paper Series

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October 2015

http://www.brunel.ac.uk/economics

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Abstract

This paper estimates and predicts the within-sector failure rate and dependence of public listed firms on the London Stock Exchange. The main novelty of the paper is to offer an additive lognormal frailty model that accounts for both unobserved factors and regime changes. Three main results emerge. First, the frailty factor is always significant across all the model specifications. Second, the adjustment factor in hazard rate during distressed market periods is also significant. Lastly, the additive lognormal frailty model tends to estimate and predict within-sector failure rates and dependencies better than the multiplicative gamma frailty model when moving away from normal market conditions.

Keywords: Additive lognormal frailty; Hazard rates; London stock exchange firms; Within-sector failure rate; Within-sector dependence.

JEL Classification: G33, C51, C41.

1 Introduction

The dynamics of firm failure correlation play a central role in contemporary risk management for corporations, regulators, academics and investors. They offer important information about credit ratings of firms provided by rating agencies and the interdependencies of economic cycles and corporate default risk (Duffie et al., 2007; Duffie, 2011). Furthermore, they can be used as an input in determining minimum capital requirements by banks and bank regulators (Duffie, 2011). In this regard, sovereign entities such as governments or their representatives across the world may design favourable macroeconomic policies for various sectors.

In this paper, we explore the dynamics of corporate failure dependence and its variations across various sectors on the London stock exchange (LSE) over the period 1985-2012. To this end, we use a multivariate frailty model that accounts for unobserved factors.

Literature broadly groups credit risk models into structural and reduced forms, given the role that information plays in modelling default risk (see e.g. Jarrow and Turnbull, 1992; Jarrow and Turnbull, 1995; Duffie and Singleton, 1999; Duffie and Lando, 2001; Jarrow and Protter; 2004; Giesecke, 2006; among others). However, reduced form approaches have received more attention than the structural ones (Jarrow, 2001, Jarrow and Protter, 2004; Duan et al. 2012; Dionne and Laajimi, 2012; Figlewski et al., 2012; Yeh et al., 2015), since these models are primarily based on the information available to the market.[1] In this paper, we employ the reduced form approach. In order to estimate the parameters of the model and within-sector dependence, we first use firm age and covariates from Duffie et al. (2007), namely one year trailing market wide return, one year trailing stock return, 3-month treasury bill rate and distance to default probability from Bharath and Shumway (2008). In addition, we use some of the covariates in Shumway (2001), such as excess return, net income to total assets, total liabilities to total assets, stock volatility.

This paper fits into the empirical literature on corporate default. Examples of this literature includes Shumway (2001), Das et al. (2007), Duffie et al. (2007), Duffie et al. (2009),

[1] For a comprehensive comparison between structural and reduced form models, see Jarrow and Protter (2004).

Lando and Nielsen (2010), Chava et al. (2011), Koopman et al. (2011, 2012), Orth (2013), and Qi et al. (2014). Shumway (2001) proposes a simple hazard model that allows for time-varying covariates to forecast firm's bankruptcy. The forecast performances of the hazard model are compared to those of a single-period classification model or static model, and the empirical results show that the hazard model outperforms the alternative models. Das et al. (2007) aim to investigate whether default events in an intensity-based setting can be modeled as "doubly stochastic", i.e. as dependent solely on exogenous factors, and Lando and Nielsen (2010) use a different specification of the intensity that allows to reject the Poisson property of the time change aggregate default process considered by Das et al. (2007). Since models that assume independence in failure rates are likely to produce inaccurate estimates, as highlighted in Das et al. (2007), frailty factors are then considered to control for unobserved effects. For instance, Duffie et al. (2009) develop a single economy-wide dynamic frailty model and showed that models with frailty factor(s) are likely to outperform those without these factors. In related studies, Koopman et al. (2011, 2012) show the importance of incorporating frailty factors in hazard rate models and how these factors may improve the predictive performance of the models. In addition, Chava et al. (2011) argue that incorporating sector unobserved effects is likely to improve the predictive ability of these hazard rate models. Hence, they proposed a multivariate frailty model that controls for two different regimes where firms in a sector share the same frailty factor. Orth (2013) takes a different approach to dealing with default predictions. The framework proposed does not need a covariate forecasting model and involves the estimation of just one parameter vector. The model is applied to North American public firms data. Following Duffie et al. (2009), Qi et al. (2014) show that accounting for unobserved risk factors in a model enhances the in-sample predictive accuracy at firm, rating group and aggregate levels, and argue that the unobserved risk factors play a more significant role in predicting default risk as compared to the observed risk factors.

This paper makes some contributions to the literature on corporate finance. First, we propose an additive lognormal frailty model with two regime changes (distressed and normal regime). While the literature predominately features gamma distribution (see e.g. Chava et al., 2011; Wienke, 2011), we use the lognormal distribution as it offers much more flexibility in modelling the dependence structures within a multivariate context (see e.g. Hougaard, 2000; Duchateau and Janssen, 2008; Wienke, 2011). The lognormal distribution is positively skewed and the dependence measure (or association) is directly proportional to the skewness of the distribution: the higher the value of association, the greater the skewness which makes the right tail longer (Lee and Wang, 2003). As the data on corporate failure is highly skewed, a power transformation of the frailtes as to make them normal-like may help to better capture the dependence on a log-scale (Hougaard, 2000). Therefore, under extreme market conditions, the lognormal tends to properly explain the frailties and the corresponding dependence structures, and the additive lognormal frailty model may provide more accurate information on corporate failure dependence as compared to models which use gamma distribution.

Second, we investigate the dynamics of corporate failure dependence on the London Stock Exchange (LSE). To the best of our knowledge, this is the first paper to look at corporate failure dependence in the UK. We also test the robustness of our model under different levels of sector distress (degrees of departure from normal market conditions), given the fact that the effects of unobserved sector specific factors tends to be more pronounced when markets move to severe distressed conditions, and compare its performances with those of the model by Chava et al. (2011). Lastly, and this is another novelty of the paper, we investigate one-step ahead predictive performances of our model and the model by Chava et al. (2011), using a naive recursive extraction approach.

Our empirical analysis delivers three main results. First, the frailty factor is always significant across all the model specifications as in previous studies (see Duffie et al., 2009; Chava et al., 2011; Koopman et al., 2011, 2012; Qi et al., 2014). Further, the significance of the within-sector frailties provides evidence of firm failure clustering, which tends to occur more during distressed market conditions. Second, the adjustment factor in hazard rate during distressed market periods is also significant, and this implies that firms on the London Stock exchange are more inclined to move faster towards failure. Lastly, the additive lognormal frailty model tends to better estimate and predict within-sector frailties and dependencies than the multiplicative gamma frailty when moving away from normal market conditions. This seems to favour the use of the additive lognormal frailty model when estimating and predicting correlations and failure rates among firms during distressed market conditions in the UK.

The rest of the paper is organized as follows. Section 2 presents methodology and data. Section 3 discusses the empirical findings and Section 4 concludes.

2 Methodology and data

In this section, we first present our additive lognormal frailty model and multiplicative gamma frailty of Chava et al. (2011), and then we describe the data.

2.1 Additive lognormal frailty model

Our additive lognormal frailty model is based on the approach of Clayton (1978). Let $T \in [0, \infty)$ be the time to event or time until a firm either fails or leaves the sample as a result of non-failure event (e.g. mergers and acquisitions). Our data set contains s clusters (sectors) and in each cluster there are n_i members (firms) (see Duchateau and Janssen, 2008). In our sample, the sum of firms across all the sectors is the total number of firms, $n = \sum_{i=1}^{s} n_i$. Given a time horizon $[0, T^*]$, staggered firm entry is allowed and some firms may leave the sample period due to non-failure events. In addition, some firms may experience failure event or survive beyond the end of the sample period, T^*, and a firm is considered censored if it leaves the sample period through non-failure reasons or survives beyond T^*. The information consists of the set $(T_{ij}, \delta_{ij}, X_{ij}(t), \tilde{u}_i)$ for $i = 1, ..., s$ and $j = 1, ..., n_i$. The term T_{ij} is the event time (either failure or censored time) of the jth firm in the ith sector, δ_{ij} is the corresponding censoring indicator which takes value 1 when T_{ij} is the failure time and 0 if T_{ij} is censoring time, and $\delta_i = \sum_{j=1}^{n_i} \delta_{ij}$ is the total number of failures in the ith sector. The vector $X_{ij}(t)$ is the set of time-varying covariates for the jth firm in the ith sector in the counting process style of input. Finally, $\tilde{u}_i$ is the unobserved information or the frailty term for ith sector.[2]

We use the classical shared frailty modelling approach of Clayton (1978) to derive our additive lognormal frailty model. The classical shared frailty model is based on the Cox PH semi-parametric framework and is defined as follows:

$$h_{ij}(t) = h_0(t)\tilde{u}_i exp(X_{ij}(t)\beta), \tag{1}$$

where $h_{ij}(t)$ is the conditional hazard rate for the jth firm in the ith sector (conditional on the frailty factor, $\tilde{u}_i$), $h_0(t)$ is an arbitrary baseline hazard and β is a p-dimensional vector of coefficients of the covariates, $X_{ij}(t)$. We rewrite the frailty factor $\tilde{u}_i$ in terms of a random effect or log-frailty as: $\tilde{w}_i = \log \tilde{u}_i$ or $\tilde{u}_i = exp(\tilde{w}_i)$. Then, equation (1) becomes:

$$\begin{aligned} h_{ij}(t) &= h_0(t) \exp(log(\tilde{u}_i)) exp(X_{ij}(t)\beta) \\ &= h_0(t) exp(X_{ij}(t)\beta + \tilde{w}_i). \end{aligned} \tag{2}$$

Equation (2) represents the classical lognormal shared frailty model. It contains two terms: the fixed effects term, which involves the covariates, and the random term, $\tilde{w}_i$, with an expected value, $E(\tilde{W}) = 0$ and a finite variance, $Var(\tilde{W}) = \gamma$. We follow Chava et al. (2011) to construct the log-frailty term as a combination of sector-specific log-frailty term, w_i, and a time-varying sector distress indicator, $Z_i(t)$, which takes value 1 for distressed sectors at time t and 0 otherwise. As such, we have:

$$\begin{aligned} \tilde{w}_i(t) &= \log \tilde{u}_i = \log(u_i \Delta^{Z_i(t)}) \\ &= \log u_i + \log \Delta Z_i(t). \end{aligned} \tag{3}$$

[2]The frailty factor is "a random component designed to account for variability due to unobserved individual-level factors that is otherwise unaccounted for by the other predictors in the model" (Kleinbaum and Klein, 2012, page 326).

Equation (3) can be re-written as:

$$\tilde{w}_i(t) = \pi Z_i(t) + w_i, \tag{4}$$

where $\pi = \log(\Delta)$ is the additive factor in the regime-switch lognormal frailty context that accounts for the extra variations in hazard rates induced by distressed market periods. Using equation (4), we have that the hazard function in equation (2) is:

$$h_{ij}(t) = h_0(t)exp(X_{ij}(t)\beta + \pi Z_i(t) + w_i), \tag{5}$$

and define the additive lognormal frailty model (regime-switch lognormal frailty model) as:

$$h_{ij}(t) = \begin{cases} h_0(t)exp(X_{ij}(t)\beta + \pi Z_i(t) + w_i) & \text{if sector } i \text{ is distressed,} \\ h_0(t)exp(X_{ij}(t)\beta + w_i) & \text{otherwise.} \end{cases} \tag{6}$$

The classical shared lognormal frailty model is a special case of our additive lognormal frailty model when $\pi = 0$. The shared lognormal frailty model does not incorporate regime changes in the impact of the lognormal frailties. Although the multiplicative gamma frailty model may show high predictive power (see Chava et al., 2011), we argue that our additive lognormal frailty model offers much more flexibility than the gamma frailty model due to the properties of the lognormal distribution within the multivariate context (Hougaard, 2000; Duchateau and Janssen, 2008; Wienke, 2011). This flexibility stems from the dependence between the right tail of the distribution and the association parameter (Lee and Wang, 2003), and its power transformation property (Hougaard, 2000).

To estimate the parameters in equation (6), we use the penalised partial likelihood (PPL) approach of McGilchrist and Aisbett (1991):

$$l_p(\beta, \pi, \gamma|w) = l_{part}(\beta, \pi|w) - l_{pen}(\gamma|w), \tag{7}$$

where

$$l_{part}(\beta, \pi|w) = \sum_{i=1}^{s}\sum_{j=1}^{n_i}\delta_{ij}\left(X_{ij}(t)\beta + \pi Z_i(t) + w_i - \log\left(\sum_{j\in R(T_{ij})} exp(X_{ij}(t)\beta + \pi Z_i(t) + w_j)\right)\right), \tag{8}$$

which is the conditional likelihood given the log-frailties and

$$l_{pen}(\gamma|w) = \frac{1}{2\gamma}\sum_{i=1}^{s} w_i^2, \tag{9}$$

represents the penalised term (the distribution of the log-frailties). This term penalises the likelihood by subtracting large values of the penalty term from the full data log likelihood if the real values of the log frailties are far from their mean (see Duchateau and Janssen, 2008). The term $R(T_{ij})$ in equation (8) is the risk set (the set of surviving firms or firms still at the risk of an event). The PPL is independent of the baseline hazard function, making it possible to estimate the parameters in the likelihood without knowing the shape of the baseline hazard rate. This characteristic of PPL makes our estimates robust irrespective of the shape of the baseline hazard rate (see e.g. Cox, 1975; Duchateau and Janssen, 2008; Allison, 2010), although estimates can be to some extent not fully efficient, but this inefficiency is normally immaterial (see Efron, 1977). However, the estimates are consistent and asymptotically normal (see e.g. Cox, 1975; Allison, 2010).

Let $\beta^* = (\beta, \pi)$ be the coefficients of the following covariates $X = (X_{ij}(t), Z(T_{ij}))$. Equation (8) can be the re-written as follows:

$$l_{part}(\beta^*|w) = \sum_{i=1}^{s}\sum_{j=1}^{n_i}\delta_{ij}\left(X\beta^* + w_i - \log\left(\sum_{j\in R(T_{ij})} exp(X\beta^* + w_j)\right)\right). \tag{10}$$

Therefore equation (7) becomes:

$$l_p(\beta^*, \gamma|w) = \sum_{i=1}^{s}\sum_{j=1}^{n_i}\delta_{ij}\left(X\beta^* + w_i - \log\left(\sum_{j\in R(T_{ij})} exp(X\beta^* + w_j)\right)\right) - \frac{1}{2\gamma}\sum_{i=1}^{s} w_i^2. \tag{11}$$

For any value of the log-frailty variance, γ, we employ the marginal log-likelihood in Ripatti and Palmgren (2000) (see also Therneau and Grambsch, 2000; Therneau et al., 2003; SAS/STAT 13.2) to derive the extended PPL of equation (11):

$$l_m(\beta^*, \gamma) = -\frac{1}{2}\log(\gamma I) + \log(\int exp[l_p(\beta^*, \gamma)]dw), \tag{12}$$

where I is the identity matrix of order $s \times s$ and s is the number of sectors in the sample. We use the approximation of Ripatti and Palmgren (2000) to derive the likelihood in equation (12):

$$l_m(\beta^*, \gamma) \approx -\frac{1}{2}\log(\gamma I) + \log(|H_{22}(\beta^*, \gamma, w^*)|) - l_p(\beta^*, \gamma, w^*), \tag{13}$$

where H is the negative Hessian of the PPL for a given value of γ. We use the PHREG procedure in SAS to maximize the likelihood in equation (13). For the variance of the frailty, $\tilde{u}_{it}$, which follows a lognormal distribution with an expected value, $E(\tilde{U}) = 1$, and a finite variance, $Var(\tilde{U}) = \theta$, it is required that $\gamma = \log(\theta + 1)$ (see Duchateau and Janssen, 2008).

In our empirical analysis, we compare the performance of the first specification of our model with the multiplicative frailty model (see Chava et al. 2011) in order to empirically ascertain whether the additive lognormal frailty model is comparatively better than the latter. In what follows, we briefly describe the multiplicative gamma frailty model of Chava et al. (2011):[3]

$$h_{ij}(t) = \begin{cases} h_0(t)u_i\Delta^{Z_i(t)}exp(X_{ij}(t)\beta) & \text{if sector } i \text{ is distressed,} \\ h_0(t)u_i exp(X_{ij}(t)\beta) & \text{otherwise.} \end{cases} \tag{14}$$

For estimation feasibility, the authors assumed that the sector frailties follow a two parameter gamma distribution, i.e. $u_i(t) = G(A_i(t), C_i(t))$ with the shape parameter $A_i(t) = 1/\theta(t) + \sum_{j=1,T_{ij}<t}^{n_{ij}} \delta_{ij}$ and scale parameter $C_i(t) = 1/\theta(t) + \sum_{j=1,T_{ij}<t}^{n_{ij}} H(T_{ij})$, where $H(T_{ij}) = \int_0^{T_{ij}} (\Delta^{Z_i(t)}exp(X_{ij}(t)\beta)dt$. Based on the above assumption, the authors derived the sample

[3]For further details, readers can refer to Chava et al. (2011). Here, we change some of the notation in Chava et al. (2011) to ease the comparison of the two models.

marginal likelihood for all sectors as

$$l(\theta, \Delta, \beta) = \sum_{i=1}^{s} l_i(\theta, \Delta, \beta), \tag{15}$$

where

$$l_i(\theta, \Delta, \beta) = \log \Gamma(\delta_i + 1/\theta) - \log \Gamma(1/\theta) - (1/\theta)\log(\theta) + \sum_{j=1}^{n_i} \delta_{ij}(X_{ij}(T_{ij})\beta$$
$$+Z_i(T_{ij})\log(\Delta)) - (\delta_i + 1/\theta)\log(1/\theta + \sum_{j=1}^{n_i} H(T_{ij})), \tag{16}$$

for each sector i. The term $\Gamma(.)$ is the gamma function with an expected value of 1 and a finite variance, θ. For consistency, we maximise equation (15) using the penalised partial maximum likelihood procedure.

2.2 Data

Data are taken from DataStream and Worldscope for the London Stock Exchange (LSE) over the period 1985-2012. Our sample consists of 524 active firms, 174 merged or acquired firms and 191 failed firms. We categorise our data set into macro-financial, firm-specific and sector level distressed indicator variables. As for the macro-financial market covariates, we employ LSE market-wide one year trailing return, calculated by cumulating monthly market returns, and 3-month Treasury bill rate. With respect to the firm-specific covariates, we use two types of covariates, namely market driven and books (including balance sheets and income statements) of firms in the 10 major DataStream sector groupings. To study the within-sector dependencies and frailties, we employ 29 subsectors from the 10 major DataStream sectors on the LSE (see Table 1).

For market driven variables we use: one year trailing cumulated monthly returns, the standard deviation of the monthly firm's equity returns, firm age and distance to default probability (a probabilistic measure of volatility adjusted leverage in the framework of struc-

Table 1: Sector names

Sector ID	Name
1	UK-DS Oil and Gas Producers
2	UK-DS Oil Equipment and Services
3	UK-DS Alternative Energy
4	UK-DS Chemicals
5	UK-DS Basic Resource
6	UK-DS Construction and Materials
7	UK-DS Aerospace and Defence
8	UK-DS General Industrials
9	UK-DS Electronic and Electrical Equipment
10	UK-DS Industrial Engineering
11	UK-DS Industrial Transportation
12	UK-DS Support Services
13	UK-DS Automobiles and Parts
14	UK-DS Food and Beverage
15	UK-DS Personal and Household Goods
16	UK-DS Health Care Equipment and Services
17	UK-DS Pharmaceuticals and Biotechnology
18	UK-DS Retail
19	UK-DS Media
20	UK-DS Travel and Leisure
21	UK-DS Fixed Line Telecommunications
22	UK-DS Mobile Telecommunications
23	UK-DS Electricity
24	UK-DS Gas, Water and Multiutilities
25	UK-DS Insurance
26	UK-DS Real Estate
27	UK-DS Financial Services(3)
28	UK-DS Software and Computer Services
29	UK-DS Technology Hardware and Equipment

Notes: We choose the 29 sub-sector due to data availability and the similarity between some of the sub-sectors. Financial Services (3) is a subsector of firms that provide financial services. This group excludes banks, real estate and insurance firms.

tural model of Merton, 1974).[4] We adopt the approach of Bharath and Shumway (2008) to construct this measure because (i) it is much easier to implement in practice since this does not require solving complex equations iteratively in the classical Merton's (1974) method; (ii) it has slightly better in and out sample predictive power as compared to Merton's Distance-to-Default metric (Bharath and Shumway, 2008). In addition, we use the ratio of net income to total assets, and total liabilities to total assets.

In order to test the robustness of our model to different levels of sector distress (degrees of departure from normal market conditions), we construct five sector level distress indicators following Gilson et al. (1990), Opler and Titman (1994) and Acharya et al. (2007).[5]

Let $r(t)$ be the median equity return of a sector during a given year t and $\varepsilon(n)$ be a real number that only takes on the values -0.10, -0.15, -0.20, -0.25, and -0.30 for the integer $n = 1, ..., 5$ respectively. We define a sector level distress indicator as:

$$Z(n) = \begin{cases} 1 & \text{if } r(t) < \varepsilon(n) \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

For example, the first sector level distress indicator is Z(1), which takes 1 if the median equity return of a sector during a given year in the sample period of our analysis is less than -10 percent and 0 otherwise. Explicitly, this sector level distress is said to occur if the returns of over half of the number of stocks within a given sector is less than -10 percent in a particular year. The third sector level distress indicator, Z(3), corresponds to Chava et al. (2011) sector level distress indicator. This indicator takes 1 if the median equity return of a sector during a given year is less than -20 percent and 0 otherwise. By our construction, the sector distress indicator 3 represents a more severe market conditions than sector distress indicator 1. All these indicators are used to control regime changes in the sample period of our analysis.

As regards the definition of failure, we follow the convention of legal definition of failure

[4] Age is defined as the period between the time a firm is listed and the time of an event.

[5] Chava et al. (2011) also follows the same authors when constructing one sector distress indicator. Here, we take a step further and construct four extra sector distress indicators.

Table 2: Descriptive statistics

Variable	Mean	Std. Dev.	Min	25th P.	Median	75th P.	Max
Distance to Default Prob.	0.692	0.263	0.000	0.666	0.778	0.852	1.000
Stock Return(%)	8.760	27.998	-91.520	-6.818	7.406	20.986	220.557
LSE Return (%)	10.922	16.940	-22.167	2.590	13.170	24.080	57.840
3-month T-bill rate (%)	5.746	3.253	0.434	4.480	5.150	6.850	14.332
ln(Age)	1.971	0.898	0.000	1.386	2.079	2.708	3.296
ln(Equity)	16.716	1.965	11.920	15.509	16.706	17.936	21.964
Inverse of Volatility	3.704	1.847	0.975	2.406	3.359	4.600	10.331
Excess Return (%)	1.355	32.428	-93.948	-15.188	0.000	14.005	169.800
Stock Volatility (%)	0.340	0.181	0.077	0.216	0.296	0.413	1.025
ln(Face Value of Debt)	10.027	2.779	2.606	8.338	10.164	11.936	16.764
ln(Total Assets)	12.488	2.132	7.498	11.150	12.420	13.781	18.368
Total Liabilities to Total Assets	0.487	0.285	0.006	0.264	0.498	0.664	1.444
Net Income to Total Assets	0.849	0.830	-0.055	0.090	0.683	1.298	3.825
Sector Distress Indicator	0.073	0.259	0.000	0.000	0.000	0.000	1.000

Notes: We follow Bharath and Shumway (2008) to calculate the covariates and define the face value of firm's debt, as the sum of debt in current liabilities and half of long term debt (see Vassalou and Xing, 2004). The 3-month T-bill rate represents the risk free interest rate. All covariates are winsorized at 1 and 99 percentiles, except distance to default probability covariate.

(see, e.g., Charitou et al., 2004; Christidis and Gregory 2010; Tinoco and Wilson, 2013) and select firms in this category. Given our sample, we specifically employ the UK insolvency Act 1986 to select failed firms. The Act states that "A company is insolvent (unable to pay its debts) if it either does not have enough assets to cover its debts (i.e. value of assets is less than amount of liabilities), or if it is unable to pay its debts as they fall due" and such a company has the option to go into either (*i*) administration, (*ii*) company voluntary arrangement (CVA), (*iii*) receivership, (*iv*) liquidation or (*v*) dissolution. We select the failed firms from the DataStream "DeadUK" category and cross-checked at Bloomberg bankruptcy segment, Wall Street Journal (European segment) and the UK Bankruptcy & Insolvency Website for companies with at least four years firm-specific data from DataStream and Worldscope. To ensure that our results are not affected by outliers, we winsorized all the variables at 1 and 99 percentiles except distance to default probability (see e.g. Shumway, 2001; Bharath and Shumway, 2008). By construction, the distance to default probability is [0,1] bounded.

Table 2 presents the summary statistics of the covariates used to estimate the coefficients of the additive lognormal and multiplicative gamma frailty models in Section 3.

3 Empirical analysis

In this section, we present our empirical analysis based on the additive lognormal frailty model in equation (13). We proceed as follows. First, we estimate the coefficients of our model using three sets of covariates from Duffie et al. (2007), Shumway (2001) and Bharath and Shumway (2008), respectively (see Sections 3.1 and 3.2).[6] Then, we compare the performance of the first specification of our model with that of the multiplicative gamma frailty model to empirically ascertain whether our model is comparatively better than the latter. In this respect, we also study the impact of various degrees of departure from market normality on the within-sector frailties, associations and the predictive characteristics of the covariates (see Section 3.3). Finally, we investigate one step-ahead forecasts for the within-sector failure rates and the corresponding dependencies using a naive recursive extraction approach.

3.1 Parameter estimation using covariates of Duffie et al. (2007)

The first set of variables that we use in our first specification of the model are ln(age), distance to default probability, one year trailing market return, one year trailing stock return, and 3-month Treasury bill rate. Models 1 and 2 (M1 and M2) represent the classical shared frailty models whilst models 3 and 4 (M3 and M4) denote the additive lognormal frailty models (see Table 3).

Findings show that the estimated coefficient of distance to default probability is positive and statistically significant. In addition, one year trailing stock return, 3-month Treasury bill rate, and ln(age) are all negative and statistically significant, while one year trailing LSE stock return is unexpectedly positive and statistically significant. The frailty variance of each model (M1-M4) is a measure of the within-sector dependence or correlation between lifetimes of firms in the sectors. We argue that older firms with high stock returns are more likely to survive than younger firms with low stock returns on the LSE (see e.g. Shumway, 2001). In

[6] We replace the distance to default covariate in Duffie et al.(2007) with distance to default probability of Bharath and Shumway (2008) for the reasons in section 2.2

Table 3: Additive lognormal frailty model. Dependent variable: time to event

	Lognormal Shared Frailty		Additive Lognormal Frailty	
	M1	M2	M3	M4
Frailty Variance	0.306	0.246	0.307	0.288
	(0.150)	(0.131)	(0.126)	(0.147)
Additive Factor			2.472	2.422
			(0.251)	(0.250)
Distance to Default Prob.	1.771	1.971	1.703	1.885
	(0.473)	(0.469)	(0.467)	(0.464)
Stock Return	−0.017	−0.016	−0.015	−0.015
	0.003	(0.003)	(0.003	(0.003)
Market Return(LSE)	0.785	0.786	0.762	0.763
	(0.065)	(0.065)	(0.062)	(0.062)
3-Month Treasury Bill Rate	−1.419	−1.373	−0.938	−0.897
	(0.194)	(0.194)	(0.174)	(0.174)
ln(Age)		−0.392		−0.360
		(0.103)		(0.105)
Marginal Log Likelihood	-632.873	-626.172	-589.175	-583.697
Likelihood ratio Test	522.766	531.960	610.471	619.239
Wald Test	325.698	330.184	376.084	382.385

Notes: The parameter estimation is done using covariates from Duffie et al. (2007). The Exact approximation is used to control for ties in the survival times of firms in our sample when deriving the penalised partial likelihood. The standard errors are in parenthesis. The parameters are adjusted for the within-sector dependencies or correlations. The Likelihood ratio and Wald Tests are significant.

addition, firms closer to default tends to exhibit higher probabilities of distance to default. As for the 3 month Treasury bill rate, results show that this covariate tend to decrease the hazard rate All in all, our results related to overall market are in line with those in Duffie et al. (2007, 2009) who argued that the unexpected positive sign of a market index should "not be an evidence that a good year in the stock market may in itself be bad news for default risk." This could be attributed to the fact that, in the subsequent years of a boom, a firm's distance to default probability is likely to overstate its financial prospects.

3.2 Parameter estimation using covariates of Shumway (2001) and Bharath and Shumway (2008)

The second set of covariates is taken from Shumway (2001). They are the logarithm of total assets (ln(total assets)), excess return, total liabilities to total assets, stock volatility and net income to total assets. In Table 4, model 5 (M5) is the classical frailty model, whilst model 7 (M7) is the additive lognormal frailty model. The estimates of these models show that the

Table 4: Additive lognormal frailty model. Dependent variable: time to event

	Lognormal Shared Frailty		Additive Lognormal Frailty	
	M5	M6	M7	M8
Frailty Variance	0.200 (0.100)	0.20 (0.098)	0.240 (0.120)	0.223 (0.118)
Additive Factor			2.396 (0.202)	2.393 (0.198)
Distance to Default Prob.		0.846 (0.349)		0.715 (0.358)
ln(Equity)		−0.499 0.054		−0.487 (0.056)
Inverse of Volatility		−0.283 (0.066)		−0.271 (0.066)
Excess Return	−0.012 (0.002)	−0.009 (0.002)	−0.010 (0.002)	−0.006 (0.002)
Stock Volatility	1.748 (0.330)		1.756 (0.336)	
ln(Face Value of Debt)		0.125 (0.042)		0.146 (0.044)
ln(Total Assets)	−0.318 (0.040)		−0.267 (0.040)	
Total liab. to Total Assets	0.910 (0.326)		0.822 (0.247)	
Net Income to Total Assets	−0.429 (0.113)	−0.305 (0.111)	−0.388 (0.115)	−0.274 (0.113)
Marginal Log Likelihood	-727.920	-710.766	-659.434	-639.839
Likelihood Ratio test	327.435	360.117	467.586	505.431
Wald test	353.269	348.813	484.286	469.825

Notes: see notes in Table 3.

coefficients of excess return, net income to total assets and ln(Total Assets) are negative and statistically significant, whilst total liabilities to total assets and stock volatility are positive and statistically significant.[7]

As for the last specification of the model, we use the following variables: distance to default probability, logarithm of face value of debt (ln(Face Value of Debt)), logarithm of equity (ln(Equity)), excess return, inverse of firm volatility, and net income to total assets (for the covariates see Bharath and Shumway, 2008). In Table 4, model 6 (M6) is the classical shared frailty model, whereas model 8 (M8) is the additive lognormal frailty model.

The results show that ln(Equity), excess return, inverse of firm volatility and net income to total assets have a negative and significant impact on hazard rates, whereas ln(Face Value of Debt) and distance to default probability covariates have a positive and significant effect

[7]We also estimated models as in Table 4 with ln(age), but this covariate was found to be negative and statistically insignificant. The results are not reported here.

on hazard rates. Again, the estimates of all the models in these specifications are adjusted for the within-sector dependencies. The two specifications, though having slightly different covariates, produce similar results. After accounting for unobserved sector effects, the results show (i) firms with low income levels and high liabilities are more likely to fail than firms with high income levels and low liabilities; (ii) firms characterised with high past returns, bigger firms and less volatile firms have high survival rates than small firms, volatile firms with low past returns. All these findings may be informative for the stakeholders (i.e. stock investors, regulators, etc.) on LSE for their decision-making process in the short-run period.

3.3 Impact of sector distress on within-sector dependence

In this section, we explore the robustness of the additive lognormal frailty and multiplicative gamma frailty models under various levels of distressed market conditions. Specifically, we employ the five different levels of severity conditions, namely $Z(1)$, $Z(2)$, $Z(3)$, $Z(4)$, and $Z(5)$ (see section 2.2). These conditions are in order of severity. For instance, the distressed market period $Z(1)$ is less severe than the distressed market period $Z(3)$.

We use the same set of covariates in specification M4 (see Table 3) to estimate the parameters of the additive lognormal frailty and the multiplicative gamma frailty models, respectively, using each of the five different distressed market conditions. For example, ALFM1 and MGFM1 are the additive lognormal frailty and the multiplicative gamma frailty models under distressed market condition $Z(1)$, respectively (see Table 5).

The estimation results show that the coefficients of the covariates in all the regressions are similar as expected. However, the scale factor increases as the degree of severity of sector distress rises. For example, in a less severe distressed period, the estimated value of the scale factor is 1.864 for the additive lognormal frailty model (ALFM1), while this value is 2.422, in a more severe distressed period (see ALFM3). As for multiplicative gamma frailty model, the estimated values of the scale factor are 1.853 and 2.408 (see MGFM1 and MGFM3), respectively. Therefore, the scale factor for the extra variations in the hazard rates for both

Table 5: The impact of levels of sector distress on within-sector dependence

Panel A: Additive Lognormal Frailty Model

	ALFM1	ALFM2	ALFM3	ALFM4	ALFM5
Frailty Variance	0.281 (0.144)	0.321 (0.157)	0.288 (0.147)	0.284 (0.147)	0.300 (0.153)
Scale Factor	1.864 (0.253)	2.159 (0.250)	2.422 (0.250)	2.851 (0.256)	3.218 (0.282)
Distance to Default Prob.	1.924 (0.467)	1.895 (0.465)	1.885 (0.464)	1.839 (0.463)	1.809 (0.458)
Stock Returns	−0.015 (0.003)	−0.015 (0.003)	−0.015 (0.003)	−0.014 (0.003)	−0.013 (0.003)
Market Returns	0.752 (0.062)	0.764 (0.062)	0.763 (0.062)	0.745 (0.062)	0.752 (0.062)
3-Month Treasure Rate	−1.097 (0.189)	−0.969 (0.179)	−0.897 (0.175)	−0.823 (0.169)	−0.757 (0.166)
ln(Age)	−0.364 (0.105)	−0.361 (0.105)	−0.360 (0.105)	−0.357 (0.106)	−0.354 (0.106)
Marginal Log likelihood	-598.894	-591.425	-583.697	-570.804	-563.708
Likelihood Ratio Test	588.594	606.143	619.239	644.525	659.174
Wald's Test	353.609	368.564	382.385	410.846	404.346

Panel B: Multiplicative Gamma Frailty Model

	MGFM1	MGFM2	MGFM3	MGFM4	MGFM5
Frailty Variance	0.220 (0.108)	0.257 (0.122)	0.228 (0.112)	0.225 (0.111)	0.220 (0.110)
Scale Factor	1.853 (0.251)	2.144 (0.250)	2.408 (0.249)	2.840 (0.256)	3.218 (0.284)
Distance to Default Prob.	1.935 (0.468)	1.908 (0.466)	1.897 (0.465)	1.855 (0.464)	1.832 (0.459)
Stock Returns	−0.015 (0.003)	−0.015 (0.003)	−0.015 (0.003)	−0.014 (0.003)	−0.013 (0.003)
Market Returns	0.751 (0.062)	0.763 (0.062)	0.761 (0.062)	0.743 (0.062)	0.752 (0.062)
3-Month Treasure Rate	−1.100 (0.190)	−0.973 (0.180)	−0.900 (0.175)	−0.820 (0.169)	−0.749 (0.166)
ln(Age)	−0.368 (0.104)	−0.365 (0.105)	−0.362 (0.105)	−0.358 (0.106)	−0.354 (0.106)
Marginal Log likelihood	-797.804	-784.520	-776.700	-763.991	-756.789
Likelihood Ratio Test	586.363	604.063	616.886	642.677	656.858
Wald's Test	350.689	365.668	379.738	409.226	400.407

Notes: see notes in Table 3.

models increases as the market conditions becomes more severe. However, our model seems to be robust to different market conditions, as it appropriately accounts for the extra randomness induced by the distressed periods, and it performs better than the multiplicative gamma frailty model (MGFM) in measuring the within-sector dependence (see frailty variances in Table 5) during distressed market periods.

For robustness of analysis, we also estimate the within-sector failure rates (frailties) and random effects (log-frailties)(see section 2.1) using our model, ALFM3, and model of Chava et al. (2011), MGFM3. The results are presented in Figure 1 (see panels A and B).

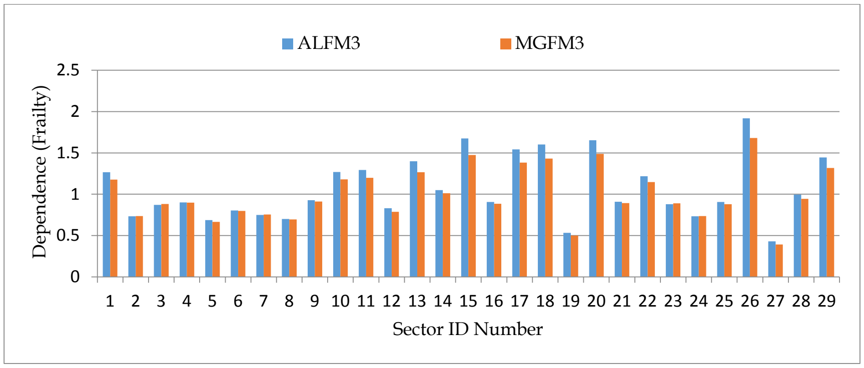
It emerges that firms in sectors with frailties larger than one tend to fail faster than firms with frailties smaller than one. For instance, firms in Real Estate sector (see sector ID. 26 in Table 1) with a frailty of 1.918 for ALFM3 (1.676 for MGFM3) are likely to fail faster than firms in fixed line Telecommunications sector (see Sector ID. 21 in Table 1) with a frailty of 0.9068 for ALFM3 (0.890 for MGFM3). Therefore, these figures confirm the results in Table 5, and they seem to suggest that, under distressed market periods, the additive lognormal frailty model is likely to outperform the multiplicative gamma frailty model.

3.4 Out-of-sample extraction of failure rates

The accuracy of the estimates of failure rates plays a central role in stakeholders' decisions. In this section we use an out-of-sample parameter extraction approach to extract sector-level failure rates (frailties are not observable). We present the results of one step-ahead extracts by using our model and the multiplicative gamma frailty model. More, specifically we consider one-year horizon, as often required by most regulatory requirements (see for instance the Bank for International Settlements), and compute the additional deviations from the expected future values. We then evaluate the accuracy of the extraction by using the root mean square of the deviations: the higher the value of this metric, the higher the accuracy.

We proceed as follows. We use a naive recursive scheme for one-step ahead extraction over the following years: 2010, 2011 and 2012. We do this in line with Shumway (2001). For

Panel A: Within-Sector Dependences



Panel B: Within-Sector Random Effects

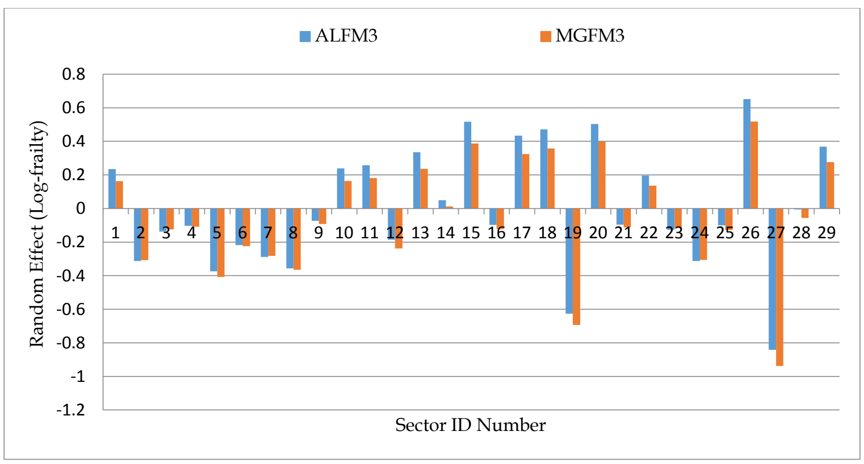


Figure 1: Estimated failure rates and log-failure rates for models ALFM3 and MGFM3.

instance, to extract the within-sector frailty (or failure rate) and the corresponding dependence for 2010, we define a sample from 1985 to 2010 and estimate the parameters using the period 1985 - 2009 by holding out 2010. In this way, we obtain the frailties at the beginning of 2010. We do the same for 2011 and 2012. This naive extraction scheme is repeated for all the sectors under consideration. Finally, for each sector i, we construct the root mean square deviation ($RMSD_i$) as follows: $RMSD_i = \sqrt{\sum_{t=1}^{3}(\hat{y}_{i,t} - E(\tilde{u}_{i,t}))^2/3}$, where $\hat{y}_{i,t}$ denotes the extracted value, $E(\tilde{u}_{i,t})$ is the expected value of frailty for sector $i = 1, ..., 29$ and $t = 1, 2, 3$, where $t = 1$ indicates the year 2010, $t = 2$ is the year 2011, and $t = 3$ is the year 2012.[8] We use the expected value of the frailty as the actual value since it is not observable at the end of 2010. For instance, $\hat{y}_{5,2}$, the extracted failure rate for the UK Basic Resource (ID.5) for the year 2011. Table 6 presents the additive lognormal and multiplicative gamma frailty model extraction based on the within-sector dependencies (see Table 7). Results for the RMSD are illustrated in Table 8.

The results in Table 6 show that there are differences in the extracted values over time and across sectors for both models. This extraction allows us to distinguish between firms in sector which are likely to fail faster or slower in the event of firm failure clustering. Firms in sectors with estimates larger than 1 (fast-failure regime) are likely to fail faster, whilst those with estimates smaller than 1 (slow-failure regime) are likely to fail slower. For instance, firms in the UK Oil and Gas Production Sector (ID. 1) are likely to fail faster, while those in the UK Health Equipment and Services sector (ID. 16) are likely to fail slower. Furthermore, these results reveal some interesting trends in firm failure. First, in a fast-failure regime, the multiplicative gamma frailty model tends to underestimate these rates across sectors, while the additive lognormal frailty tends to predict these rates more accurately. For instance, for the UK Real Estate Sector (ID. 26), the extractions of the failure rates for the multiplicative model are 1.469, 1.631 and 1.676, whereas those for the additive lognormal frailty model are 1.753, 1.853 and 1.918, respectively. In addition, these dynamics also hold for a mixed regime, where

[8]The impact of frailties on hazard rates during distressed periods tends to be more pronounced and hence we construct a metric for capturing the additional variations in hazard rates across the years for each sector. Therefore, high values of our metric are desirable.

Table 6: Within-sector failure rate extractions, $\hat{y}_{i,t}$.

Additive Lognormal Frailty Model				Multiplicative Gamma Frailty Model			
Sec. ID	2010	2011	2012	Sec. ID	2010	2011	2012
1	1.437	1.246	1.313	1	1.235	1.163	1.210
2	0.848	0.746	0.727	2	0.894	0.754	0.730
3	0.966	0.893	0.870	3	0.979	0.906	0.882
4	0.974	0.904	0.899	4	0.976	0.901	0.893
5	0.733	0.682	0.656	5	0.772	0.663	0.630
6	0.832	0.801	0.801	6	0.869	0.799	0.795
7	0.778	0.749	0.744	7	0.837	0.757	0.748
8	0.788	0.732	0.699	8	0.845	0.733	0.693
9	0.889	0.924	0.933	9	0.906	0.909	0.913
10	1.015	1.297	1.285	10	0.990	1.201	1.188
11	1.052	1.244	1.299	11	1.020	1.163	1.200
12	0.640	0.797	0.800	12	0.653	0.760	0.757
13	1.147	1.417	1.424	13	1.079	1.274	1.279
14	0.964	1.014	1.044	14	0.961	0.983	1.003
15	1.471	1.655	1.680	15	1.285	1.459	1.473
16	0.739	0.952	0.924	16	0.800	0.930	0.899
17	1.364	1.533	1.540	17	1.210	1.377	1.379
18	1.375	1.604	1.667	18	1.231	1.432	1.474
19	0.813	0.534	0.519	19	0.830	0.503	0.483
20	1.595	1.626	1.687	20	1.397	1.467	1.510
21	1.147	0.927	0.906	21	1.078	0.914	0.890
22	1.300	1.243	1.224	22	1.159	1.164	1.149
23	0.910	0.883	0.878	23	0.941	0.895	0.888
24	0.824	0.759	0.731	24	0.875	0.768	0.735
25	0.949	0.928	0.915	25	0.951	0.903	0.886
26	1.735	1.853	1.918	26	1.469	1.631	1.676
27	0.451	0.432	0.425	27	0.462	0.394	0.384
28	1.121	1.011	0.992	28	1.058	0.963	0.940
29	1.411	1.333	1.441	29	1.236	1.232	1.311

Notes: The reported estimates denotes the failure rate with an expected value of 1. In the event of failure clustering, firms with estimates larger (lower) than 1 are likely to failure faster (slower). These estimates are adjusted for the within-sector dependencies or correlations in Table 7.

Table 7: Out-of-sample within-sector dependence extracts

Additive Lognormal Frailty Model				Multiplicative Gamma Frailty Model			
Sec. ID	2010	2011	2012	Sec. ID	2010	2011	2012
1	0.234	0.202	0.196	1	0.123	0.152	0.148
2	0.339	0.334	0.358	2	0.206	0.334	0.369
3	0.392	0.401	0.428	3	0.206	0.332	0.367
4	0.307	0.313	0.335	4	0.177	0.265	0.288
5	0.218	0.193	0.205	5	0.166	0.205	0.224
6	0.268	0.285	0.307	6	0.178	0.268	0.291
7	0.312	0.336	0.366	7	0.207	0.334	0.369
8	0.315	0.297	0.308	8	0.207	0.305	0.331
9	0.240	0.259	0.278	9	0.159	0.222	0.238
10	0.197	0.181	0.191	10	0.130	0.137	0.146
11	0.266	0.266	0.287	11	0.155	0.188	0.199
12	0.132	0.112	0.118	12	0.115	0.108	0.114
13	0.361	0.371	0.393	13	0.176	0.217	0.233
14	0.251	0.235	0.250	14	0.157	0.193	0.203
15	0.144	0.140	0.149	15	0.089	0.101	0.107
16	0.297	0.228	0.228	16	0.207	0.195	0.202
17	0.186	0.144	0.150	17	0.109	0.106	0.112
18	0.131	0.118	0.121	18	0.086	0.089	0.091
19	0.176	0.155	0.151	19	0.133	0.182	0.177
20	0.088	0.082	0.082	20	0.060	0.065	0.065
21	0.286	0.261	0.274	21	0.155	0.223	0.240
22	0.291	0.271	0.285	22	0.146	0.191	0.205
23	0.366	0.396	0.432	23	0.206	0.332	0.367
24	0.329	0.340	0.361	24	0.206	0.334	0.370
25	0.246	0.201	0.210	25	0.156	0.177	0.188
26	0.103	0.087	0.089	26	0.066	0.066	0.068
27	0.138	0.099	0.102	27	0.140	0.115	0.117
28	0.150	0.127	0.132	28	0.102	0.112	0.118
29	0.187	0.140	0.136	29	0.108	0.110	0.106

Notes: The estimates represent the dependence or correlation between the lifetimes of firms in the sectors.

firms are likely to fail slower in some years and faster in others (see e.g. sector ID. 14). Second, in the slow-failure periods, the multiplicative gamma frailty tends to overestimate the rates, whilst additive lognormal frailty model predicts (extracts) these rates more accurately. For example, for the UK Alternative Energy Sector (ID. 8), the predictions of the multiplicative gamma model for the rates are 0.979, 0.906 and 0.882 and those of the additive model are 0.966, 0.893 and 0.870, respectively. The results of our model seem to offer a more accurate classification of firms in terms of failure speed, and they may be useful for an appropriate portfolio reshuffling.

Table 8: Root mean square deviations

Sec ID	ALFM3	MGFM3
1	0.313	0.210
2	0.273	0.270
3	0.130	0.118
4	0.101	0.107
5	0.344	0.370
6	0.199	0.205
7	0.256	0.252
8	0.301	0.307
9	0.067	0.087
10	0.285	0.188
11	0.299	0.200
12	0.200	0.243
13	0.424	0.279
14	0.044	0.003
15	0.680	0.473
16	0.076	0.101
17	0.540	0.379
18	0.667	0.474
19	0.481	0.517
20	0.687	0.510
21	0.094	0.110
22	0.224	0.149
23	0.122	0.112
24	0.269	0.265
25	0.085	0.114
26	0.918	0.676
27	0.575	0.616
28	0.008	0.060
29	0.441	0.311

When comparing the RMSD of the two models for each sector, the additive lognormal frailty model has slightly higher values than those by the multiplicative gamma frailty model (see Table 8). These findings seem to confirm the relevance of our distribution assumption on the frailties, as the additive lognormal frailty model fits the data better than the multiplicative gamma frailty model during distressed market periods.

4 Conclusions

We use a multivariate lognormal regime-switch frailty model to estimate and predict within-sector failure rates and the corresponding dependencies of listed firms on London Stock Exchange (LSE) over period 1985-2012. The model is particularly suitable for dealing with distressed market periods. In relation to a set of observable predictive factors of failure rates, we find significant evidence of unobserved sector-specific source of default rates amongst the listed firms. Neglecting these unobserved sector-specific factors may likely lead to underestimation of the hazard rates.

We also account for the adjustment factor in hazard rates and investigate the dynamics of this relative to a set of crucial firm failure predictive factors when moving away from normal market conditions. The scalar adjustment increases when moving from less to more severe distressed market conditions, whilst the desirable impact of distance to default probability (volatility adjusted leverage) with a substantial predictive power for hazard rates averagely deteriorates. However, all the other covariates also experience slight changes in their magnitudes as expected. Interestingly, we also found that the distance to default probability of firms is likely to overstate the financial prospects of these firms after a boom on LSE.

We also compare our model with the multiplicative gamma frailty model of Chava et al. (2011). It results that the former outperforms the latter both in-sample and out-sample estimates, as it offers much flexibility in accounting for extra variations in hazard rates induced by departure from market normality and unobserved sector factors. Therefore, we argue that the additive lognormal frailty is likely to produce better estimates and predictions of hazard rates, within-sector failure rates and dependencies.

Our findings have some important implications for stakeholders on LSE. Specifically, in the event of failure clustering on LSE, the within-sector failure rates of our model could be used by investors and other stakeholders to discriminate amongst firms or sectors, which are likely to fail faster or slower. In this respect, investors may effectively rebalance their portfolios and obtain good estimates of their portfolio risks. On the other hand, regulators may rank firms into various risk profiles in order to suitably design new or enhance existing regulatory requirements to make firms more risk sensitive. Finally, market participants are highly recommended not to be conservative on firms' distance to default probability after a market boom on LSE. Failing to account for this may likely lead to underestimation of default rates, within-sector failure rates and dependencies of firms.

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