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LONG MEMORY AND DATA FREQUENCY IN FINANCIAL MARKETS

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Abstract

This paper investigates persistence in financial time series at three different frequencies (daily, weekly and monthly). The analysis is carried out for various financial markets (stock markets, FOREX, commodity markets) over the period from 2000 to 2016 using two different long memory approaches (R/S analysis and fractional integration) for robustness purposes. The results indicate that persistence is higher at lower frequencies, for both returns and their volatility. This is true of the stock markets (both developed and emerging) and partially of the FOREX and commodity markets examined. Such evidence against the random walk behavior implies predictability and is inconsistent with the Efficient Market Hypothesis (EMH), since abnormal profits can be made using specific option trading strategies (butterfly, straddle, strangle, iron condor, etc.).

Keywords: *Persistence, Long Memory, R/S Analysis, Fractional Integration*

JEL Classification: *C22, G12*

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1. Introduction

The Efficient Market Hypothesis (EMH), according to which asset prices should follow a random walk and therefore not exhibit long memory (see Fama, 1970) has been for decades the dominant paradigm in financial economics. However, the available empirical evidence is quite mixed. Mandelbrot (1972), Greene and Fielitz (1977), Booth et al. (1982), Helms et al. (1984), Caporale et al. (2014), Mynhardt et al. (2014) among others all provided evidence of long-memory behaviour in financial markets. By contrast, Lo (1991), Jacobsen (1995), Berg and Lyhagen (1998), Crato and Ray (2000), Batten et al. (2005) and Serletis and Rosenberg (2007) did not find long-memory properties in financial series. A possible reason for such different findings is that the degree of persistence might change over time as argued by Corazza and Malliaris (2002), Glenn (2007) and others.

The present study aims to examine this possible explanation by estimating persistence in financial time series at three different frequencies (daily, weekly and monthly. The analysis is carried out for various financial markets (stock markets, FOREX, commodity markets), for both returns and their volatility, over the period from 2000 to 2016 using two different long memory approaches (R/S analysis with the Hurst exponent method and fractional integration) for robustness purposes. The hypothesis to be tested is that persistence is higher at lower frequencies.

The layout of the paper is the following. Section 2 describes the data and outlines the empirical methodology. Section 3 presents the empirical results. Section 4 provides some concluding remarks.

2. Data and Methodology

The R/S method was originally applied by Hurst (1951) in hydrological research and improved by Mandelbrot (1972), Peters (1991, 1994) and others analysing the fractal nature of financial markets. Compared with other approaches it is relatively simple and suitable for programming as well as visual interpretation.

For each sub-period range R (the difference between the maximum and minimum index within the sub-period), the standard deviation S and their average ratio are calculated. The length of the sub-period is increased and the calculation repeated until the size of the sub-period is equal to that of the original series. As a result, each sub-period is determined by the average value of R/S. The least square method is applied to these values and a regression is run, obtaining an estimate of the angle of the regression line. This estimate is a measure of the Hurst exponent, which is an indicator of market persistence. More details are provided below.

1. We start with a time series of length M and transform it into one of length $N = M - 1$ using logs and converting prices into returns (or volatility):

$$N_i = \log\left(\frac{Y_{t+1}}{Y_t}\right), \quad t = 1,2,3,...\,(M-1) \qquad (1).$$

2. We divide this period into contiguous A sub-periods with length n, so that $A_n = N$, then we identify each sub-period as I_a, given the fact that a = 1, 2, 3. . . , A. Each element I_a is represented as N_k with k = 1, 2, 3. . . , N. For each I_a with length n the average e_a is defined as:

$$e_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a}, \quad k = 1,2,3,...N, \quad a = 1,2,3...,A \qquad (2).$$

3. Accumulated deviations $X_{k,a}$ from the average e_a for each sub-period I_a are defined as:

$$X_{k,a} = \sum_{i=1}^{k}(N_{i,a} - e_a). \qquad (3)$$

The range is defined as the maximum index $X_{k,a}$ minus the minimum $X_{k,a}$, within each sub-period (I_a):

$$R_{Ia} = \max(X_{k,a}) - \min(X_{k,a}), \quad 1 \leq k \leq n. \qquad (4)$$

4. The standard deviation S_{Ia} is calculated for each sub-period I_a:

$$S_{Ia} = \left((\frac{1}{n}) \sum_{k=1}^{n} (N_{k,a} - e_a)^2 \right)^{0,5}. \qquad (5)$$

5. Each range R_{Ia} is normalised by dividing by the corresponding S_{Ia}. Therefore, the re-normalised scale during each sub-period I_a is R_{Ia}/S_{Ia}. In the step 2 above, we obtained adjacent sub-periods of length n. Thus, the average R/S for length *n* is defined as:

$$(R/S)_n = (1/A) \sum_{i=1}^{A} (R_{Ia}/S_{Ia}). \qquad (6)$$

6. The length *n* is increased to the next higher level, *(M - 1)/n*, and must be an integer number. In this case, we use *n*-indexes that include the initial and ending points of the time series, and Steps 1 - 6 are repeated until *n = (M - 1)/2*.

7. Now we can use least square to estimate the equation *log (R / S) = log (c) + Hlog (n).* The angle of the regression line is an estimate of the Hurst exponent *H*. This can be defined over the interval [0, 1], and is calculated within the boundaries specified below (for more detailed information see Appendix C):

- $0 \leq H < 0.5$ – the data are fractal, the EMH is not confirmed, the distribution has fat tails, the series are anti-persistent, returns are negatively correlated, there is pink noise with frequent changes in the direction of price movements, trading in the market is riskier for individual participants.
- $H = 0.5$ – the data are random, the EMH is confirmed, asset prices follow a random Brownian motion (Wiener process), the series are normally distributed, returns are uncorrelated (no memory in the series), they are a white noise, traders cannot «beat» the market using any trading strategy.
- $0.5 < H \leq 1$ – the data are fractal, the EMH is not confirmed, the distribution has fat tails, the series are persistent, returns are highly correlated, there is black noise and a trend in the market.

There are different approaches to calculate the Hurst exponent (see Appendix A). In most cases de-trended fluctuation analysis (DFA) produces the best results (Weron, 2002; Grech and Mazur, 2004), but for financial series the R/S analysis seems to be the most appropriate (see Appendix B), and therefore is used here. The interpretation of the Hurst exponent is as follows: the higher it is, the lower the efficiency of the market is (Cajueiro and Tabak, 2005).

In order to analyse persistence, we also estimate parametric/semiparametric fractional integration or I(d) models. This type of models were originally proposed by Granger (1980) and Granger and Joyeux (1980); they were motivated by the observation that the estimated spectrum in many aggregated series exhibits a large value at the zero frequency, which is consistent with nonstationary behaviour; however, this becomes close to zero after differencing, which suggests over-differentiation. Examples of applications of fractional integration to financial time series data can be found in Barkoulas and Baum (1996), Barkoulas et al. (1997), Sadique and Silvapulle (2001), Henry (2002), Baillie et al. (2007), Caporale and Gil-Alana (2004, 2014) and Al-Shboul and Anwar (2016) among many others.

In this study we adopt the following specification:

$$(1 - L)^d x_t = u_t , \quad t = 0, \pm 1, ..., \tag{7}$$

where d can be any real value, L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is I(0), defined for our purposes as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Note that H and d are related through the equality H = d – 0.5.

In the semiparametric model no specification is assumed for u_t.The most common approach is based on the log-periodogram (see Geweke and Porter-Hudak, 1983). This method was later extended and improved by many authors including Künsch (1986), Robinson (1995a), Hurvich and Ray (1995), Velasco (1999a, 2000) and Shimotsu and

Phillips (2002). In this paper, however, we will employ instead another semiparametric method, which is essentially a local 'Whittle estimator' defined in the frequency domain using a band of high frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_{d} \left(\log \overline{C(d)} - 2\, d\, \frac{1}{m} \sum_{s=1}^{m} \log \lambda_s \right), \qquad (8)$$

$$\overline{C(d)} = \frac{1}{m} \sum_{s=1}^{m} I(\lambda_s)\, \lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{T}, \qquad \frac{m}{T} \to 0,$$

where m is a bandwidth parameter, and $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} x_t\, e^{i \lambda_s t} \right|^2,$$

and $d \in (-0.5, 0.5)$. Under finiteness of the fourth moment and other mild conditions, Robinson (1995b) proved that:

$$\sqrt{m}\, (\hat{d} - d_o) \to_d N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroscedasticity and is more efficient than other more recent semiparametric competitors. Recent refinements of this procedure can be found in Velasco (1999b), Velasco and Robinson (2000), Phillips and Shimotsu (2004, 2005) and Abadir et al. (2007).

Estimating d parametrically along with the other model parameters can be done in the frequency domain or in the time domain. In the former, Sowell (1992) analysed the exact maximum likelihood estimator of the parameters of the ARFIMA model, using a recursive procedure that allows a quick evaluation of the likelihood function. Other parametric methods for estimating *d* based on the frequency domain were proposed, among others, by Fox and Taqqu (1986) and Dahlhaus (1989) (see also Robinson, 1994 and Lobato and Velasco, 2007 for Wald and LM parametric tests based on the Whittle function).

We analyse both returns and their volatility. Returns are computed as follows:

$$R_i = (\frac{Close_i}{Open_i}\text{-}1) \times 100\% , \tag{9}$$

where R_i – returns on the *i-th*day inpercentage terms;

$Open_i$ – open price on the *i-th*day;

$Close_i$ – close price on the *i-th*day.

Volatility is defined as follows:

$$R_i = (\frac{High_i}{Low_i}\text{-}1) \times 100\% , \tag{10}$$

where R_i – returns on the *i-th*day in percentage terms;

$High_i$ – maximum price on the *i-th*day;

Low_i – minimum price on the *i-th*day.

Data from different financial markets (stock markets, FOREX and commodity markets) are used for the empirical analysis. Specifically, the following financial series are analysed: Dow Jones Index, FTSE index, NIKKEI for the developed stock markets (USA, Great Britain and Japan respectively) and MICEX and PFTS for the emerging ones (Russian and Ukraine respectively); the EUR/USD and USD/JPY exchange rates for the FOREX; Gold and Oil futures for the commodity markets). The sample period goes from 2000 to 2016 (in some cases it differs because of data unavailability).

3. Empirical Results

The results of the R/S analysis for the various financial markets are presented in Appendix D. As can be seen, in the case of stock markets returns are more persistent the lower the frequency is. The results for the commodity markets are more mixed. In the case of gold higher persistence is still found at lower frequencies, but in the case of oil the Hurst exponent is the same at the daily and monthly frequency, whilst it is higher at the weekly frequency, suggesting an increase in the degree of persistence at lower frequencies. In the FOREX, persistence of returns is the same across frequencies, except for the USDJPY exchange rate, whose monthly returns are much more persistent then daily ones.

Overall it appears that the evidence for returns is most consistent with the EMH in the case of the FOREX and least so in the case of stock markets. The observation that persistence is higher at lower frequencies suggests that for prediction purposes using data at such frequencies is most useful. Whilst most daily series follow a random walk, monthly ones exhibit long-memory properties seemingly inconsistent with the EMH. Concerning the results for volatility, we find that the daily series also follow a random walk, whilst the weekly and monthly ones have long memory and are persistent, this being true of the stock and FOREX markets, whilst in the case of the commodity markets persistence at the daily frequency is replaced by anti-persistence at the weekly and monthly ones. This suggests that markets are noisy and that abnormal profits can be made through volatility trading by using specific option trading strategies (butterfly, straddle, strangle, iron condor etc.).

The results for the fractional integration methods are presented in Appendix E. First, we display in Table E.1 the estimates of d along with their corresponding 95% confidence interval using a parametric method (Robinson, 1994). As before, the hypothesis that persistence is higher at lower frequencies cannot be rejected for the stock market series, since the estimated value of d increases as one moves from daily to weekly and monthly data. By contrast, no significant differences across frequencies emerge for the FOREX and commodity markets. As for the volatility series, there is evidence of long memory (i.e., $d > 0$) in all cases but no evidence of a higher degree of persistence at lower frequencies.

Appendix F focuses on the semi-parametric approach, first for the return series (Table F.1) and then for their volatilities (Table F.2). We find again higher persistence at lower frequencies for the stock markets considered, but not the FOREX and the commodity ones.

4. Conclusions

This paper uses both the Hurst exponent and parametric/semiparametric fractional integration methods to analyse the long-memory properties of financial data at different frequencies. The hypothesis of interest is that lower frequencies correspond to higher persistence. Daily, weekly and monthly (return and volatility) series from different financial markets (stock markets, FOREX and commodity markets) are analysed for the period from 2000 to 2016.

The findings suggest that in the case of returns daily data usually follow a random walk, consistently with the EMH, whilst at lower frequencies persistence is higher, which implies predictability and the possibility of making abnormal profits using appropriate trading strategies. This is true for the stock markets (both developed and emerging) and partially for the FOREX and commodity market considered. The results for the volatility series in the case of stock market are similar to those for returns, namely lower frequencies are associated to higher persistence, whilst in the commodity markets lower-frequency data are characterised by anti-persistence.

Very similar results are obtained when using fractional integration methods, be they parametric or semi-parametric: for returns the estimated value of d is higher at lower frequencies for the stock markets analysed, though basically the same across frequencies for the other markets examined. However, for the FOREX and commodity markets, we do not find significant differences across frequencies. For the volatility series, the observed long-memory properties (i.e., $d > 0$) are also unaffected by the data frequency. Obviously in all cases when persistence is higher at lower frequencies there exist profit opportunities (through appropriately designed trading strategies) that are inconsistent with market efficiency.

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Appendix A

Table A.1: Methodology for the Hurst exponent calculations: general review

Author(s)	Methodology*	Results
Taqquetal., (1995)	R/S, DFA	R/S overestimates the Hurst exponent, DFA underestimates it.
Weron, R. (2002)	R/S, DFA	DFA exceeds R/S
Kantelhardtetal., (2002)	MF -DFA	MF -DFA estimations are better than those from the R/S – analysis
Couillard and Davison, (2005)	R/S analysis	No long memory in financial data is detected.
Grechand Mazur, (2004)	DFA, DMA	DFA exceeds DMA
Teverovsky, Taqqu, Willinger (1999)	R/S	A variety of shortcomings in the R/S methodology are detected
Lo (1991)	R/S (modified)	Using the modified R/S analysis short-term memory is detected instead of long-term memory. In general the results provide evidence in favour of the EMH.

* rescaled range analysis (R/S), generalized Hurst exponent approach (GHE), detrended moving average (DMA), detrended fluctuation analysis (DFA), multifractal generalization (MF-DFA)

Appendix B

Hurst exponent in financial data: general overview

Table B.1: Hurst exponent calculation methodology applied for financial data

Author	Methodology	Data and period	Results
Barunik, Jozef & Kristoufek, Ladislav, (2010)	R/S, GHE, DMA, DFA, MF-DFA	S&P 500 Index (1983-2009)	GHE methodology provides better results. R/S-analysis is stable for the fat tails in the data. MF- DFA and DMA are inappropriate for data with fat tails.
Hja Su, LinYang (2003)	R/S	Chinese Stock Market (1991-2001)	Short-term memory is detected but there is no long-term dependence in the data
Greene and Fielitz (1977)	R/S	US Stock Market (NYSE)	Substantial evidence in favour of long-term dependency
Peters (1991) and Peters (1994)	R/S	S&P 500 Index (1950 – 1988)	Hurst exponent equals 0.78 for the monthly returns in S&P 500 data. Evidence in favour of persistence in data
Corazza and Malliaris (2002).	R/S	FOREX (1972-1994)	Hurst exponent statistically differs from 0.5 and in not stable over time
Glenn (2007)	R/S	NASDAQ	Hurst exponent for daily data equals 0.59 but increases to 0.87 for annual data
Lento, Camillo (2013)	R/S	DJIA (1998-2008)	Hurst exponent can identify the persistence properties in the data
Onali, Enrico and Goddard, John (2011)	R/S	Mibtel (Italy) andPX-Glob (Czech Republic).	Evidence in favour of long-term dependence in logarithm returns
Serletis and Rosenberg (2009)	R/S	US Stock Market	No long-term dependence
Batten, Elli, and Fetherston (2005)	R/S	Nikkei Index (1980 -2000)	No long memory is detected
Berg, Lennart and Lyhagen, Johan (1998)	R/S	Swedish Stock Market (1980-1995)	Evidence in favour of the long-term dependence in data ais not clear
Lo (1991)	R/S (modified)	US Stock Market (1872-1986)	No long-term dependence
Ding et al. (1993)	R/S	S&P 500 Index	Evidence of long-term memory in returns
Jacobsen, Ben (1995)	R/S	European, USA and Japan Stock Markets	No long-memory is detected
Barkoulas, Labys, and Onochie (1997)	R/S	Futures markets	Stable evidence of long memory in futures returns
Crato and Ray (2000)	R/S	Commodities (1977-1997)	No persistence in the case of returns, but evidence of long memory in volatility.

Appendix C

Hurst exponent interval characteristics

Table C.1: Hurst exponent interval characteristics

Interval	Hypothesis	Distribution	«Memory» of series	Type of process	Trading strategies
$0 \leq H < 0,5$	Data is fractal, FMH is confirmed	"Heavy tails" of distribution	Anti-persistent series, negative correlation in instruments value changes	Pink noise with frequent changes in direction of price movement	Trading in the market is more risky for an individual participant
$H = 0,5$	Data is random, EMH is confirmed	Movement of asset prices is an example of the random Brownian motion (Wiener process), time series are normally distributed	Lack of correlation in changes in value of assets (memory of series)	White noise of independent random process	Traders cannot "beat" the market with the use of any trading strategy
$0,5 < H \leq 1$	Data is fractal, FMH is confirmed	"Heavy tails" of distribution	Persistent series, positive correlation within changes in the value of assets	Black noise	Trend is present in the market

Appendix D

R/S analysis

Table D.1: Results of the R/S analysis for the different financial markets, 2004-2016

Financial market	Instrument	Return	Volatility
i) Daily data			
FOREX	EURUSD	0,55	0,48
	USDJPY	0,56	0,43
Stock market	Dow Jones	0,51	0,46
	FTSE	0,47	0,47
	NIKKEI	0,54	0,68
	MICEX	0,55	0,46
	PFTS	0,67	0,46
Commodities	Oil	0,57	0,62
	Gold	0,54	0,66
ii) Weekly data			
FOREX	EURUSD	0,56	0,36
	USDJPY	0,57	0,43
Stock market	Dow Jones	0,56	0,53
	FTSE	0,52	0,56
	NIKKEI	0,57	0,51
Commodities	Oil	0,64	0,46
	Gold	0,56	0,40
iii) Monthly data			
FOREX	EURUSD	0,55	0,38
	USDJPY	0,66	0,42
Stock market	Dow Jones	0,73	0,63
	FTSE	0,74	0,46
	NIKKEI	0,68	0,57
	MICEX	0,61	0,42
	PFTS	0,73	0,53
Commodities	Oil	0,57	0,34
	Gold	0,63	0,41

Appendix E

Fractional integration. Parametric method

Table E.1: Estimates of d using uncorrelated (white noise) errors

Financial market	Instrument	Return	Volatility
i) Daily data			
FOREX	EURUSD	-0.01 (-0.03, 0.01)	0.26 (0.25, 0.28)
	USDJPY	-0.03 (-0.05, -0.01)	0.25 (0.23, 0.27)
Stock market	Dow Jones	-0.08 (-0.10, -0.06)	0.36 (0.34, 0.38)
	FTSE	-0.15 (-0.17, -0.13)	0.33 (0.30, 0.34)
	NIKKEI	-0.05 (-0.08, -0.03)	0.34 (0.32, 0.36)
	MICEX	-0.02 (-0.04, 0.00)	0.39 (0.37, 0.41)
	PFTS	0.10 (0.08, 0.12)	
Commodities	Oil	-0.01 (-0.03, 0.01)	0.26 (0.24, 0.27)
	Gold	-0.02 (-0.04, 0.00)	0.27 (0.26, 0.29)
ii) Weekly data			
FOREX	EURUSD	0.01 (-0.03, 0.06)	0.31 (0.28, 0.35)
	USDJPY	-0.03 (-0.06, 0.02)	0.26 (0.23, 0.30)
Stock market	Dow Jones	-0.06 (-0.10, -0.01)	0.39 (0.35, 0.44)
	FTSE	-0.12 (-0.15, -0.07)	0.42 (0.38, 0.48)
	NIKKEI	-0.04 (-0.08, 0.00)	0.37 (0.33, 0.42)
Commodities	Oil	0.01 (-0.03, 0.06)	0.35 (0.32, 0.38)
	Gold	-0.02 (-0.05, 0.02)	0.60 (0.55, 0.66)
iii) Monthly data			
FOREX	EURUSD	-0.01 (-0.09, 0.10)	0.30 (0.24, 0.38)
	USDJPY	0.02 (-0.06, 0.12)	0.28 (0.20, 0.39)
Stock market	Dow Jones	0.03 (-0.07, 0.15)	0.28 (0.20, 0.39)
	FTSE	0.02 (-0.07, 0.12)	0.29 (0.21, 0.40)
	NIKKEI	0.08 (-0.01, 0.21)	0.31 (0.23, 0.42)
	MICEX	0.11 (0.01, 0.26)	0.47 (0.39, 0.58)
	PFTS	0.21 (0.08, 0.41)	
Commodities	Oil	-0.01 (-0.10, 0.11)	0.45 (0.39, 0.54)
	Gold	-0.07 (-0.14, 0.01)	0.49 (0.42, 0.60)

Appendix F

Semi-parametric method

Table F.1: Estimates of d for the return series

i) Daily data										
		56	58	60	62	64	66	68	70	72
FOREX	Euro	0.015	0.005	0.016	0.016	0.013	-0.008	-0.001	-0.006	0.000
	DJPY	0.129	0.107	0.112	0.111	0.104	0.121	0.110	0.101	0.102
Stock Market	D & J	**-0.041**	**-0.037**	-0.030	-0.025	-0.009	-0.020	-0.020	-0.009	-0.001
	FTSE	-0.214	-0.228	-0.228	-0.215	-0.233	-0.240	-0.240	-0.247	-0.237
	Nikkei	0.010	0.002	0.004	0.002	0.009	0.004	0.001	0.009	0.022
	MICEX	0.113	0.107	0.078	0.082	0.079	0.051	0.059	0.070	0.073
Comm.	Oil	-0.040	-0.036	-0.036	-0.038	-0.036	-0.030	-0.030	-0.032	-0.031
	Gold	0.042	0.018	0.015	-0.020	-0.054	-0.043	-0.063	-0.076	-0.075
ii) Weekly data										
		22	24	26	28	30	32	34	36	38
FOREX	Euro	0.047	0.001	0.014	0.020	0.008	0.042	0.027	0.025	0.032
	DJPY	-0.030	-0.015	-0.014	0.014	0.033	0.063	0.080	0.095	0.130
Stock Market	D & J	0.091	0.029	0.072	0.102	0.121	0.079	0.044	0.080	0.063
	FTSE	0.207	0.122	0.074	0.115	0.067	0.093	0.073	0.061	-0.009
	Nikkei	0.014	0.050	0.046	0.082	0.073	0.116	0.091	0.103	0.125
Comm.	Oil	-0.069	-0.042	-0.013	0.032	0.033	0.050	0.000	0.004	-0.009
	Gold	0.097	0.106	0.098	0.107	0.141	0.105	0.067	0.056	0.009
iii) Monthly data										
		11	12	13	14	15	16	17	18	19
FOREX	Euro	-0.121	-0.114	-0.066	-0.059	-0.045	-0.019	0.025	0.072	0.089
	DJPY	0.306	0.285	0.262	0.260	0.220	0.208	0.129	-0.004	-0.009
Stock Market	D & J	0.127	0.120	0.132	-0.100	-0.035	0.015	-0.004	-0.018	0.023
	FTSE	0.265	0.124	0.058	0.062	0.019	0.040	0.047	0.090	0.108
	Nikkei	0.076	0.035	0.039	0.002	0.049	0.101	0.002	-0.037	-0.020
	MICEX	-0.098	-0.082	-0.057	-0.084	-0.045	-0.019	-0.036	-0.054	-0.066
Comm.	Oil	-0.085	-0.103	-0.054	-0.101	-0.070	-0.087	-0.114	-0.096	-0.151
	Gold	0.175	0.222	0.215	0.147	0.155	0.111	0.102	0.097	0.101

Table F.2: Estimates of d for the volatility series

i) Daily data										
		56	58	60	62	64	66	68	70	72
FOREX	Euro	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	DJPY	0.448	0.462	0.483	0.493	0.500	0.500	0.500	0.500	0.500
Stock Market	D & J	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	FTSE	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	Nikkei	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	MICEX	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
Comm.	Oil	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	Gold	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
ii) Weekly data										
		22	24	26	28	30	32	34	36	38
FOREX	Euro	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	DJPY	0.403	0.444	0.429	0.448	0.443	0.375	0.376	0.396	0.426
Stock Market	D & J	0.362	0.365	0.392	0.373	0.409	0.401	0.399	0.400	0.412
	FTSE	0.417	0.421	0.420	0.411	0.403	0.437	0.429	0.446	0.447
	Nikkei	0.450	0.461	0.499	0.444	0.449	0.434	0.449	0.444	0.418
Comm.	Oil	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	Gold	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
iii) Monthly data										
		11	12	13	14	15	16	17	18	19
FOREX	Euro	0.484	0.448	0.436	0.461	0.500	0.500	0.500	0.483	0.475
	DJPY	0.306	0.285	0.262	0.260	0.220	0.208	0.129	-0.004	-0.009
Stock Market	D & J	0.391	0.362	0.316	0.306	0.331	0.305	0.326	0.308	0.337
	JTSE	0.065	-0.120	-0.058	-0.062	-0.019	0.040	0.047	0.090	0.108
	Nikkei	0.076	0.035	-0.039	0.002	0.049	0.101	0.002	-0.037	-0.020
	MICEX	-0.098	-0.082	-0.057	-0.084	-0.045	-0.019	-0.036	-0.054	-0.066
Comm.	Oil	-0.085	-0.103	-0.054	-0.101	-0.070	-0.087	-0.114	-0.096	-0.151
	Gold	0.175	0.222	0.215	0.147	0.155	0.111	0.102	0.097	0.101

In bold, statistical evidence of long memory (d > 0) in the volatility processes. Please note that d can only be estimated in the case of stationarity (i.e., d>0.5) and is set equal to 0.5 otherwise.