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VOLATILITY FORECASTS FOR THE RTS STOCK INDEX: OPTION-IMPLIED VOLATILITY VERSUS ALTERNATIVE METHODS

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Abstract

This paper compares volatility forecasts for the RTS Index (the main index for the Russian stock market) generated by alternative models, specifically option-implied volatility forecasts based on the Black-Scholes model, ARCH/GARCH-type model forecasts, and forecasts combining those two using a mixing strategy based either on a simple average or a weighted average with the weights being determined according to two different criteria (either minimizing the errors or maximizing the information content). Various forecasting performance tests are carried out which suggest that both implied volatility and combination methods using a simple average outperform ARCH/GARCH-type models in terms of forecasting accuracy.

Keywords: option-implied volatility, ARCH-type models, mixed strategies

JEL Classification: C22, G12

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1. Introduction

Derivatives, and options in particular, have become increasingly sophisticated financial instruments designed to deal with the uncertainty resulting from volatile asset prices. A popular method for forecasting their volatility is the option-implied volatility (IV) approach introduced by Black and Scholes (1972), who analysed the efficiency of options market and derived the most commonly applied formula for the estimation of European option prices. The evidence on the forecasting performance of implied volatility is rather mixed, partly because of the different forecasting techniques used by researchers. Doidge and Wei (1998) reported that in the case of the Canadian stock market non-simultaneity of prices and a non-competitive trading environment led to a poor performance of the IV estimator. Canina and Figlewski (1993) and Day and Lewis (1992) showed that for S&P 100 stock index implied volatility does not contain any valuable information, and mentioned model misspecification and expiration day effects as possible reasons; they also concluded that the Treasury bill rate is not a good proxy for the rate faced by an options arbitrageur.

In contrast, other studies found that IV forecasts outperform time series volatility forecast. For example, Christensen and Prabhala (1998) reached this conclusion for the S&P 100 stock index, and argued that using non-overlapping samples was the reason for the efficiency and unbiasedness of IV forecasts. Neely (2005) detected a strong linkage between changes in implied volatility and important economic events for the three-month eurodollar interest rates.

This paper focuses on volatility forecasting in the case of the RTS Index, one of the most traded stock indices in Russia for which no previous evidence is available. Specifically, it carries out various tests to compare the volatility forecasts generated by alternative models, namely option-implied volatility forecasts based on the Black-Scholes model, ARCH/GARCH-type model forecasts, and forecasts combining the former two using a mixing strategy based either on a simple average or a weighted average with the weights being determined according to two different criteria (either minimizing the errors or maximizing the information content).

The rest of the paper is organised as follows: Section 2 outlines the methodology; Section 3 presents the forecasting performance results; finally, Section 4 offers some concluding remarks.

2. Methodology

2.1 Sampling procedure

We choose the RTS Index as the underlying asset for a number of reasons. First, existing studies have typically analysed the forecasting performance of implied volatility in the case of the main American and European markets, whilst there is no evidence concerning the Russian one. Second, the RTS Index is one of the most representative ones for the Russian market. It is a free-float capitalization-weighted composite index of Russian stocks of the largest and dynamically developing issuers traded on the Moscow Exchange. The stocks included are the 50 most liquid ones, which are reviewed on a quarterly basis (see the Moscow Exchange website, https://www.moex.com/en/index/rtsi). Third, options on the RTS Index futures contracts are plain vanilla, which is one of the assumptions of the Black-Scholes model.

The RTS Index was launched on 1 September 1995. It is calculated on a real time basis and is denominated in US dollars. The options included have the following characteristics:

- They are futures-style options on RTS Index futures contracts;
- Expiration occurs on the third Thursday of every quarter (March, June, September, December). On the Moscow Exchange there are RTS options that expiry quarterly, monthly and even weekly; however, the present study focuses only on options with quarterly expiration because of their high liquidity;
- They are American-style options, i.e. they allow holders to exercise them at any time prior to and including the maturity date.

The frequency is daily and the sample period goes from 5 January 2014 to 31 October 2018. This is a sufficiently long span of data for carrying out appropriate tests; moreover, it includes the Russian financial crisis of 2014-2015, which is particularly interesting for testing the predictive power of implied volatility. Finally, as a proxy for the risk-free rate we use daily observations on the 1-month MosPrime Rate. This rate is calculated by the National Foreign Exchange Association together with Thomson Reuters on the basis of the offer rates of deposits denominated in rubles quoted by the leading money market participants to the superior financial institutions (see the Central Bank of the Russian Federation website cbr.ru/eng/hd_base/mosprime). Given the findings of the existing literature, the analysis is based on non-overlapping samples, and uses the closest expiration term of the option series and the prices of option contracts at the end of each trading day.

2.2. Implied volatility forecasts

A European-style call (put) option gives the right, but not the obligation, to purchase (sell) an asset at a strike price at maturity date (Poon and Granger, 2003). For pricing such options the well-known Black–Scholes formula can be used (Black and Scholes, 1973). This is a partial differential equation, based on the idea that one can hedge by trading the underlying asset, which involves modelling a call option price C or a put option price P as follows (see Hull, 2008):

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0, \text{ where} \qquad (1)$$

where V (S,t) is the price of an option as a function of time and stock prices;

$t \in [0;T]$ stands for time in years;

S is the price of the underlying asset;

r is the annualized continuously compounded risk-free interest rate;

σ is the volatility of the underlying asset.

According to the Black and Scholes model and using Itô's lemma, the logarithm of the price of the underlying asset should have the following dynamic specification:

$$dlnS = (\mu - \frac{\sigma^2}{2})dt + \sigma dz \qquad (2)$$

or alternatively:

$$lnS_t = lnS_0 \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}, \textit{ where} \qquad (3)$$

where lnS_t is the natural logarithm of the price level of the underlying asset;

lnS_0 is the natural logarithm of the initial price level of the underlying asset;

μ is the mean value;

σ is the standard deviation;

W_t is a standard Wiener process (Brownian motion).

In other words, the logarithm of the price of the underlying asset is assumed to follow a normal distribution with parameters μ and σ.

In the case of the American-style options on RTS Index futures contracts considered in the present study, the standard Black and Scholes model should be modified as follows. First, the equality defined before for European-style options becomes an inequality of the following form:

$$\begin{cases} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV \le 0 \\ terminal\ and\ boundary\ conditions: \\ \quad V(S,T) = H(S) \\ \quad V(S,t) \ge H(S) \end{cases}, \text{ where} \quad (4)$$

$H(S)$ - the payoff of the option when the price of the underlying asset is S.

Second, for futures contract the spot price of the underlying asset is replaced by the discounted futures price (Black, 1976):

$$S = e^{-r(T-t)}F \quad (5)$$

The model discussed above allows to determine the option price as a function of a number of variables:

- F_t – the current price of the futures contract;
- X – the strike price of the option;
- r – the risk-free interest rate;
- T-t – time remaining to maturity;
- σ – volatility of the underlying asset over the time remaining to maturity.

Note that F_t , X, r and (T-t) can be observed, and that the option price is also known, either as a quote or because there was a transaction; here we use the price from real trades. Using backward induction, one can derive the volatility implied by the formula above which is used by market participants to determine the quote; this is the variable being examined.

The amended call and put option prices are the following:

$$C^{BS}(T-t, X, F_t, r, \sigma) = e^{-r(T-t)} \cdot (F_t \cdot N(d_1) - X \cdot N(d_2)) \quad (6)$$

$$P^{BS}(T-t, X, F_t, r, \sigma) = e^{-r(T-t)} \cdot (X \cdot N(-d_2) - F_t \cdot N(-d_1)),\ where \quad (7)$$

$d_1 = \frac{\ln\left(\frac{F_t}{X}\right) + (T-t)\cdot\frac{\sigma^2}{2}}{\sigma\cdot\sqrt{(T-t)}}$ and $d_2 = d_1 - \sigma \cdot \sqrt{T-t}$

C^{BS} – call option price;

P^{BS} – put option price;

N(d) – standard normal distribution function.

Under the assumption of market efficiency the market price and that implied by the Black and Scholes equations should be the same, i.e.

$C^{Market} = C^{BS}(T-t, X, S_0, \gamma, r, \sigma^{IV})$ and $P^{Market} = P^{BS}(T-t, X, S_0, \gamma, r, \sigma^{IV})$.

This equality is the basis for calculating implied volatility and then assessing its forecasting properties. Note that the Black and Scholes (1973) model is based on the following assumptions:

- the volatility of the underlying asset, σ, is constant;
- the risk-free interest rate, r, is known and constant over time;
- the underlying price follows the log-normal distribution;
- the index pays no dividends;
- there are no transaction costs or taxes;
- the underlying asset is divisible;
- there are no restrictions for short-selling;
- there is continuous trading without arbitrage.

These are "ideal conditions" and violation of any of them will result in some inaccuracy in the estimated theoretical price. In practice at least some of them are not satisfied. For instance, consider the assumption of constant volatility. Options with different strikes but with the same time to maturity typically give different values for the implied volatility for the same underlying asset; in particular, options either deep in-the-money or out-of-the-money tend to produce higher values for the implied volatility; this phenomenon is known as "volatility smile". Let us analyse it in the specific case of the RTS options considered here, which expire quarterly (on the 3rd Thursday of March, June, September, December). Figures 1 to 4 show the relationship between implied volatility and the central strike, i.e. the strike closest to the settlement price of RTS futures, on two different dates, in the case of call and put options respectively. The tick size for the RTS option contract is 2500, so shift +1 stands for the central strike + 2500.

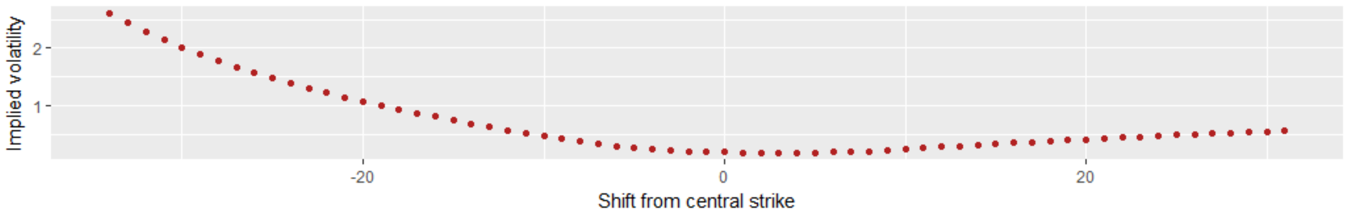

Figure 1. Volatility smile of the call RTS option on the 16th of November 2017

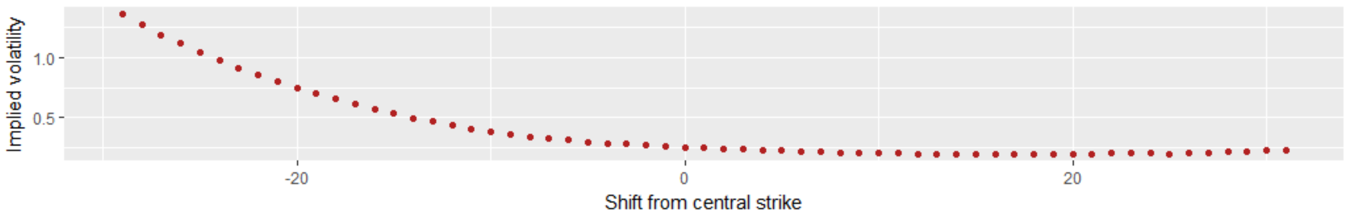

Figure 2. Volatility smile of the call RTS option on the 15th of May 2017

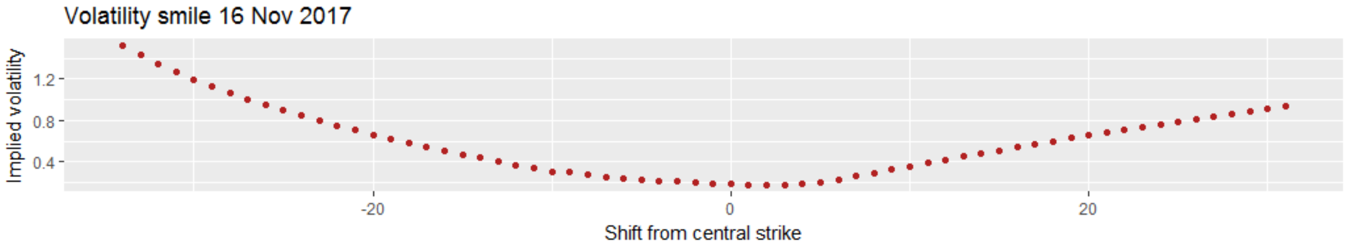

Figure 3. Volatility smile of the put RTS option on the 16th of November 2017

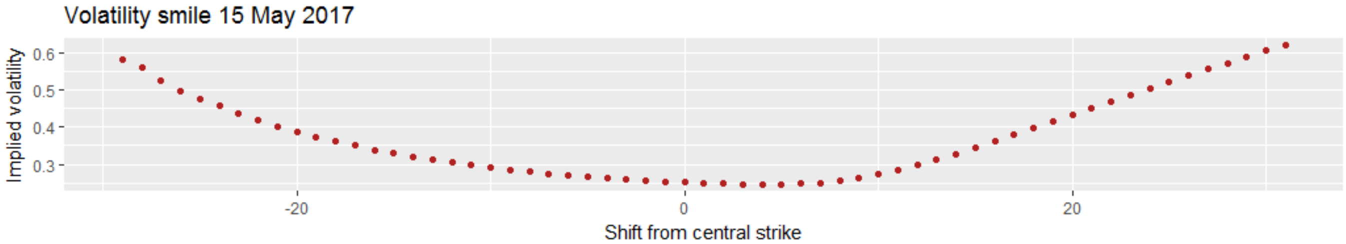

Figure 4. Volatility smile of the put RTS option on the 15th of May 2017

Poon and Granger (2003) and Christensen and Prabhala (1998) highlight four possible reasons for this “puzzle”:

Distributional assumptions. In the Black and Scholes model the price of the underlying asset is assumed to follow the lognormal distribution (3). However, numerous studies provide evidence of leptokurtic tails (see, e.g., Blattberg and Gonedes, 1974; Fama, 1965), which results in overestimating the implied volatility at very low and very high strikes.

Stochastic volatility. The underlying asset might have its own dynamics and volatility. To avoid this type of problem researchers tend to use at-the-money options for forecasting.

Market microstructure and measurement errors. The no-arbitrage, zero transaction cost and continuous trading conditions of an ideal trading environment are normally not met in practice, which leads to market inefficiencies and option prices deviating from their theoretical price.

Investor risk preferences. In the Black and Scholes model investor risk preferences are irrelevant in option pricing. However, in practice these affect option prices, and in turn their volatility.

2.3 Comparisons of Historical and IV Forecasts

IV Forecasts

IV forecasts are market-based volatility forecasts reflecting the expectations of market participants and are therefore an ex-ante measure. In order to compute the ex-post volatility over the remaining time to maturity the sample standard deviation for daily index returns can be used:

$$\sigma_t = \sqrt{\frac{1}{N-1} \cdot \sum_{t=1}^{N} (R_t - \bar{R})^2}, \tag{8}$$

where

$\bar{R}$ is the mean RTS index return;

N is the total number of observations.

Canina and Figlewski (1993) suggested a forecasting horizon of 35 days, whilst Doidge and Wei (1998) assumed that the optimal number of days to compute the historical variance is 100. The calculations in the present study are based on 75 trading days or a quarter, arguably a time horizon more relevant to investors who trade options with quarterly expiration as in the case of this paper. Usually this type of investors rollover at the maturity date, when they reinvest a mature contract into a new issue of the same underlying asset.

To assess the information content of the implied volatility the following regression can be run (*Figure 5-6. A*):

$$\sigma_t^{Act} = a_0 + a_1 \cdot \sigma_t^{IV} + e_t^{IV} \tag{9}$$

where σ_t^{Act} stands for actual volatility, and σ_t^{IV} for implied volatility. Three hypotheses can then be tested. The first one is whether or not implied volatility is informative about future volatility; in this case the coefficient on σ_t^{IV} should be different from zero:

$$H_0: \alpha_1 \neq 0$$

The second hypothesis of interest is whether or not forecasts based on implied volatility are unbiased. This implies that the coefficient on σ_t^{IV} should be 1, while the constant term should be 0:

$$H_0: \begin{cases} \alpha_0 = 0 \\ \alpha_1 = 1 \end{cases}$$

Finally, one can test whether the implied volatility is an efficient forecasting measure. In that case, the error term should follow a white noise process and be uncorrelated with σ_t^{IV}:

$$H_0: \begin{cases} e_t \sim WN \\ Cov(e_t, \sigma_t^{IV}) = 0 \end{cases}$$

If all three hypotheses are satisfied and the model is found to be data congruent, the estimated coefficients and the residuals can then be used in mixed strategies for forecasting the actual volatility index.

ARCH/GARCH-type Forecasts

Next we analyse forecasts based on historical data generated from ARCH/GARCH models. As a first step we use the standard GARCH framework introduced by Engle (1982) and Bollerslev (1986). In line with most of the existing literature, a parsimonious GARCH(1,1) specification is assumed to be sufficient to capture the stochastic behaviour of the volatility index.

The GARCH(1,1) model consists of 2 equations, one of which is a conditional mean equation (10), and the other a conditional variance equation (11):

$$R_t = \theta_0 + \theta_1 \cdot R_{t-1} + \epsilon_t \tag{10}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \cdot \epsilon_{t-1}^2 + \beta_1 \cdot \sigma_{t-1}^2 \tag{11}$$

$$\epsilon_t | \Psi_{t-1} \sim N(0, \sigma_t^2),$$

where

- Ψ_{t-1} is the information set that contains all information which is available at time t-1;
- $N(0, \sigma_t)$ is the normal distribution function.

In order to obtain a well-defined process two restrictions should be satisfied. The first one rules out non-stationarity, and it requires the sum of the coefficients to be inside the unit circle: $\alpha_1 + \beta_1 < 1$. The second ensures non-negativity of volatility, and it implies that all coefficients should not be less than 0: $(\alpha_0, \alpha_1, \beta_1) \geq 0$.

Then we also examine the forecasts obtained using a number of alternative ARCH/GARCH specifications suggested in the literature:

- GJR-GARCH (Glosten, Jagannathan and Runkle (1993)), which captures asymmetries in the ARCH process since there is empirical evidence that negative shocks have a stronger impact on returns volatility than positive shocks, which is known as the "leverage effect";
- T-ARCH (Glosten (1993), Zakoian (1994)), which is similar to GJR GARCH, but uses the standard deviation instead of the conditional variance;
- TS-ARCH (Taylor (1986), Schwert (1989)), which is a standard GARCH specification, but again using the standard deviation rather than the variance;

- EGARCH (Nelson (1991), which implies an exponential rather than quadratic leverage effect;
- IGARCH, which restricts the parameters of the standard GARCH model to sum up to one.

After obtaining the estimates for volatility based on the different ARCH/GARCH specifications, one can then run the same regression as before to assess the information content of the historical volatility for predicting the ex-post volatility (*Figure 5-6. B*):

$$\sigma_t^{Act} = a_2 + a_3 \cdot \sigma_t^G + e_t^G \tag{12}$$

where σ_t^G stands for the ARCH/GARCH-type volatility estimate. As before, the estimated coefficients and the residuals are then used in the mixed strategies.

Our preferred specification, which minimizes the information criteria (AIC, BIC) and maximizes the R-squared, is the IGARCH model; therefore this is the one used in the forecasting comparison exercise.

Specification	AIC	BIC	R_squared
GARCH	-911.7083	-899.1189	0.3067808
GJR-GARCH	-932.8368	-920.2474	0.3359784
T-ARCH	-871.7767	-859.1873	0.2480474
TS-ARCH	-915.4036	-902.8143	0.3119785
EGARCH	-897.4221	-884.8328	0.2863147
IGARCH	-944.6279	-932.0386	0.3517346

Table 1. Comparison of the ARCH-type Models for Call Options

Specification	AIC	BIC	R_squared
GARCH	-899.0506	-886.4858	0.3072366
GJR-GARCH	-928.6212	-916.0564	0.3480495
T-ARCH	-866.6262	-854.0614	0.2595424
TS-ARCH	-903.5805	-891.0157	0.3136505
EGARCH	-891.0926	-878.5278	0.2958232
IGARCH	-931.8367	-919.2719	0.3523399

Table 2. Comparison of the ARCH-type Models for Put Options

Mixed Strategies

Mixed strategies aim at producing more accurate forecasts than those based on either implied volatility or ARCH/GARCH-type models, both of which have been shown to perform relatively poorly (see, e.g., Beckers (1981), Canina and Figlewski (1993), Doidge and Wei (1998)). The idea is to combine the information provided by the two approaches considered so far in order to obtain better forecasts.

To begin with, we apply the simple method developed by Vasilellis and Meade (1996). An equal weight is assigned to each of the two volatility measures (*Figure 5-6. C*):

$$\omega_t^G = \omega_t^{IV} = 0.5$$

$$\sigma_t^{simple\ weighted} = \omega_t^G \cdot \sigma_t^G + \omega_t^{IV} \cdot \sigma_t^{IV}, \qquad (13)$$

where

σ_t^G is volatility estimated using the IGARCH(1,1) model at time t;

σ_t^{IV} is volatility estimated using the Black and Scholes model at time t.

It is a simple average of the two measures that assumes that they are both equally informative and that their respective informational content is constant over time.

By contrast, below we give a larger weight to the procedure with the lowest error term (e_t^{IV} from (9) and e_t^G from(12)), where the weight ω_t of each method is calculated using the share of its inverse error ($1/e_t$) in the cumulative inverse error term (*Figure 5-6. D*):

$$\omega_t^G = \frac{1/e_t^G}{1/e_t^{IV} + 1/e_t^G} \qquad (14)$$

$$\omega_t^{IV} = \frac{1/e_t^{IV}}{1/e_t^{IV} + 1/e_t^G} \qquad (15)$$

Alternatively, one can assign weights using a different criterion, i.e. by maximizing the information content rather than minimizing the error term. In this case, the weights will be a function of the standardized estimated coefficients a_1 and a_3 from equations (6) and (9) respectively, namely (*Figure 5-6. E*):

$$\omega_t^G = \frac{a_3}{a_3 + a_1} \qquad (16)$$

$$\omega_t^{IV} = \frac{a_1}{a_3 + a_1} \quad (17)$$

Actual and implied volatility in each case are shown in Figure 5 and 6 for call and put options respectively:

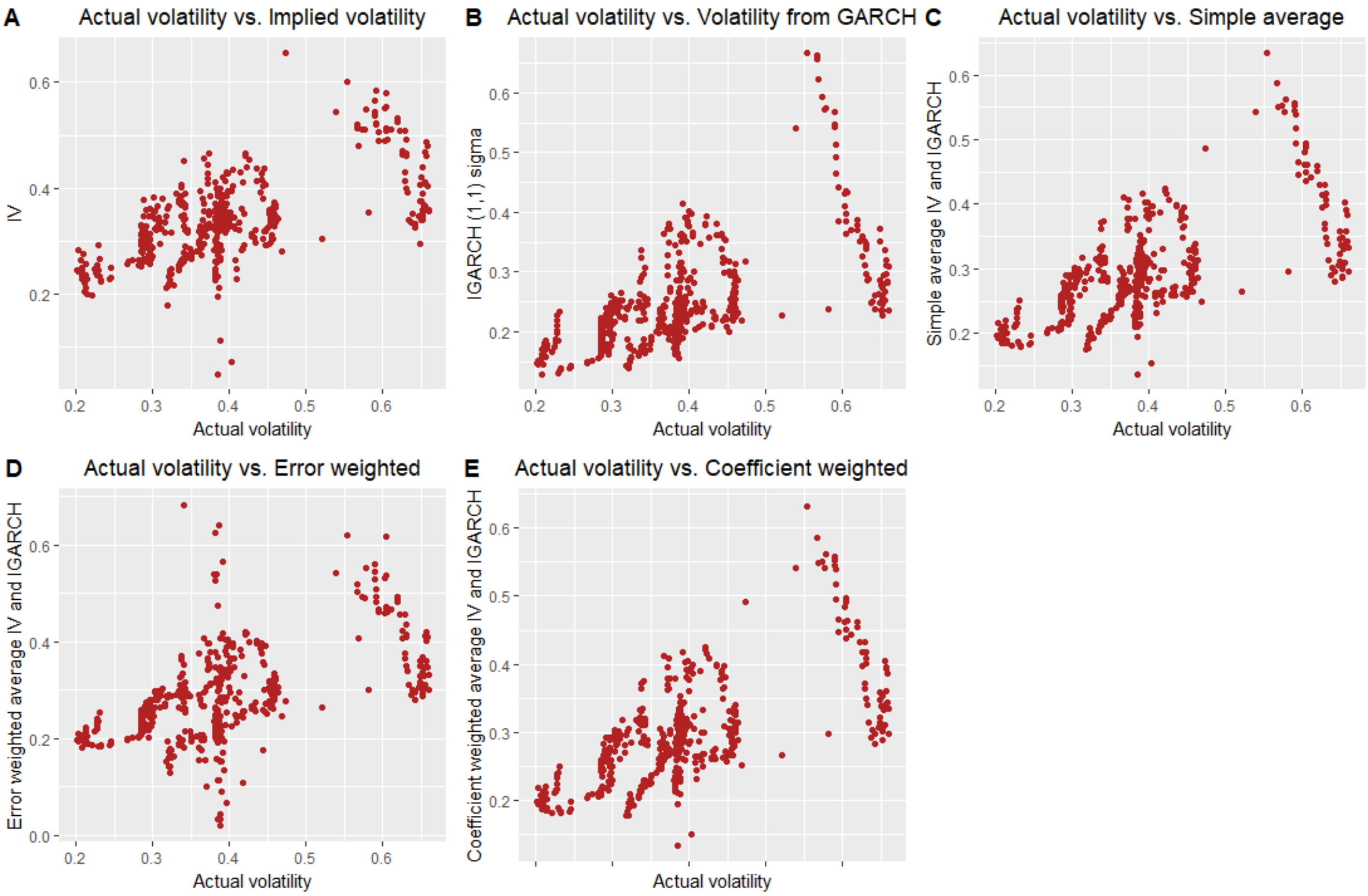

Figure 5. Regressions for Actual Volatility of the RTS Index Based on Data from Call Options

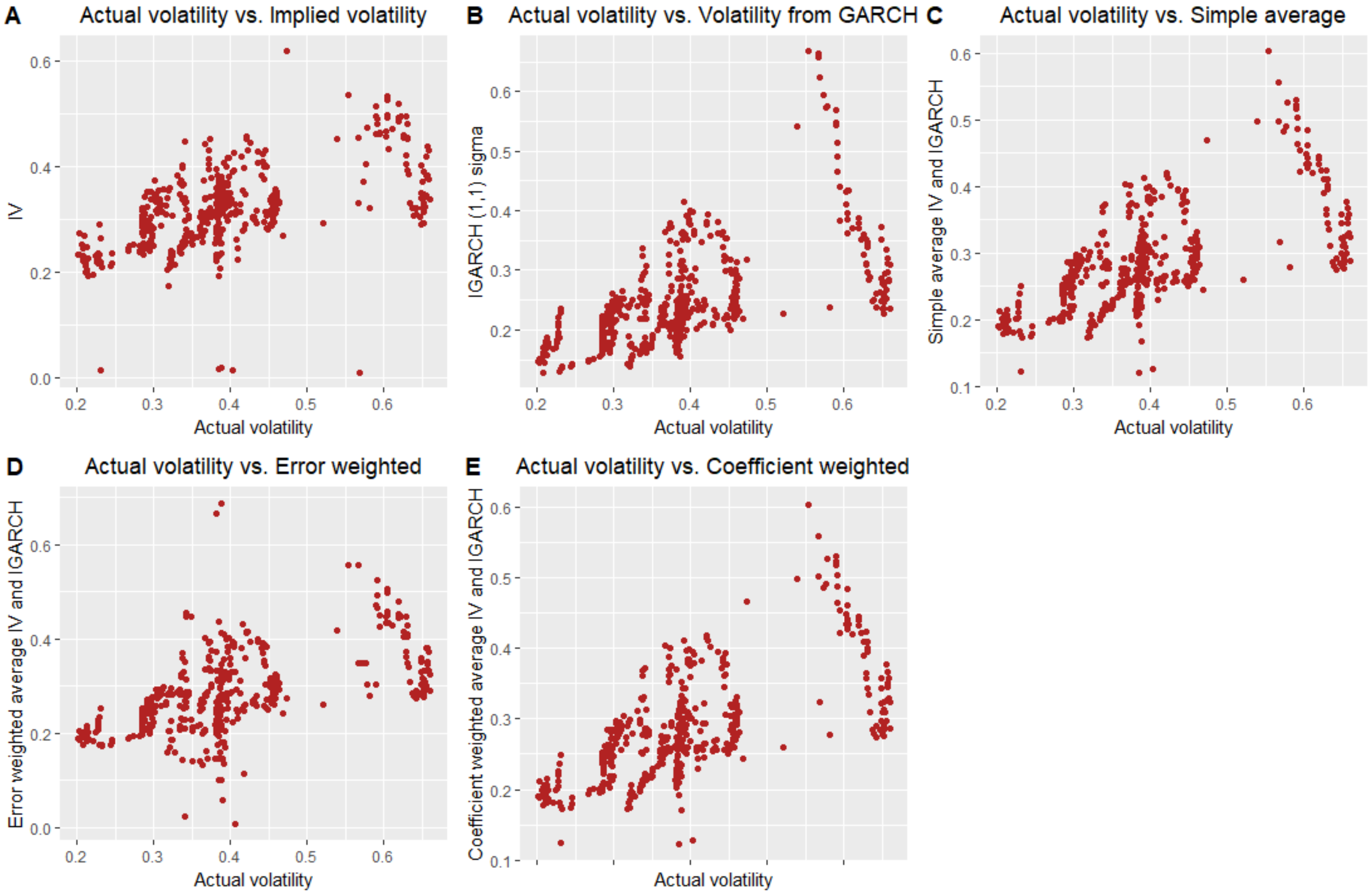

Figure 6. Regressions for Actual Volatility of the RTS Index Based on Data from Put Options

3. Forecasting Performance Comparisons

In order to assess the forecasting performance of the different methods considered above the sample is split into two: the first part of the sample period (from 5 January 2014 to 31 December 2016) is used to obtain in-sample estimates of the model parameters, and then out-of-sample forecasts (for the period from 1 January 2017 to 31 October 2018) are generated using a rolling window and are compared to the actual volatility measures.

Visual inspection of the estimated residuals in percentage terms (see Figure 8) suggests that predicted volatility is overestimated compared to the actual one by all methods.

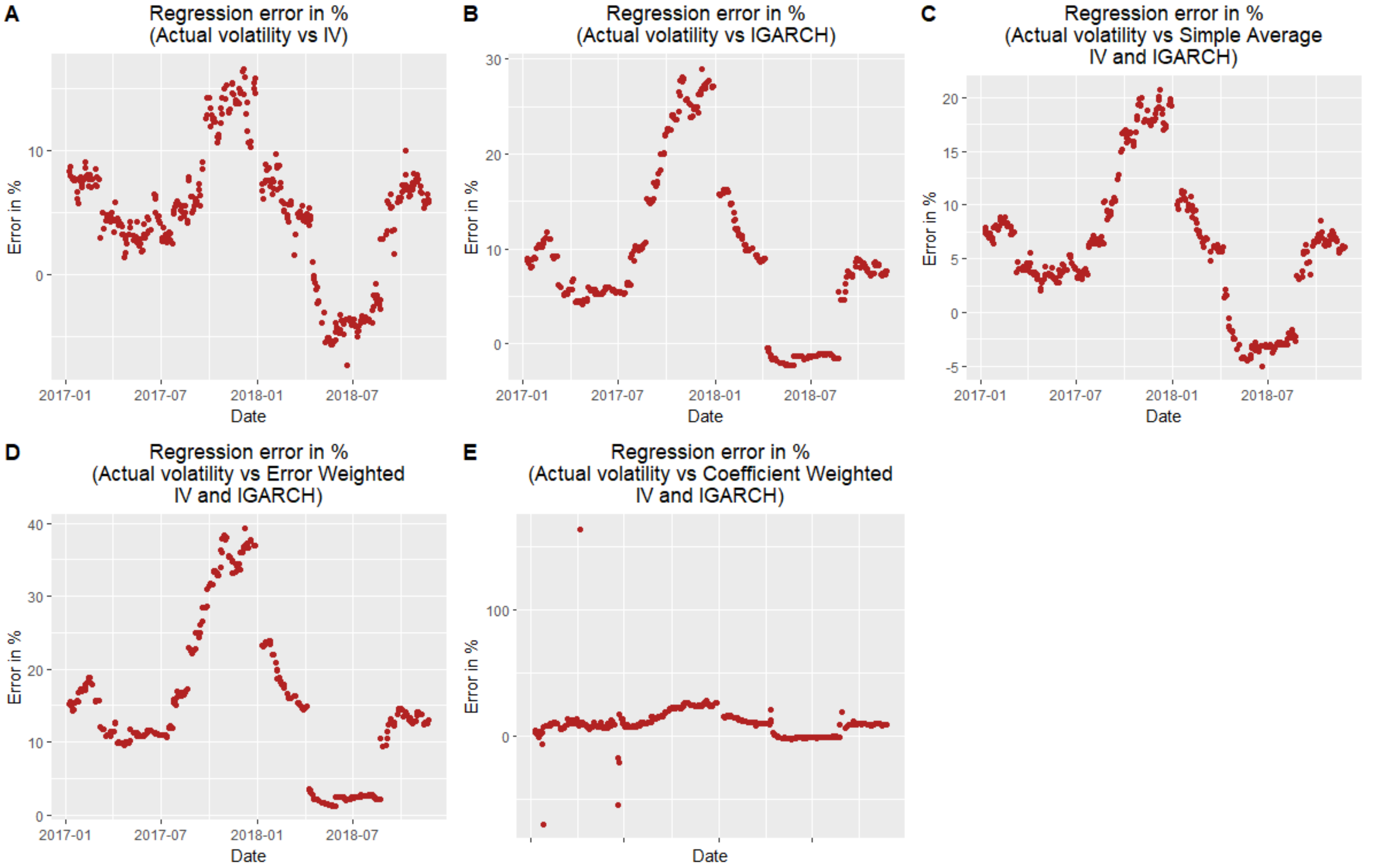

Figure 7. Error Term of Linear Regressions Based on Data from Call Options

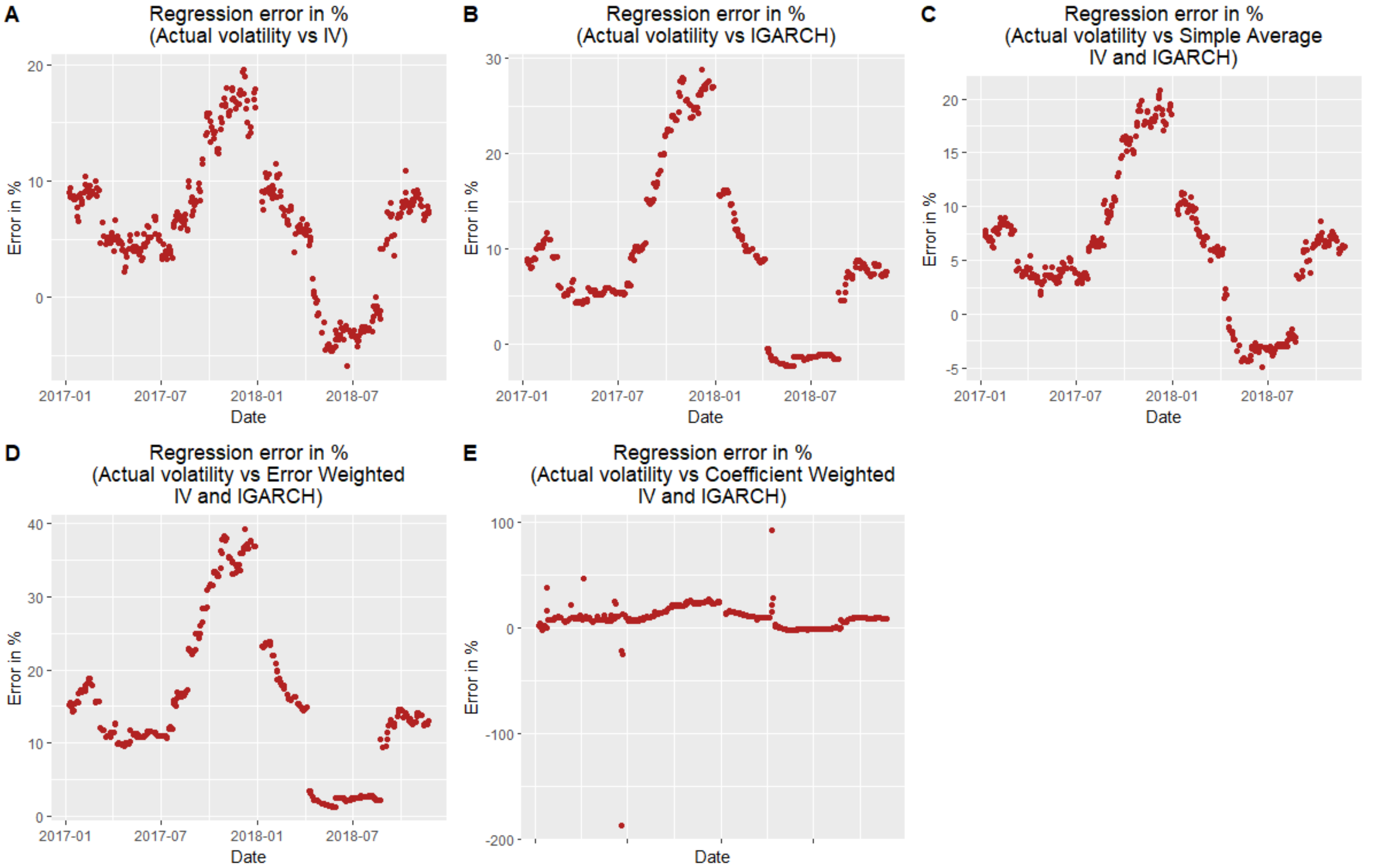

Figure 8. Error Term of Linear Regressions Based on Data from Put Options

The following forecasting performance tests are then carried out:

• Mean absolute error: $MAE = \frac{1}{n} \cdot \sum_{i=1}^{n} |e_i|$;

• Mean absolute percentage error: $MAPE = \frac{100}{n} \cdot \sum_{i=1}^{n} \left|\frac{e_i}{\sigma_i^{Act}}\right|$;

• Root mean square error: $RMSE = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} e_i^2}$,

where n is the number of predicted values, and N the total number of observations.

The results for call and put options are shown in Table 3 and 4, respectively.

Method	MAE	MAPE	RMSE
Implied Volatility	0.06790527	0.3178851	0.07169616
IGARCH(1,1)	0.09555701	0.4913620	0.10810776
Simple average IV and IGARCH	0.07485739	0.3673446	0.08018339
Error weighted average IV and IGARCH	0.15347463	0.7695998	0.16780812
Coefficient weighted average IV and IGARCH	0.11541527	0.5518777	0.19503414

Table 3. Forecasting Tests for RTS Call Options

Method	MAE	MAPE	RMSE
Implied Volatility	0.07693223	0.3699959	0.08143614
IGARCH(1,1)	0.09555701	0.4913620	0.10810776
Simple average IV and IGARCH	0.07485739	0.3673446	0.08018339
Error weighted average IV and IGARCH	0.15347463	0.7695998	0.16780812
Coefficient weighted average IV and IGARCH	0.11541527	0.5518777	0.19503414

Table 4. Forecasting Tests for RTS Put Options

The test statistics suggest that the implied volatility forecasts outperform the IGARCH forecasts, and that the combined forecasts based on the simple average of the former two are at times even more accurate.

5. Conclusions

This paper compares the volatility forecasts for the RTS Index (the main index for the Russian stock market) generated by alternative models, specifically, option-implied volatility forecasts based on the Black-Scholes model, IGARCH (1.1) forecasts (where this is the preferred specification selected from a variety of ARCH/GARCH models), and combined forecasts based on a mixing strategy with the weights being determined using either a simple average or a weighted average method (with the latter either minimizing the errors or maximizing the information content). Various forecasting performance tests are carried out which show that both implied volatility forecasts or mixing strategy forecasts based on a simple average of IV and IGARCH forecasts have higher predictive power than IGARCH forecasts for the volatility of the underlying asset.

These findings are of interest not only to academics but also to financial market participants for designing hedging strategies for which volatility forecasts are needed. They provide evidence that the widely used ARCH/GARCH models do not perform particularly well in terms of forecasting accuracy; practitioners should exploit the information from the options market to forecast the volatility of stock indices. Implied volatility (despite its computational complexity, the lower liquidity of the derivatives market compared to the spot one, and the strict assumptions of the Black and Scholes model) has some important informational content that should not be disregarded, and either by itself or in combination with IGARCH forecasts can generate more accurate volatility forecasts.

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