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Stock Market Indices and Interest Rates in the US and Europe: Persistence and Long-Run Linkages

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STOCK MARKET INDICES AND INTEREST RATES
IN THE US AND EUROPE:
PERSISTENCE AND LONG-RUN LINKAGES

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Abstract

This paper uses fractional integration/cointegration methods to analyse (i) the persistence of the S&P500 and DAX stock indices as well as of the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, and (ii) the long-run linkages between stock prices and interest rates in the US and Europe respectively. The data are monthly and the sample period goes from January 1999 to December 2022. The results can be summarised as follows. All series examined are nonstationary: stock prices are found to be I(1) while interest rates display orders of integration substantially above 1, and therefore all four series are highly persistent, and mean reversion does not occur in any case. Moreover, the fractional cointegration analysis suggests that stock prices and interest rates are not linked in the long run.

Keywords: Stock market prices; interest rates; persistence; fractional integration; fractional cointegration

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1. Introduction

The aim of this paper is to examine the degree of persistence of some representative interest rate and stock price series for the US and Europe as well as the possible existence of long-run equilibrium linkages between these two variables in each case. More specifically, the two interest rate series used for the empirical analysis are the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, whilst the stock indices are the S&P500 and the German DAX; the former includes the 500 stocks with the largest market cap that are traded in the US and covers a wide variety of sectors: information and technology (Oracle, Microsoft, Mastercard), health care (Johnson & Johnson), financial (JPMorgan Chase & Co., Berkshire Hathaway, consumer discretionary (Starbucks), etc.; the latter comprises 40 companies with German headquarters chosen on the basis of their market cap as well as liquidity conditions.

The Fed's Effective Federal Funds rate is the interest rate charged to banks when they lend money to each other overnight (it is also known as the overnight rate), whilst the ECB's Marginal Lending Facility rate is the rate banks pay when they borrow from the ECB overnight (a collateral being required). Therefore in both cases an interest rate rise will decrease profitability by making debt more expensive and thus reducing the capital available for investment; in addition, it will make savings accounts and fixed income securities more attractive to investors, who will become less inclined to invest in equity; for both these reasons, one would expect a negative effect of higher interest rates on stock prices. However, the financial industry (banks, brokerages, mortgage companies, and insurance companies) benefits from an increase in interest rates by being able to charge more for lending; therefore the total effect on stock prices of higher interest rates could be positive instead if the financial industry dominates. Interestingly, Bernanke and

Kuttner (2005) concluded that the effects of unanticipated monetary policy actions on expected excess returns account for the largest part of the response of stock prices.

Note that causality could also run in the opposite direction. For instance, Rigobon and Sack (2003) used an identification method based on heteroscedasticity and reported that a 5 percent rise (fall) in the S&P 500 index increased the likelihood of a 25 basis point tightening (easing) by the Fed by about a half. Hashemzadeh and Taylor (1988) carried out the Granger–Sims test and also found that causality runs from interest rates to stock prices. Bjørnland and Leitemo (2009) estimated a Vector AutoRegressive (VAR) model and found bidirectional causality between the S&P500 and the Federal Funds rate.

The present study aims to shed further light on the behaviour of interest rates and stock indices, as well as their possible linkages, by using a fractional integration/cointegration approach, which is more general and flexible than the standard framework based on the I(0) versus I(1) (stationary versus non-stationary) dichotomy used in most previous studies since it allows for fractional values of the differencing (cointegration) parameter, and thus it encompasses a much wider range of stochastic processes and of adjustment mechanisms towards the long-run equilibrium.

The layout of the paper is the following: Section 2 describes the data; Section 3 outlines the methodology; Section 4 discusses the empirical results; Section 5 summarises the main findings and offers some concluding remarks.

2. Data Sources and Description

The four series analysed are the S&P500 and DAX stock indices, the Fed's effective Federal Funds rate, and the ECB's Marginal Lending Facility rate. The frequency is monthly, and the sample goes from January 1999 to December 2022, for a total of 288 observations. The source for the S&P500 and the DAX is Yahoo finance; specifically, we use the adjusted closing price (the results are almost the same using the closing price instead). The interest rate series have been obtained from the FRED webpage and are displayed in Figures 1-4 below.

<table>
<tr><td>FED</td><td>ECB</td></tr>
<tr><td>

Figure 1

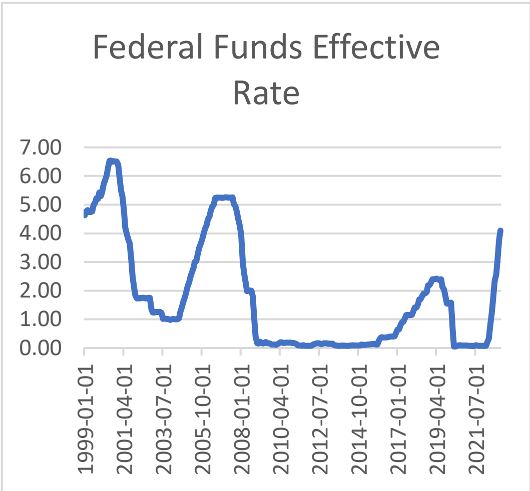

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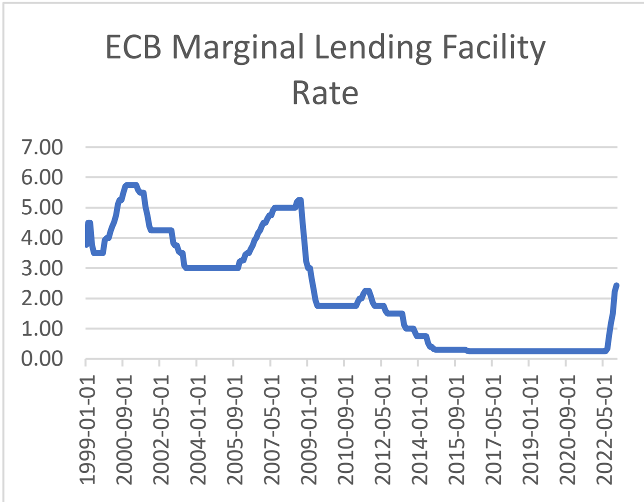
Figure 2



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<tr><td>S&P500</td><td>DAX</td></tr>
<tr><td>

Figure 3

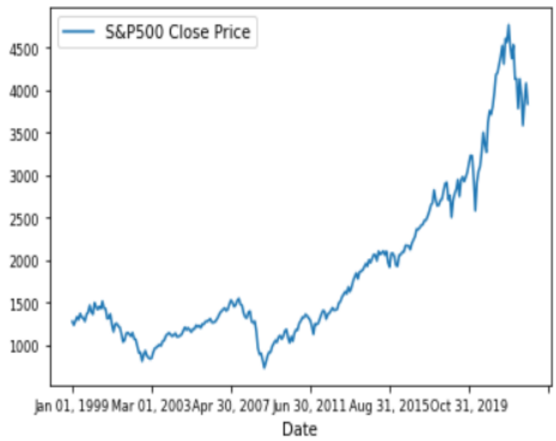

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Figure 4

</td></tr>
</table>

It is noteworthy that the ECB lowered interest rates to stimulate the economy much later than the Fed in the wake of both the DOTCOM and the Global Financial Crisis (GFC), and also kept them at a higher level compared to the Fed. Then, at the onset of the Covid-19 pandemic in 2020, unlike the Fed, it was not able to reduce rates since these had been very close to 0 from 2014. Most recently, in response to a surge in inflation, the Fed increased interest rates in March 2022 whilst the ECB did so in July 2002. At the end of 2022, the ECB's Marginal Lending Facility rate was 2.75% whilst the Fed's Federal Funds Effective Rate was 4.33%. Figures 3 and 4 show that both stock market indices exhibit volatility but have increased significantly since 1999 and peaked in December 2021, before starting to decrease and then to rebound.

3. Methodology

For the empirical analysis we use fractional integration methods to model the series as I(d) processes, where d is the order of integration, which can be any real value, including fractional ones, as proposed by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981). Such a process x_t can be represented as follows:

$$(1 - L)^d x_t = u_t \qquad t = 1, 2, ..., \qquad (1)$$

where L is the lag operator, u_t is assumed to be stationary I(0) and d can be a fractional value (see Gil-Alana and Robinson, 1997 for an empirical application to the 14 macroeconomic variables analysed in Nelson and Plosser, 1992). Note that the parameter d can be interpreted as a measure of persistence, since the polynomial on the left-hand side of (1) can be expressed in terms of its Binomial expansion, such that for all real d,

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \ldots, \qquad (2)$$

and thus, if d is a fractional value, x_t can be expressed in terms for all its past history, i.e.,

$$x_t \quad = \quad dx_{t-1} \quad + \quad \frac{d(d-1)}{2} x_{t-2} \quad - \quad \dots \quad + \quad u_t. \qquad (3)$$

As already mentioned, the parameter d provides a measure of persistence, higher values of d corresponding to a higher degree of dependence between the observations.

The estimated model is the following:

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, \dots, \qquad (4)$$

where α is a constant, β is the slope coefficient, and x_t is the error that follows the process given by equation (1). Combining equations (1) and (2) one obtains the following framework:

$$y_t = \alpha + \beta t + x_t, \quad (1-L)^d x_t = u_t, \quad t = 1, 2, \dots, \qquad (3)$$

The parameter d is then estimated under three different assumptions for the errors: White Noise, Bloomfield-type and Seasonal MA(1) errors. In the first case no time dependence structure is imposed; in the second the adopted specification is used to approximate ARMA structures; in the third, given the monthly nature of the data, a seasonal MA(1) process is assumed which can be represented as:

$$u_t = \rho u_{t-12} + \varepsilon t, \quad t = 1, 2, \dots. \qquad (4)$$

In each of those three cases, three model specifications are estimated:

i) without either a constant or a trend, i.e., imposing α = β = 0 in equation (2).

ii) with a constant but without a trend, i.e., with β = 0 a priori in equation (2).

iii) with a constant and a (linear) time trend

Note that if there exists a linear combination of two (fractionally integrated) variables that displays an order of integration smaller than that of the individual series these are said to be (fractionally) cointegrated. Specifically, we follow the two-step approach originally developed by Engle and Granger (1982), testing first

i) If x_{1t} (stock prices) and x_{2t} (interest rates) are both integrated of a given order, say d, and then

ii) Regressing each stock price series on the corresponding interest rate series,

$$x_{1t} = \delta + \gamma x_{2t} + \varepsilon t, \quad t = 1, 2, ...$$

And testing if the estimated residuals are integrated of a smaller order, i.e., d – b, with b >0, which would imply cointegration (see Engle and Granger, 1987, and more recently Cheung and Lai, 1993, and Gil-Alana, 2003).

4. Empirical Results

As a first step we carry out ADF, Phillips and Perron (1988), Kwiatkowski et al. (1992) or Elliot et al. (1996) unit root tests, all of which imply that the series are nonstationary. (these results are not reported for reasons of space). However, it is well known that these tests have low power against fractional alternatives (see Diebold and Rudebusch, 1991; Hassler and Wolters, 1993; Lee and Schmidt, 1996). This motivates the fractional integration approach we adopt to estimate the differencing parameter d using the three previously mentioned specifications for the error term: white noise (Table 2), Bloomfield-type errors (Table 3) and seasonal AR (Table 4). Each table reports the estimated values of d (and the corresponding 95% confidence intervals) for the three cases of no deterministic terms (2nd column), a constant only (3rd column), and both a constant and a linear trend (last column) in the regression model. The coefficients in bold are those from the specification selected on the basis of the statistical significance of the regressors.

Table 2 shows that for the DAX the estimated value of d is 0.96 with a confidence interval of (0.88, 1.06), whilst the corresponding value for the S&P500 is 0.94 with a confidence interval of (0.88 and 1.01). For the logged series the corresponding estimates are 1.02 and 1.01 respectively, and the confidence intervals still contain 1, so the null of d = 1 cannot be rejected, which represents evidence in favour of the Efficient Market Hypothesis (EMH). For the ECB rate the estimated value of d is 1.45 with a confidence interval of (1.36, 1.57), and for the Fed rate it is 1.56 with a confidence interval of (1.48, 1.66), and thus the null of d = 1 is decisively rejected for both interest rate series.

Table 2: Estimates of d. White noise errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
DAX	0.96 (0.88, 1.05)	**0.96 (0.88, 1.06)**	0.96 (0.88, 1.06)
S&P500	0.91 (0.85, 0.98)	**0.94 (0.88, 1.01)**	0.93 (0.87, 1.00)
Log DAX	0.98 (0.91, 1.08)	**1.02 (0.94, 1.12)**	1.02 (0.94, 1.12)
Log S&P500	0.98 (0.91, 1.07)	**1.01 (0.94, 1.10)**	1.01 (0.94, 1.14)
ECB	1.27 (1.19, 1.38)	**1.45 (1.36, 1.57)**	1.45 (1.36, 1.59)
FED	1.25 (1.18, 1.33)	**1.56 (1.48, 1.66)**	1.56 (1.48, 1.66)

Under the assumption of Bloomfield-type errors (Table 3) the estimated value of of d is 0.88 with a confidence interval of (0.77, 1.05) for the DAX, and 1.03 with a confidence interval of (0.94, 1.17) for the S&P500, both of them being higher than in the previous case. The corresponding estimates for the logged series are 0.97 with a confidence interval of (0.84, 1.13) for the DAX, and a 1 with a confidence interval of (0.89, 1.14) for the S&P500. Those for the ECB and Fed rates are 1.23 and 1.45 with corresponding confidence intervals of (1.06, 1.40) and (1.31, 1.60) respectively, these values being lower than in previous case.

Table 3: Estimates of d. Bloomfield errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
DAX	0.96 (0.83, 1.20)	**0.88 (0.77, 1.05)**	0.89 (0.77, 1.05)
S&P500	1.05 (0.93, 1.22)	**1.03 (0.94, 1.17)**	1.03 (0.94, 1.17)
Log DAX	0.97 (0.86, 1.14)	**0.97 (0.84, 1.13)**	0.97 (0.83, 1.13)
Log S&P500	0.98 (0.85, 1.15)	**1.00 (0.89, 1.14)**	1.00 (0.89, 1.15)
ECB	1.17 (1.07, 1.37)	**1.23 (1.06, 1.40)**	1.23 (1.06, 1.40)
FED	1.35 (1.22, 1.52)	**1.45 (1.31, 1.60)**	1.45 (1.31, 1.60)

As can be seen, the estimates under the assumption of MA(1) errors (Table 4) are almost the same as those in the case of white noise errors.

Table 4: Estimates of d. Seasonal MA(1) errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
DAX	0.96 (0.88, 1.05)	**0.96 (0.88, 1.05)**	0.95 (0.88, 1.05)
S&P500	0.91 (0.86, 0.99)	**0.93 (0.88, 1.01)**	0.93 (0.87, 1.00)
Log DAX	0.98 (0.90, 1.08)	**1.02 (0.94, 1.12)**	1.02 (0.94, 1.12)
Log S&P500	0.98 (0.90, 1.08)	**1.01 (0.94, 1.10)**	1.01 (0.94, 1.10)
ECB	1.27 (1.19, 1.38)	**1.45 (1.36, 1.57)**	1.45 (1.36, 1.59)
FED	1.25 (1.18, 1.33)	**1.56 (1.48, 1.66)**	1.56 (1.48, 1.66)

The next step is to check for the existence of a long-run relationship between the S&P500 and the Fed rate, as well as between the DAX and the ECB rate, using the cointegration approach of Engle and Granger (1987). Table 5 displays the OLS estimates of α and β for these two regressions. Both intercepts are positive, whilst both slope coefficients are negative, and all of them are statistically significant.

Table 5: Estimates of the coefficients in the regression model

Regression model	Intercept (t-value)	Regr. Coefficient (t-value)
S&P500 / FED	3.2575 (212.67)	-0.0209 (-3.59)
DAX / ECB	4.0340 (316.99)	-0.0741 (-16.54)

Table 6: Estimates of d for the regression errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
i) White noise errors			
S&P500 / FED	**1.08 (1.01, 1.17)**	1.08 (1.01, 1.16)	1.08 (1.01, 1.16)
DAX / ECB	**1.12 (1.04, 1.23)**	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)
ii) Bloomfield (autocorrelated) errors			
S&P500 / FED	**1.07 (0.95, 1.20)**	1.09 (0.98, 1.24)	1.09 (0.98, 1.24)
DAX / ECB	**1.13 (0.96, 1.29)**	1.13 (0.98, 1.33)	1.13 (0.98, 1.33)

Table 6 reports the estimates d based on the errors in the above regression models. For cointegration to hold it is necessary that d = 0. Again three model specifications are used (with $\alpha=\beta=0$, $\beta = 0$, α and β different from 0 respectively). The intercept and the time trend coefficients are found to be statistically insignificant and the estimates of d are above 1 in all four cases. When assuming white noise errors the estimates of d are significantly higher than 1, while under the assumption of autocorrelation the unit root null hypothesis cannot be rejected. The hypothesis of mean reversion ($d < 1$) is rejected in all four cases.

Since the residuals are nonstationary, least squares and generalized least squares estimates will be inconsistent (see Robinson and Hidalgo, 1997). Robinson (1994) proposed a semi-parametric NBFDLS estimator which uses OLS on a degenerated band of frequencies around the origin. An improved version of the test for the stationary case is given in Christensen and Nielsen (2006).

In the two-variable case, the NBFDLS estimator proposed in Robinson (1994) is given by:

$$\hat{\beta} = \left\{ \frac{1}{m} \sum_{j=1}^{m} \mathrm{Re}\left[I_{y_1 y_1}(\lambda_j) \right] \right\}^{-1} \times \frac{1}{m} \sum_{j=1}^{m} \mathrm{Re}\left[I_{y_1 y_1}(\lambda_j) \right] \quad (5)$$

which is asymptotically distributed as:

$$\sqrt{m}\lambda_m^{d_e - d}\left(\hat{\beta} - \beta_0\right) \xrightarrow{D} N\left[0, \frac{g_e(1-2d)^2}{2g_{y1}(1-2d-2d_e)}\right] \quad (6)$$

where g_{y1} and g_e are the elements of a G diagonal 2×2 matrix. From (6), normality is ensured as long as $d + d_e < 0.5$ (Christensen and Nielsen, 2006). Note that this estimator crucially depends on the value of the bandwidth parameter m.

Table 7: Estimates of d in the regression errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
S&P500 / FED			
i) White noise errors			
m = 0.5	**0.99 (0.91, 1.08)**	1.05 (0.99, 1.13)	1.05 (0.99, 1.14)
m = 0.6	**0.97 (0.90, 1.06)**	0.95 (0.89, 1.05)	0.95 (0.88, 1.05)
m = 0.7	**0.99 (0.91, 1.08)**	1.05 (0.98, 1.13)	1.05 (0.98, 1.13)
ii) Bloomfield (autocorrelated) errors			
m = 0.5	**1.00 (0.86, 1.17)**	1.08 (0.96, 1.22)	1.09 (0.96, 1.23)
m = 0.6	**1.00 (0.86, 1.14)**	1.08 (0.96, 1.11)	1.09 (0.97, 1.11)
m = 0.7	**1.00 (0.86, 1.17)**	1.09 (0.97, 1.22)	1.09 (0.97, 1.23)
DAX / ECB			
i) White noise errors			
m = 0.5	**1.00 (0.93, 1.10)**	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)

m = 0.6	**0.96 (0.89, 1.06)**	1.04 (0.96, 1.14)	1.04 (0.96, 1.14)
m = 0.7	**1.00 (0.93, 1.10)**	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)
ii) Bloomfield (autocorrelated) errors			
m = 0.5	**0.99 (0.87, 1.15)**	1.13 (0.98, 1.33)	1.13 (0.98, 1.32)
m = 0.6	**0.95 (0.84, 1.13)**	0.96 (0.84, 1.12)	0.97 (0.83, 1.12)
m = 0.7	**0.99 (0.87, 1.16)**	1.13 (0.98, 1.33)	1.13 (0.98, 1.33)

Table 7 reports the results based on this estimator, again for the three cases of no regressors, an intercept only, and an intercept as well as a time trend, for three different bandwidth parameters, m = 0.5, 0.6 and 0.7. In all cases the estimates are again very close to 1 and the unit root null hypothesis cannot be rejected, which again provides evidence against (fractional) cointegration.

In the cointegration analysis it is implicitly assumed that all variables are stochastic. In what follows we depart from this assumption by assuming that interest rates are exogenous, and therefore estimate the following regressions with lagged rates:

$$S\&P\ 500_t = \alpha + \beta IR_{t-k} + x_t \qquad (7)$$

$$DAX_t = \alpha + \beta IR_{t-k} + x_t \qquad (8)$$

where k is the lag index, and x_t is assumed again to be an I(d) process as in equation (1).

Table 8: Estimates in a regression of SP500(t) on FED(t-k)

K	d (95% band)	a (t-value)	b (t-value)
k = 1	1.00 (0.88, 1.14)	7.117 (157.79)	0.0006 (0.60)
k = 2	1.02 (0.90, 1.15)	7.159 (161.04)	0.0038 (0.38)
k = 3	1.01 (0.90, 1.18)	7.200 (160.37)	0.0043 (0.42)
k = 4	1.01 (0.90, 1.16)	7.173 (163.10)	0.0070 (0.60)
k = 5	1.01 (0.90, 1.14)	7.225 (165.08)	0.0007 (0.70)
k = 6	1.00 (0.90, 1.17)	7.192 (165.44)	0.0006 (0.68)
k = 7	0.93 (0.89, 1.15)	7.186 (166.27)	0.0068 (0.70)

k = 8	0.99 (0.88, 1.15)	7.157 (165.58)	0.0168 (0.70)
k = 9	0.99 (0.89, 1.16)	7.217 (166.68)	0.0068 (0.70)
k = 10	0.99 (0.90, 1.17)	7.733 (161.14)	0.0064 (0.66)
k = 11	1.00 (0.89, 1.18)	7.292 (170.71)	0.0061 (0.63)
k = 12	0.99 (0.90 1.17)	7.242 (170.24)	0.0062 (0.64)

Table 8 reports the estimated values of d, α and β for the regression of S&P500 on the Fed rate. The estimates of d are very close for all values of k, and the confidence intervals contain 1, therefore the hypothesis d = 1 cannot be rejected. Note that the estimates of α, but not those of β, are statistically significant.

Table 9: Estimates in a regression of DAX(t) on ECB(t-k)

K	d (95% band)	a (t-value)	b (t-value)
k = 1	0.96 (0.85, 1.14)	8.492 (137.77)	0.0095 (0.70)
k = 2	0.99 (0.85, 1.17)	8.491 (140.23)	0.0055 (0.40)
k = 3	0.99 (0.84, 1.15)	8.490 (141.06)	0.0077 (0.56)
k = 4	0.97 (0.84, 1.15)	8.535 (142.09)	0.0087 (0.64)
k = 5	0.97 (0.84, 1.14)	8.590 (143.44)	0.0097 (0.72)
k = 6	0.96 (0.83, 1.16)	8.540 (142.98)	0.0098 (0.73)
k = 7	0.95 (0.84, 1.13)	8.373 (144.48)	0.0100 (0.75)
k = 8	0.96 (0.83, 1.14)	8.552 (143.80)	0.0100 (0.73)
k = 9	0.95 (0.83, 1.14)	8.624 (145.47)	0.0098 (0.74)
k = 10	0.95 (0.85, 1.13)	8.686 (145.44)	0.0098 (0.62)
k = 11	0.96 (0.86, 1.14)	8.846 (150.91)	0.0083 (0.55)
k = 12	0.96 (0.85, 1.14)	8.832 (150.52)	0.0072 (0.55)

Table 9 reports the corresponding results for the regression of the DAX index on the ECB rate. The estimates of d are slightly below 1 but once more the unit root null hypothesis cannot be rejected; similarly to the previous case, only the intercepts are statistically significant.

5. Conclusions

This paper has used fractional integration/cointegration methods to analyse (i) the persistence of the S&P500 and DAX stock indices as well as of the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, and (ii) the long-run linkages between stock prices and interest rates in both the US and Europe. The data are monthly and the sample period goes from January 1999 to December 2022.

The results can be summarised as follows. All series examined are nonstationary: stock prices are found to be I(1) while interest rates display orders of integration substantially above 1, and therefore all four series are highly persistent, and mean reversion does not occur in any case. Moreover, the fractional cointegration analysis suggests that stock prices and interest rates are not linked in the long run.

Future work should extend the analysis in two ways. First, a multivariate model including other relevant variables such as inflation, money supply, exchange rates etc. should be estimated to shed further light on the linkages between interest rates and stock prices. Second, expectations and announcement effects should be incorporated into the model. It is well known that stock prices can react to anticipated interest rate changes or monetary announcements even before these take place. Because investors have already discounted those changes the observed correction at the time of their implementation will then be smaller, and so will be the estimated impact. Therefore, not allowing for expectation and announcement effect could result in underestimating the strength of the linkages between monetary policy and stock markets.

References

Bjørnland, H.C., and K. Leitemo, (2009). Identifying the interdependence between US monetary policy and the stock market. Journal of Monetary Economics, 56(2), 275-282. doi:10.1016/j.jmoneco.2008.12.001

Bernanke, B. S. and K.N. Kuttner (2005). What explains the stock market's reaction to federal reserve policy? Journal of Finance, 60(3), 1221-1257. doi:10.1111/j.1540-6261.2005.00760.x

Cheung, Y.W. and K. Lai (1993) A fractional cointegration analysis of purchasing power parity, Journal of Business and Economic Statistics 11, 103-112.

Christensen, B. J. and M. Ø. Nielsen, (2006), 'Asymptotic normality of narrow-band least squares in the stationary fractional cointegration model and volatility forecasting', Journal of Econometrics 133(1), 343-371.

Dickey, D. A., and W. A. Fuller, (1979). Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association, 74(366), 427-431. doi:10.1080/01621459.1979.10482531

Dickey, D.A. and S.G. Pantula, (1987), Determining the order of differencing in autoregressive processes, Journal of Business and Economic Statistics 5(4), 455-461. doi:10.1080/07350015.1987.10509614

Diebold, F. X. and G.D. Rudebusch, 1991: On the power of Dickey-Fuller tests against fractional alternatives. *Economics Letters*, 35, 155-160. https://doi.org/10.1016/0165 1765(91)90163-F

Elliot, G., Rothenberg, T.J., and Stock, J.H. (1996). Efficient Tests for an Autoregressive Unit Root, *Econometrica*, 64, 813-836.

Engle, R., and Granger, C.W.J. (1987). Cointegration and error correction. Representation, estimation and testing, Econometrica 55, 251-276.

Gil-Alana, L.A. (2003). Testing of fractional cointegration in macroeconomic time series, Oxford Bulletin of Economics and Statistics 65, 4, 517-529.

Gil-Alana, L. A., and P.M. Robinson, (1997). Testing of unit root and other nonstationary hypotheses in macroeconomic time series. Journal of Econometrics, 80(2), 241-268. doi:10.1016/S0304-4076(97)00038-9

Granger, C.W.J. (1986). Developments in the study of cointegrated economic variables. Oxford Bulletin of Economics and Statistics, 48(3), 213-228. doi:10.1111/j.1468-0084.1986.mp48003002.x

Granger, C.W.J., (1980), Long memory relationships and the aggregation of dynamic models. Journal of Econometrics 14, 227–238.

Granger, C.W.J., (1981), Some properties of time series data and their use in econometric model specification. Journal of Econometrics 16, 121–130.

Granger, C.W.J. and R. Joyeux (1980), An introduction to long memory time series and fractional differencing, Journal of Time Series Analysis 1, 1 ,15-29

Hashemzadeh, N., and P. Taylor, (1988). Stock prices, money supply, and interest rates: The question of causality. Applied Economics, 20(12), 1603-1611. doi:10.1080/00036848800000091

Hassler, U., and Wolters, J., (1994). On the power of unit root tests against fractional alternatives. Economic Letters 45, 1–5.

Hosking, J.R.M. (1981) Modelling persistence in hydrological time series using fractional differencing. Water Resources Research 20, 1898–1908

Kwiatkowski, D., Phillips, P.C.D., Schmidt, P., and Y. Shin (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?, Journal of Econometrics 54, 159–178.

Lee, D., and Schmidt, P. (1996). On the power of the KPSS test of stationary against fractionally integrated alternatives, Journal of Econometrics 73, 285-302.

Nelson C.R. and C.I. Plosser (1982), Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10, 139-162.

Phillips, P.C.B., and Perron, P. (1988). 'Testing for a unit root in time series regression', *Biometrika*, Vol. 75, No. 2, pp.335–346.

Rigobon, R. and B. Sack, (2003), Measuring the reaction of monetary policy to the stock market. Quarterly Journal of Economics 118, 639–669. DOI:1162/003355303321675473

Robinson, P. M. (1994), 'Semiparametric analysis of long-memory time series', Annals of Statistics 22, 515—539.

Robinson, P. M. and F.J. Hidalgo, (1997), 'Time series regression with long-range dependence', Annals of Statistics 25, 77—104.