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Guglielmo Maria Caporale, Ana Rubio-Bezares, and Luis A. Gil-Alana

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Guglielmo Maria Caporale
Brunel University London, UK

Ana Rubio-Bezares
University of Navarra, Pamplona, Spain

Luis A. Gil-Alana
University of Navarra, Pamplona, Spain
and Universidad Francisco de Vitoria, Madrid, Spain

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Abstract

This paper applies fractional integration methods to analyse persistence in stock market returns and volatility at different frequencies in the case of various European indices, specifically the French CAC, the Spanish IBEX 35, the German DAX, the British FTSE 100, and the Euro Stoxx 50, over the period from January 2018 to January 2023. Returns are calculated as the first differences of the logged prices, whilst absolute and squared returns are used as volatility proxies. The results vary depending on the data frequency. More precisely, evidence of long memory is found for returns at the daily frequency only, and for both volatility proxies at the weekly as well as the daily frequency, but not at the monthly one.

Keywords: Stock market prices; volatility; fractional integration; persistence

JEL Classification: C22, G12

Corresponding author: Professor Guglielmo Maria Caporale, Department of Economics and Finance, Brunel University London, Uxbridge, UB8 3PH, UK. Email: Guglielmo-Maria.Caporale@brunel.ac.uk; https://orcid.org/0000-0002-0144-4135

1. Introduction

Investors are interested in both stock market returns and their volatility. The latter is a measure of the uncertainty or risk associated with the value of an investment and can be caused by a range of factors, including the overall economic environment, shifts in market sentiment, changes in interest rates, and unexpected events such as natural disasters, geopolitical tensions, and financial crises.

An important issue for understanding the long-run behaviour of both returns and their volatility is whether or not they exhibit long memory, which is a property of time series decaying slowly over time, or mean reversion, which is the tendency to return to a long-run average over time. According to the Efficient Market Hypothesis (EMH - Fama, 1970), stock prices should reflect all available information and follow a random walk. However, numerous empirical studies have found evidence of persistence in both returns and their volatility, which implies that markets might not be efficient. Various volatility measures have been used in the literature, including estimates based on the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982) and on the generalized autoregressive conditional heteroskedasticity (GARCH) one of Bollerslev (1986), as well as realized volatility and simple proxies such as absolute and squared returns.

Ding et al. (1993) proposed the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model to analyse long memory in stock market returns. This is a generalized version of the Autoregressive Integrated Moving Average (ARIMA) model, which allows for fractional integration. They found evidence that past returns have persistent effects on future ones, even after accounting for short-term dependencies. This suggests that stock prices exhibit a degree of predictability, which can be exploited by investors. Granger and Hyung (2013) showed that, when a time series contains occasional breaks, the long-memory property may be distorted or even disappear. To address this

issue, they proposed a new model, called the Occasional Long Memory (OLM) one, which allows for occasional breaks in the data while still capturing the long-memory property; it is a combination of the Fractional Integrated GARCH (FIGARCH) and the Autoregressive Conditional Heteroskedasticity (ARCH) models, and it has been shown to outperform other models not accounting for occasional breaks. Marra et al. (2021) analysed the persistence and predictability of stock return volatility (using both ARCH and GARCH estimates) in the G7 countries (Canada, France, Germany, Italy, Japan, the UK, and the US) from 2000 to 2018, and found that stock return volatility was persistent and predictable in all cases, and also that there was evidence of volatility clustering.

The present study examines the long-memory properties of both returns and their volatility (proxied by absolute and squared returns) in the case of the five main European stock indices: the IBEX 35, the DAX, the CAC, the EuroStoxx 50, and the FTSE 100, at the daily, weekly, and monthly frequency, over the period from January 2018 to January 2023. For this purpose, it applies very general, fractional integration techniques to estimate the differencing parameter d, which is allowed to take any real value, including fractional ones, and represents a measure of persistence. This parameter provides information about whether the series of interest exhibit either long memory or mean reversion. Specifically, a value of d greater than 0 indicates the presence of long memory in the series, whilst a value below 1 implies mean reversion, with the effects of shocks being transitory.

The layout of the paper is the following: Section 2 describes the data; Section 3 outlines the econometric methodology; Section 4 presents the empirical results. Section 5 offers some concluding remarks.

2. Data Description

The series analysed are the French CAC, the Spanish IBEX 35, the German DAX, the British FTSE 100, and the Euro Stoxx 50 stock market indices, at the daily, weekly, and monthly frequency, over the period from January 2018 to January 2023. The data sources are Investing.com and Yahoo Finance. The five indices are constructed as follows:

IBEX 35

The IBEX 35 is the Spanish benchmark stock market index and measures the combined performance of the 35 most traded companies among those listed on the Electronic Stock Exchange Interconnection System (SIBE) on the four Spanish stock exchanges (Madrid, Barcelona, Bilbao and Valencia). It comprises stocks of companies from different sectors, including banking, energy, telecommunications, insurance, and construction, among others, which best meet the parameters of capitalization, liquidity, and traded volume required by the technical advisory committee. The weighting of each company in the index is based on its market capitalization and liquidity.

Euro Stoxx 50

The Euro Stoxx 50 is a market capitalization-weighted stock index of 50 large, blue-chip European companies operating within 12 eurozone countries, and its composition is reviewed quarterly or semi-annually. These companies belong to 19 different sectors, including banking, oil and gas, mining and technology, with the financial institutions' sector having the biggest weight. Some of the companies included in this index are: AXA Group, SAP, Unilever, Banco de Santander and Daimler. This index captures about 60% of the free-float market cap of the EURO STOXX Total Market Index (TMI).

DAX 30

The Deutscher Aktienindex (DAX 30) is the German stock index for the Frankfurt Stock Exchange. It includes the 30 most influential German companies (such as BMW, Adidas, Bayer, and Allianz), again from various sectors, but it is especially well-known for those in the automotive, textile, and chemical sectors. It is weighted by market capitalization, and the list of stocks included is subject to a quarterly review. To qualify for index membership, a company must be included in the German Prime Standard market, and none can have a weight greater than 20%. The composition of the index is reviewed annually in September.

CAC 40

CAC stands for Continuous Assisted Quotation - its value varies continuously every working day every 15 seconds, and it is made up of 40 stocks of companies belonging to the top 100 in terms of capitalization. Many of those included are multinationals, and thus 45% of the index is represented by companies whose headquarters are in other countries, with German ones accounting for 21% of the index, followed by British, Japanese, and American ones. The top 10 companies included in this index are LVMH, Total Energies, Sanofi, L'Oréal, Schneider Electric, Air Liquide, Airbus, BNP Paribas Act, Essilor Luxottica, and Vinci.

FTSE 100

FTSE 100 is an acronym for Financial Times Stock Exchange 100. It is colloquially known as the Footsie 100 and is the benchmark index of the London Stock Exchange and one of the top 5 European stock indices. It began operating in 1984 including a total of 1,000 UK companies, but now only comprises the top 100 in terms of market

capitalization (representing 70% of the total value of the London stock market), on which weights are based. It is reviewed on a quarterly basis.

Tables 1, 2 and 3 report descriptive statistics for these five indices at the monthly, weekly and daily frequency respectively. It can be seen that at the monthly frequency (Table 1) there are sizeable differences between maximum and minimum values (especially in the case of the DAX), which suggests the presence of significant fluctuations in stock prices. The IBEX 35 and FTSE 100 are the most and least volatile indices on the basis of their standard deviation.

Instead, at both the weekly (Table 2) and daily (Table 3) frequency, high and low values do not vary significantly across markets, and all stock indices have similar standard deviations. Again, the IBEX 35 and FTSE 100 indices have the highest and lowest standard deviation respectively.

Table 1: Descriptive statistics for the European stock indices at monthly frequency

Index	Maximum	Minimum	Std. Deviation	Mean
CAC	0.08202	-0.07961	0.02444	-0.00232
DAX	0.077990	-0.06074	0.02476	-0.00168
IBEX 35	0.109100	-0.09754	0.02583	0.00059
FTSE 100	0.064533	-0.05058	0.017984	-0.00079
Eurostoxx 50	0.077256	-0.07211	0.024436	-0.00159

Table 2: Descriptive statistics for the European stock indices at the weekly frequency

Index	Maximum	Minimum	Std. Deviation	Mean
CAC	0.0961636	-0.0441424	0.013199	-0.0004011
DAX	0.0969766	-0.0449588	0.013539	-0.0002063
IBEX 35	0.1015286	-0.0542041	0.013436	0.0002958
FTSE 100	0.0807443	-0.0329721	0.010642	-0.0000882
Eurostoxx 50	0.0968489	-0.0451393	0.013279	-0.0002168

Table 3: Descriptive statistics for the European stock indices at the daily frequency

Index	Maximum	Minimum	Std. Deviation	Mean
CAC	0.0568854	-0.03499	0.005656	-0.00008
DAX	0.0566965	-0.04523	0.005869	-0.00004
IBEX 35	0.0658007	-0.0357219	0.005683	0.000061
FTSE 100	0.0499978	-0.0376395	0.004909	-0.0000039
Eurostoxx 50	0.0575028	-0.0383667	0.005764	-0.0000451

3. Econometric Methodology

The empirical analysis in this study uses fractional integration methods which are more general than the standard ones based on the dichotomy between stationarity and non-stationarity. Note that a series is said to be integrated of order 0, or I(0), if it is covariance stationary and the infinite sum of its autocovariances is finite. That is, given a covariance stationary process, x_t, t = 0, ±1, ±2, …, , with autocovariance function $\gamma_u = E[x_t - \mu)(x_{t+u} - \mu)]$, where μ is the mean of the process, x_t is said to exhibit short memory or be I(0) if:

$$\sum_{u=-\infty}^{\infty} \gamma_u < \infty. \qquad (1)$$

This definition includes a wide range of processes, for example white noise ones, but also those characterised by weak time dependence, such as the stationary and invertible AutoRegressive Moving Average (ARMA) class of models.

In the standard approach, if a series is nonstationary first differences are taken, i.e. $(1 - L)x_t$ is modelled, where L is the lag-operator defined as $Lx_t = x_{t-1}$. In such a case, the original series is said to be integrated of order 1. However, in the 1980s, authors such as Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981) noticed that many aggregated series that apparently were nonstationary I(1) once they were differenced became over-differenced, which suggested that the appropriate degree of differentiation might be fractional.

A series is said to be integrated of order d, and denoted as I(d), if it can be represented as:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{2}$$

where u_t is a short-memory or I(0) process, and d is a fractional number, which is a measure of persistence. If d is positive, x_t exhibits the long-memory property because the infinite sum of its autocovariances is infinite, i.e.,

$$\sum_{u=-\infty}^{\infty} \gamma_u = \infty. \tag{3}$$

Specifically, the fractional integration model we estimate is the following one:

$$x_t = \alpha + \beta t + z_t, \quad (1 - L)^d z_t = u_t, \qquad t = 1, 2, \ldots, \tag{4}$$

where x_t is the observed time series (in logs), α and β are unknown coefficients, *t* is a linear time trend, and the regression errors, z_t, are I(d), such that u_t is an I(0) process which is assumed to be weakly autocorrelated and for which the exponential spectral model of Bloomfield (1973) is applied. This is a non-parametric method to approximate ARMA models by using a log-spectral density that is similar to that of AR structures. The parameters are estimated using the Whittle function in the frequency domain (Dahlhaus, 1989) as in the testing approach developed in Robinson (1994), which is widely used in empirical applications (see, e.g., Gil-Alana and Robinson, 1997; Gil-Alana and Moreno, 2007; Abbritti et al., 2016, 2023; etc.).

4. Empirical Results

Table 4, 5 and 6 report the results for returns and the two volatility proxies (absolute and squared returns) at the monthly, weekly and daily frequency respectively. Each table displays the estimates of d (and the corresponding 95% confidence intervals) for three different model specifications:

1) without deterministic terms (i.e., setting $\alpha = \beta = 0$ in Equation (4)),

2) with an intercept only (i.e., setting $\beta = 0$)

3) with both an intercept and a linear time trend

The values in bold in the tables are those for the specification selected on the basis of the statistical significance of the regressors.

Concerning the monthly results (Table 4), the selected model for returns includes both deterministic terms in the case of the CAC, FTSE 100, and IBEX 35 indices, and an intercept only in the case of DAX and EuroStokk50 ones. Further, the estimated coefficients for the differencing parameter d are in all cases in the I(0) interval, i.e., their confidence intervals include the value 0, which supports the null hypothesis of short-memory or I(0) returns. This implies that stock prices are I(1) and thus follow a random walk, consistently with the Efficiency Markets Hypothesis (EMH). As for volatility persistence, for both absolute and squared returns the estimates of d are all negative, and once more all of them are within the I(0) interval, and thus there is no evidence of long-memory behaviour in the volatility series.

Table 4: Estimates of d. Monthly data

i) Stock market returns			
Series	No terms	An intercept	An intercept and a linear time trend
CAC	-0.28 (-0.59, 0.16)	-0.28 (-0.59, 0.17)	**-0.43 (-0.78, 0.10)**
DAX	-0.39 (-0.64, 0.13)	**-0.42 (-0.78, 0.13)**	-0.41 (-0.79, 0.16)
EURO STOKK50	-0.38 (-0.69, 0.24)	**-0.42 (-0.83, 0.24)**	-0.42 (-0.94, 0.25)
FTSE 100	-0.28 (-0.55, 0.16)	-0.28 (-0.56, 0.17)	**-0.42 (-0.76, 0.12)**
IBEX 35	-0.39 (-0.81, 0.17)	-0.39 (-0.76, 0.17)	**-0.49 (-1.91, 0.14)**
ii) Absolute returns			
CAC	-0.17 (-0.31, 0.29)	**-0.17 (-0.42, 0.20)**	-0.23 (-0.54, 0.16)
DAX	-0.16 (-0.30, 0.18)	**-0.32 (-0.65, 0.15)**	-0.27 (-0.64, 0.20)

EUROSTOKK50	-0.16 (-0.29, 0.11)	**-0.41 (-0.79, 0.12)**	-0.41 (-0.79, 0.12)
FTSE 100	-0.15 (-0.31, 0.28)	**-0.18 (-0.42, 0.20)**	-0.23 (-0.54, 0.17)
IBEX 35	-0.18 (-0.29, 0.00)	**-0.40 (-0.67, 0.01)**	-0.40 (-0.69, 0.01)
iii) Squared returns			
CAC	-0.22 (-0.37, 0.11)	**-0.26 (-0.53, 0.12)**	-0.28 (-0.54, 0.11)
DAX	-0.19 (-0.36, 0.09)	**-0.28 (-0.57, 0.12)**	-0.27 (-0.56, 0.13)
EURO STOKK50	-0.22 (-0.38, 0.03)	**-0.36 (-0.63, 0.04)**	-0.35 (-0.65, 0.05)
FTSE 100	-0.22 (-0.39, 0.11)	**-0.26 (-0.53, 0.11)**	-0.29 (-0.56, 0.10)
IBEX 35	-0.17 (-0.34, 0.13)	**-0.22 (-0.48, 0.14)**	-0.22 (-0.48, 0.14)

In bold the estimates and confidence intervals for the selected model for each series.

Regarding the weekly data (Table 5), the selected model for returns is the one without regressors, and the estimates of d are all in the I(0) interval. By contrast, the preferred specification for absolute returns includes an intercept only, and the values of d are now all positive, ranging from 0.35 (IBEX 35) to 0.43 (DAX), which implies the presence of long memory in these series. The same conclusion is reached in the case of the squared returns, though the selected model now does not include any regressors, and the estimates of d range between 0.26 (IBEX 35) and 0.42 (FTSE 100).

Table 5: Estimates of d. Weekly data

i) Stock market returns			
Series	No terms	An intercept	An intercept and a linear time trend
CAC	**-0.12 (-0.27, 0.07)**	-0.13 (-0.28, 0.06)	-0.12 (-0.27, 0.06)
DAX	**-0.15 (-0.28, 0.04)**	-0.15 (-0.29, 0.04)	-0.15 (-0.29, 0.06)
EURO STOKK50	**-0.14 (-0.30, 0.06)**	-0.14 (-0.30, 0.06)	-0.14 (-0.28, 0.08)
FTSE 100	**-0.06 (-0.22, 0.17)**	-0.06 (-0.22, 0.17)	-0.06 (-0.25, 0.18)
IBEX 35	**-0.15 (-0.30, 0.05)**	-0.15 (-0.31, 0.05)	-0.15 (-0.31, 0.05)
ii) Absolute returns			

CAC	0.39 (0.23, 0.63)	**0.37 (0.22, 0.61)**	0.37 (0.22, 0.61)
DAX	0.45 (0.29, 0.69)	**0.43 (0.28, 0.66)**	0.43 (0.28, 0.67)
EURO STOKK50	0.42 (0.26, 0.66)	**0.42 (0.25, 0.63)**	0.42 (0.25, 0.63)
FTSE 100	0.39 (0.22, 0.62)	**0.37 (0.23, 0.62)**	0.39 (0.22, 0.62)
IBEX 35	0.37 (0.22, 0.58)	**0.35 (0.22, 0.56)**	0.35 (0.22, 0.57)
iii) Squared returns			
CAC	**0.35 (0.19, 0.60)**	0.35 (0.20, 0.60)	0.35 (0.20, 0.60)
DAX	**0.36 (0.19, 0.62)**	0.36 (0.20, 0.60)	0.36 (0.20, 0.60)
EURO STOKK50	**0.34 (0.20, 0.59)**	0.35 (0.20, 0.59)	0.35 (0.20, 0.59)
FTSE 100	**0.42 (0.23, 0.67)**	0.42 (0.23, 0.70)	0.42 (0.23, 0.70)
IBEX 35	**0.26 (0.11, 0.47)**	0.26 (0.12, 0.47)	0.26 (0.12, 0.47)

In bold the estimates and confidence intervals for the selected model for each series.

Finally, at the daily frequency (Table 6) the selected model without regressors provides evidence of long memory in the returns themselves in the case of the CAC and IBEX 35 indices, whilst for the DAX, EuroStokk50 and the FTSE 100 ones the I(0) hypothesis of short-memory behaviour cannot be rejected. Similarly to the weekly case, long memory is found instead in both volatility proxies, the preferred specification including a constant in the case of absolute returns and no regressors in the case of square returns.

Table 6: Estimates of d. Daily data

i) Stock market returns			
Series	No terms	An intercept	An intercept and a linear time trend
CAC	**0.07 (0.00, 0.14)**	0.07 (0.00, 0.14)	0.07 (0.00, 0.14)
DAX	**0.06 (-0.01, 0.15)**	0.06 (-0.01, 0.15)	0.06 (-0.01, 0.15)
EURO STOKK50	**0.06 (-0.01, 0.16)**	0.06 (-0.01, 0.16)	0.06 (-0.01, 0.16)
FTSE 100	**0.00 (-0.06, 0.07)**	0.00 (-0.06, 0.07)	0.00 (-0.07, 0.08)
IBEX	**0.11 (0.04, 0.21)**	0.11 (0.04, 0.21)	0.12 (0.04, 0.21)

ii) Absolute returns			
CAC	0.41 (0.36, 0.47)	**0.41 (0.36, 0.48)**	0.41 (0.36, 0.48)
DAX	0.38 (0.32, 0.44)	**0.38 (0.32, 0.42)**	0.36 (0.32, 0.42)
EURO STOKK50	0.39 (0.33, 0.45)	**0.39 (0.33, 0.44)**	0.39 (0.33, 0.44)
FTSE 100	0.34 (0.28, 0.39)	**0.33 (0.29, 0.39)**	0.33 (0.29, 0.39)
IBEX	0.39 (0.33, 0.45)	**0.38 (0.33, 0.45)**	0.38 (0.33, 0.45)
iii) Squared returns			
CAC	**0.45 (0.38, 0.51)**	0.43 (0.38, 0.51)	0.43 (0.38, 0.51)
DAX	**0.36 (0.30, 0.42)**	0.36 (0.30, 0.42)	0.36 (0.30, 0.42)
EURO STOKK50	**0.41 (0.34, 0.47)**	0.41 (0.35, 0.47)	0.41 (0.35, 0.47)
FTSE 100	**0.33 (0.28, 0.40)**	0.33 (0.28, 0.40)	0.33 (0.28, 0.40)
IBEX	**0.44 (0.35, 0.53)**	0.44 (0.35, 0.54)	0.44 (0.35, 0.54)

In bold the estimates and confidence intervals for the selected model for each series.

5. Conclusions

This paper applies fractional integration methods to analyse persistence in stock market returns and volatility at different frequencies in the case of various European indices, specifically the French CAC, the Spanish IBEX 35, the German DAX, the British FTSE 100, and the Euro Stoxx 50, over the period from January 2018 to January 2023. Returns are calculated as the first differences of the logged prices, whilst absolute and squared returns are used as volatility proxies. The results vary depending on the data frequency. More precisely, evidence of long memory is found for returns at the daily frequency only, and for both volatility proxies at the weekly as well as the daily frequency but not at the monthly one.

The analysis can be extended in several ways. For instance, the stochastic behaviour of alternative measures of volatility such as realized volatility can be used. Two other important issues that can be analysed are the possible presence of nonlinearities and/or breaks in the series. The former can be modelled using the orthogonal polynomials

in time of Hamming (1973) and Bierens (1997) in the context of fractional integration as in Cuestas and Gil-Alana (2016), or the Fourier functions in time as in Gil-Alana and Yaya (2021), or neural networks as in Yaya et al. (2021). To test for the latter, both the Bai and Perron (2003) approach and the methods proposed by Gil-Alana (2008) and Hassler and Meller (2014) can be used, the latter being specifically designed for the case of fractional integration. Future work will address these issues.

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