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Modelling country and group levels corporate default dependence: Evidence from the Euro area

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Abstract

This paper employs a mixed effects Cox model to estimate the failure dependence caused by firms' exposure to unobserved factors at both country and group level. We use a quarterly panel data set of 1,422 public listed firms across the Euro area over the period 1994Q1-2014Q4. The empirical analysis delivers three main results. First, when countries are grouped together, with economic and financial similar conditions, failure clustering tend to be larger, as firms are subject to an extra risk due to the impact of unobserved factors at the group level. Second, there is significant evidence of failure dependence caused by firms' exposure to country level unobserved factors. Third, models that do not account for the distance to default probability tend to perform poorly as compared with their counterparts.

Keywords: Hazard rates; mixed effects model; country and group level dependence; Eurozone

JEL Classification: G33, C51, C41.

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1 Introduction

The financial crisis and the sovereign debt crisis hit the Euro area significantly. Not only the PIIGS contries, but also Belgium and France were affected (Metiu, 2012; Arghyrou and Kontonikas, 2012; Ludwig, 2014), even though to a lesser extent.[1] As a result, banks were more conservative with their lending activities, and a large reduction in loan supply was observed, with an impact on investment activities, job creation, and sale growth (Acharya et al., 2016). Since then, business entities within the Euro have struggled to survive, and the hazard rates of these businesses have been severely affected, due to their exposure to risk factors at country and group level.

Das et al. (2007) observed excess default correlation induced by unobserved factors (or frailty factors), and showed that models based on the assumption that corporate defaults are conditionally independent after adjusting for observable factors tend to underestimate default clustering. Against this background, several studies have considered failure dependence induced by unobserved risk factors at country or industry levels using frailty factors (see e.g. Duffie et al., 2009; Chava et al., 2011; Koopman et al., 2011, 2012; Qi et al., 2014) so to yield more accurate estimates of hazard rates.

In this paper, we estimate the failure dependence of 1,422 public listed firms in 11 Eurozone countries over the period 1994Q1-2014Q4. The analysis is conducted at country and group level. As for the group level, we consider PIIGS and non-PIIGS countries, since the strong linkage between economic conditions and firm performance (see e.g Bhattacharjee et al., 2009; Bonfim, 2009; Chen, 2010; Tang and Yan, 2010; Jacobson et al., 2013). In addition, we consider three extra groups that are formed by including: only Belgium in the PIIGS (PIIGSB); only France in the PIIGS (PIIGSF); both Belgium and France in the PIIGS (PIIGSBF) (see also Giordano et al., 2013). This allows us to establish to what extent the crisis in the Euro area really affected Belgian and French firms performance along that of those firms in the PIIGS.

We use a nested frailty model that accounts for two hierarchical clustering (see Duchateau

[1] It has been argued that a contagion effect has propagated from Belgium to France, as a result of the distressed bank Dexia.

and Janssen, 2008; Wienke, 2011) within a multivariate framework of mixed effects Cox model (see Ripatti and Palmgren, 2000; Therneau and Grambsch, 2000; Therneau et al., 2003). A cluster level-specific random effect, which is assumed to follow a Gaussian distribution, is considered for country and group level clustering, and firms in each country are exposed to country (internal) and group level (external) risk (unobserved) factors. We also consider a non-nested frailty model, which accounts only for country level (internal) unobserved factors (firms are not exposed to any potential external risk factors). This model is considered for comparison purposes. It is expected that the estimates of the nested frailty model will likely be more accurate than those by the non-nested frailty model, due to the fact that the latter ignores the potential impact of externals factors. This may suggest that the total risk exposure of a firm is not only limited to country level factors, but also to bloc level factors. Therefore, during distressed economic and market conditions, the two-level dependence model tends to better gauge the riskiness of firms listed in countries with similar economic structures than the single-level dependence model does.

As for the specification of the models, we select covariates from Shumway (2001), Duffie et al. (2007), and Bharath and Shumway (2008), namely distance to default probability, one-year trailing stock return, one year trailing market return, firm age, and 3 month T-bill rate.

The empirical analysis offers three main results. First, an increase in distance to default probability pushes the firms towards a potential failure. Further, older firms with high stock returns are less likely to experience failure, as compared to younger firms with lower stock returns, and previous year's market performance tends to enhance firms' performance in the following years. Second, while there is no much difference between the estimates of the covariates among nested and non-nested models, nested frailty models tend to accurately estimate dependencies or correlations caused by both internal and external unobserved factors. This suggests that nested frailty models are more likely to produce smaller margin of error when estimating failure rates as compared to non-nested frailty models. This implies that nested frailty models tend to better explain the effect of the crises on firms' behaviour in the Euro area. Third, models that account for the distance to default probability covariate tend to outperform their counterparts, since this covariate has a higher explanatory power in default rate models.

This paper offers three contributions to the empirical literature. First, while previous papers focus on default clustering at either economy-wide or industry level within a country (see e.g. Duffie et al., 2009; Chava et al., 2011; Atsu and Costantini, 2015), we take a further step by looking at default clustering at country level and group level. In doing so, we are able to appropriately capture firms exposure to both internal and external risk factors, which may play an important role in corporate financial decision-making. For instance, when gauging the risk exposures of firms within the Eurozone, investors seeking to diversify their portfolios by investing across the bloc should not restrict their assessment to only country-based risk exposures but also the potential firm exposures emanating from the group level, especially during unfavourable macroeconomic conditions. Second, this study examines default clustering of public listed firms on 11 stock markets in the Euro area. To the best of our knowledge, this is the first study to examine default clustering at country and group levels within the Euro area. Lastly, this paper examines the impact of firms' membership to groups of the Euro area on riskiness. To this end, an index of riskiness is proposed.

The rest of the paper is organised as follows. Section 2 presents methodology and data. Empirical findings are presented and discussed in Section 3. Chapter 4 draws the conclusions.

2 Methodology and data

This section is devoted to models and data used in the empirical analysis. We first present the non-nested frailty model, which serves as a benchmark model, and then describe the nested frailty model, which accounts for country level as well as for group level unobserved factors. Lastly, data are described.

2.1 Non-nested frailty models

Let T_{ij} and δ_{ij} respectively be the event time and event indicator (censoring indicator) of firm i listed in country j among q countries. The indicator δ_{ij} takes the value 1 if T_{ij} is a failure time and 0 otherwise. Suppose that the data set of firm i follows a shared frailty model. The hazard rate of the firm is defined as follows (see Ripatti and Palmgren, 2000; Therneau et al., 2003; Duchateau and Janssen, 2008, among others):

$$\lambda_{ij}(t) = \lambda_0(t)u_j exp(X_{ij}(t)\beta), \tag{1}$$

where $\lambda_{ij}(t)$ is the hazard rate of firm i listed in country j, and $X_{ij}(t)$ and β are vectors of covariates and parameters. The frailty term u_j, which is shared among firms listed in country j, acts multiplicatively on the hazard rate. The baseline hazard function $\lambda_0(t)$ is assumed to be unknown, which makes the hazard rate in equation (1) semi-parametric.[2]

Equation (1) can be reformulated as (see Ripatti and Palmgren, 2000; Therneau et al., 2003):

$$\lambda_{ij}(t) = \lambda_0(t)exp(X_i\beta + Z_i w), \tag{2}$$

where $w = log(u_j)$, and X_i and Z_i are covariates. We define $Z_{ij} = 1$ if firm i is listed in country j, and 0 otherwise. In model (2) each firm is listed in only one country. In other terms, cross listing of firms is not allowed.

In order to incorporate time varying covariates in the estimation of equation (1), we employ the counting process input style of Andersen and Gill (1982). As a result, the pair (T_{ij}, δ_{ij}) for firm i listed in country j is substituted by $(N_i(t), Y_i(t))$, where $Y_i(t)$ assumes value 1 if firm i is still active and 0 otherwise, and $N_i(t)$ is the number of events in the period $(t_l, t_{t+1}]$ for firm i, with t_l and t_{l+1} being the beginning and the ending time of the interval. As in McGilchrist and Aisbett (1991) and McGilchrist (1993), we assume that the random effect w in equation

[2] The non-frailty model is derived from equation (1) by setting the frailty term $u_j = 1$: $\lambda_{ij}(t) = \lambda_0(t)exp(X_{ij}(t)\beta)$.

(2) is normally distributed on the log-scale, and the parameters β and w are estimated by maximizing the penalised partial likelihood (PPL):

$$PPL = PL(\beta, w; data) - g(w; \theta), \tag{3}$$

where PL is defined as the log of the classical Cox partial likelihood conditioned on the data set:

$$PL(\beta, w) = \sum_{i=1}^{n} \int_0^{\infty} \Big[Y_i(t)(exp(X_i\beta + Z_i w) - log\Big(\sum_k Y_k(t)(exp(X_k\beta + Z_k w)\Big) \Big] dN_i(t), \tag{4}$$

and the penalty term is defined by

$$g(w; \theta) = \frac{1}{2\theta} \sum_{j=1}^{q} w_i^2, \tag{5}$$

where θ is the variance of the log-frailty or random effect.[3] For a given value of the variance estimate θ, we use the expansion and approximation of Ripatti and Palmgren (2000) to derive a modified likelihood defined as:

$$\begin{aligned} l_m(\beta, \theta) &= -\frac{1}{2}\log(|D|) + \log\Big(\int exp\Big[PL(\beta, w) - \frac{1}{2} w' D^{-1/2} w\Big] dw \Big) \\ &\approx PL(\beta, \tilde{w}) - \frac{1}{2}\log\Big(\tilde{w}' D^{-1/2}\tilde{w} + log|D|) + \log(|H_{22}(\beta, \tilde{w})\Big), \end{aligned} \tag{6}$$

where $D = \theta I$ is a diagonal matrix and I is an identity matrix of order $q \times q$; q is the number of countries in the sample, and $g(w; \theta) = \tilde{w}' D^{-1}(\theta)\tilde{w}$. The term $\tilde{w} = \tilde{w}(\beta, \theta)$ solves the following equation

$$\sum_{i=1}^{n} \int_0^{\infty} (Z_{ij} - Z_j(t)) dN_i(t) - D^{-1}(\theta)\tilde{w} = 0 \tag{7}$$

[3] For details on penalised partial likelihood of a shared frailty model, see Ripatti and Palmgren (2000) and Therneau et al. (2003).

We maximize the likelihood in equation (6) over the parameters using the "*coxph*" procedure in the "survival" package in R (see Therneau (2014)).

2.2 Nested frailty model

Our sample comprises of s clusters (groups), and in each group there are n_i subclusters (countries). Further, each country contains n_{ij} members (firms) (see Duchateau and Janssen, 2008). In this setting, firms are located within countries, and countries are nested in groups. The nested frailty model is given by:

$$\begin{aligned}\lambda_{ijk}(t) =&\lambda_0(t)u_i z_{ij} exp(X_{ijk}(t)\beta) \\ =&\lambda_0(t)exp(X_{ijk}(t)\beta + w_i + v_{ij}), \end{aligned} \tag{8}$$

where $\lambda_{ijk}(t)$ is the hazard rate at time t of firm $k = 1, ..., n_{ij}$ in country $j = 1, ..., s_i$ located in group $i = 1, ..., s$. The term $\lambda_0(t)$ is the baseline hazard function at time t and β is a $p-$dimensional parameters of the set of covariates, $X_{ijk}(t)$. In addition, $w_i = logu_i$ is the random effects term for group i, whilst $v_{ij} = logz_{ij}$ is the random effects term of country j nested in group i. We define T_{ijk} as the event time of firm k listed in country j located in group i with a corresponding censoring indicator, δ_{ijk}. The latter takes value 1 if T_{ijk} is a failure time and 0 otherwise. The nested model is developed within a multivariate shared frailty framework (see Section 2.1) and equation (8) can be re-written as follows (see e.g. Ripatti and Palmgren, 2000; Therneau et al., 2003):

$$\lambda(t) = \lambda_0(t)exp(X\beta + Zb), \tag{9}$$

$$b \sim G\Big(\theta, \sum(\theta)\Big),$$

where X and Z are the time-varying covariate matrices for the fixed and random effects, respectively, and β and b are the vectors of fixed and random effects coefficients with dimensions q. The non-negative term λ_0 is the baseline hazard function and it is assumed to be unknown.

The coefficients of equation (9) can still be estimated without knowing the shape of λ_0. The random effects distribution G is a multivariate Gaussian distribution with zero mean and variance matrix $\sum$, which is a function of a vector of the parameters θ. Following Therneau and Grambsch (2000) and Therneau et al. (2003), we define the log penalised partial likelihood function as follows:

$$PPL(\beta, b, \theta) = l(\beta, b) - g(b, \theta), \tag{10}$$

where the penalty function $g(b, \theta) = b' \sum^{-1}(\theta) b/2$. The term $l(\beta, b)$ is called the partial likelihood (PL) (see Therneau, 2015) for any given value of β and b, and is defined as:

$$l(\beta, b) = \sum_{k=1}^{n_{ij}} \int_0^\infty \Big[Y_k(t)\eta_k(t) - log\Big(\sum_j Y_j(t)\eta_j(t) \Big) \Big] dN_k(t), \tag{11}$$

where $\eta_k(t) = X_k(t)\beta + Z_k(t)b$ is the linear score for firm k at time t, $X_k(t)$ and $Z_k(t)$ are the k^{th} rows of the covariate matrices X and Z, respectively. In other words, the above row matrices are the data set for firm k in country j. The term $Y_k(t)$ describes the surviving firms (or firms still at risk of default), which takes value 1 when firm k is active at time t, and 0 otherwise. Equation (10) can then be re-written as:[4]

$$PPL(\beta, b, \theta) = \sum_{k=1}^{n_{ij}} \int_0^\infty \Big[Y_k(t)\eta_k(t) - log\Big(\sum_j Y_j(t)\eta_j(t) \Big) \Big] dN_k(t) - \frac{b' \sum^{-1}(\theta) b}{2}. \tag{12}$$

The estimates of β and b, $\hat{\beta}$ and $\hat{b}$, are obtained by solving the following score equations (see Therneau et al., 2003):

$$\frac{\partial PPL}{\partial b_j} = \sum_{i=1}^{n} \int_0^\infty (Z_{ij} - Z_j(t)) dN_i(t) - \frac{\partial g(b; \theta)}{\partial b_j}, \tag{13}$$

[4]For detailed treatment, refer to Therneau et al. (2003).

$$Z_j(t) = Z_j(\beta, b, t) = \frac{\sum Z_k j Y_k [X_k \beta + Z_k b]}{\sum Y_k [X_k \beta + Z_k b]}. \tag{14}$$

We also obtain the integrated partial likelihood (IPL) by integrating out the random effects as obtained below (Therneau, 2015):

$$IPL = \frac{1}{(2\pi)^{q/2} |\sum(\theta)^{1/2}|} \int PPL(\beta, \theta) exp\Big(- b' \big(\sum \big)^{-1} (\theta) b/2 \Big) db, \tag{15}$$

where q is the number of random effects. We estimate the parameters using the "*coxme*" package in R by (Therneau, 2015).

2.3 Data

Our data are drawn from DataStream and Worldscope for public listed firms in 11 member states of the Eurozone for the period 1994Q1-2014Q4. The sample is comprised of 1,422 firms: 905 active firms, 398 failed firms and 119 acquired or merged firms, and this translates into 71,680 quarterly firm observations. The countries are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain. The choice of these countries is based on data availability.[5] Table 1 presents firms' status at country level across the 11 selected members of the Eurozone.

The definition of firm failure may differ across all the member states of the Euro area. For the sake of uniformity, we follow Altman and Narayanan (1997), who provide a definition of failure: (i) filing by a company; (ii) bond default; (iii) bank loan default; (iv) delisting of a company; (v) government intervention via special financing; and (vi) liquidation.[6] We select failed, and acquired or merged firms from the DataStream "DEAD" category for each country in conjunction with other sources (e.g. Bloomberg bankruptcy segment). For instance, DataStream items "DEADGR", "DEADBD" and "DEADFR" are the categories for dead

[5]These countries may have some accounting information disclosure differences, but Worldscope adjusts the variables for these differences.

[6]For delisting of a company, we cross check the reasons for delisting at other sources. These reasons include mergers, acquisitions and some of the reasons already stated.

Table 1: Active, failed and merged or acquired firms within the Eurozone

ID	Country	Active firms	Failed firms	Merged/Acquired firms	Total
1	Austria	33	41	4	78
2	Belgium	57	29	10	96
3	Finland	39	22	14	75
4	France	200	51	21	272
5	Germany	199	44	10	253
6	Greece	38	47	18	103
7	Netherlands	78	56	10	144
8	Ireland	31	24	2	57
9	Italy	111	32	13	156
10	Portugal	37	27	4	68
11	Spain	82	25	13	120
	Total	905	398	119	1422

firms in Greece, Germany and France, respectively.

In duration models, the dependent variable is the time taken for a subject to experience either a non-failure or failure event. Time to event is usually specified with the corresponding event indicator which takes value 1 for failure event and 0 otherwise. To incorporate time varying covariates, we use the counting process input style following Andersen and Gill (1982). For instance, let us suppose that it takes six years for a firm to experience an event. For a failure event, we construct the intervals $(0, 1]$, $(1, 2]$, $(2, 3]$, $(3, 4]$, $(4, 5]$, and $(5, 6]$ for year 1, 2, 3, 4, 5, and 6, respectively. The event indicator is 0 for the years 1, 2, 3, 4, and 5, when a firm is still active, but takes value 1 for the 6th year, when a firm fails. We can therefore simply reconstruct the intervals as follows: $(0, 1; 0]$, $(1, 2; 0]$, $(2, 3; 0]$, $(3, 4; 0]$, $(4, 5; 0]$, and $(5, 6; 1]$, where the first and second values are the beginning and the end of the year, and the last one is the event indicator. For example, $(2, 3; 0]$ indicates the dependent variable for the third year, where 2 and 3 are the beginning and end of the third year, and the third value 0 is the event indicator, since the firm is still traded at the end of the third year. For non-failure event, e.g. when a firm is delisted as a result of merger and acquisition activities, we have $(0, 1; 0]$, $(1, 2; 0]$, $(2, 3; 0]$, $(3, 4; 0]$, $(4, 5; 0]$, and $(5, 6; 0]$. The event indicator is 0 for all the intervals, since the firm is censored as a result of a non-failure event.[7]

We employ some widely used covariates in the empirical literature of corporate failure, given their explanatory power (see Shumway, 2001; Duffie et al., 2007, 2009; Duan et al., 2012; Qi et al., 2014, among others). First, we use the 3 month T-bill rate, which is a measure of short-term interest rates. Second, we consider the one year trailing stock return, which is a good predictor of firm failure (see Shumway, 2001), and is constructed by cumulating monthly stock returns. Third, we use the one year trailing market return, which is a measure of the overall market performance, and is constructed by cumulating monthly market returns. Fourth, the distance to default probability is used as a probabilistic measure of volatility adjusted leverage. In constructing this measure, we follow Bharath and Shumway (2008): firms with higher probabilities are close to default, whilst firms with lower probabilities are far from default. Lastly, we consider the age of a firm to test whether older firms are less likely to fail than the younger ones (see e.g. Gong et al., 2004; George, 2005; Aldrich and Ruef, 2006; Wiklund et al., 2010).[8]

Table 2 presents the descriptive statistics for the covariates used in the empirical analysis.

Table 2: Descriptive statistics

Variable	Mean	Std. Dev	Min.	25th P.	Median	75th P.	Max.
Distance to default prob.	0.122	0.176	0.000	0.002	0.046	0.179	1.000
Stock return (%)	12.195	46.398	-76.015	-15.195	4.340	32.028	200.690
Market return (%)	12.263	29.447	-66.066	-9.064	16.115	29.722	178.882
ln(age)	1.875	0.816	0.000	1.386	2.079	2.565	2.996
3 month T-bill rate (%)	4.418	2.528	1.123	2.598	3.788	5.211	12.144

Notes: The terms 25th P. and 75th P. represent 25th and 75th percentiles, respectively.

The distance to default probability has the minimum and maximum values of 0.000 and 1.000, respectively, with a mean of value 0.122 and a standard deviation of value 0.176. The stock return falls within the range -76.015% and 200.690%, while the minimum and maximum values of the market return variable are -66.066% and 178.882%, respectively. The stock and market returns have approximately a mean value of 12%, but the former varies more about

[7] For details on counting process, refer to Andersen and Gill (1982).

[8] Firm age is defined as the period between the time a firm is listed and the time of an event.

the mean than the latter. Additionally, the natural log of firm age is bounded by (0.000, 2.996) since the firm age falls within the interval [1, 21]. The 3 month T-bill rate ranges from 1.123% to 12.144%.

3 Empirical analysis

This section presents the empirical results obtained using non-nested and nested frailty models. More specifically, we first estimate the parameters of the non-nested frailty model, which serves as benchmark model. Then the parameters of the nested frailty model are estimated, and its performance is compared to that of the non-nested model. Lastly, we compute the total riskiness of firms in order to evaluate how firms are affected by country and group level unobserved factors.

In our analysis, we consider PIIGS countries against non-PIIGS ones along with three extra groups that are formed by including: only Belgium in the PIIGS (PIIGSB); only France in the PIIGS (PIIGSF); and both Belgium and France (PIIGSBF). In other words, we extract the country and group level frailty factors that affect listed firms for following pair of groups: (i) PIIGS versus non-PIIGS; (ii) PIIGSB versus non-PIIGSB; (iii) PIIGSF versus non-PIIGSF; and (iv) PIIGSBF versus non-PIIGSBF.

In the regression analysis, we employ distance to default probability, one year trailing stock return, one year trailing market return, ln(age), and 3 month T-bill rate as covariates. Table 3 reports the estimates for non-nested frailty models. For model 1, the hazard rate is assumed to be a function of only distance to default probability. The results in Table 3 confirm that this covariate is a good predictor (see Bharath and Shumway, 2008). Model 2 uses the distance to default probability along with stock return, market return, and log of firm age. Model 4 adds the 3 month T-bill rate to the specification of model 2. All the covariates are significant with the expected sign in both models, with the exception of 3 month T-bill rate. The significance of these covariates show that the distance to default probability is a very important predictor, but not sufficient for the prediction of failure rates (see also Bharath and Shumway, 2008). This implies that, for an appropriate default model specification, the distance to default probability should be augmented with suitable covariates. Model 3, which we consider to perform a confirmatory test on the distance to default probability, also shows the insignificance of the 3 month T-bill rate, while the other covariates are significant.

To find the best model in terms of goodness of fit across all the specifications, we use both the pseudo-deviance and the information criteria measures. For the former criterion, we have:

$$Pseudo\text{-}deviance = -\,2loglik_A + 2loglik_B, \tag{16}$$

where $-2loglik_A$ and $-2loglik_B$ are the deviance statistics for generic models A and B, respectively. The statistic in (16) says how model A performs worse than the supposed best model B, and it can be used in case of nested models. The statistic follows a chi-square distribution (χ^2), with the degrees of freedom being the difference between the number of parameters in model A and that in model B.

As for the information criteria, we consider the Akaike information criterion (AIC), the corrected Akaike information criterion (AICC), and the Bayesian information criterion (BIC):

$$AIC = -2logL + 2k, \tag{17}$$

$$AICC = AIC + \frac{2k(k+1)}{n-k-1}, \tag{18}$$

$$BIC = -2logL + klogn, \tag{19}$$

where $-2LogL$ is the partial likelihood, which is obtained by using the rank of events (Singer and Willett, 2003), k and n denote the number of parameters and events, respectively (see Raftery, 1995). As a rule of thumb, the lower the values of these information criteria, the better the fit. The information criteria are suitable for both non-nested and nested models.

Since model 2 nests model 1, and model 4 nests model 2, we use the pseudo-deviance to compare the performance of these models. In case of models 1 and 2 (the latter nests the former), the pseudo-deviance statistic is equal to $31.480(5173.580 - 5142.100)$, with 3 degrees of freedom. Since the value of the statistic is greater than the critical value, $\chi^2_{0.01(3)} = 16.266$, the null hypothesis that the coefficients of stock return, market return and ln(age) are all equal zero can be rejected. This implies an improvement of the fit due to the inclusion of these covariates, which makes model 2 the best candidate. Similarly, the values of this statistic for models 1 and 4 is 31.876, which is larger than the critical value $\chi^2_{0.01(4)} = 18.467$. Therefore, model 4 fits the data better than Model 1. Instead, a different result is found for models 2 and 4. Here, the value of the pseudo-deviance statistic, 0.396, is smaller than the critical value, $\chi^2_{0.01(1)} = 10.828$. Therefore, there is no improvement in terms of fit if one adds 3 month T-bill rate to model 4. Likewise, for models 3 and 4, the test statistic is 63.296 with $\chi^2_{0.01(1)} = 10.828$. The significance of this test suggests that model 4 fits data better than model 3. When considering non-nested models (models 2 and 3), the results from AIC, $AICC$, and BIC, show that Model 2 is the best model.

Using model 2 (the best model), we further explore the effects of covariates on the hazard rates (expected time to default) using the transformation $100(e^{\beta} - 1)$, where β is the coefficient of a given covariate. The coefficient of the distance to default probability covariate is 0.929 and produces the value $100(e^{0.929} - 1) = 153.198$. This implies that a unit increase in the distance to default probability variable leads to 153.198% increase in the instantaneous rate of default, holding other factors constant. As for the stock return, market return, and age, a 1-year increase in these covariate lead to a decrease in the hazard of failure by -25.770%, -42.822%, and -31.887%, respectively. Therefore, the expected time to failure rises with an increase in stock return, market return, and firm age covariates. These trends are robust to the effects of only country-based unobserved risk factors.

The above results have some implications for the Eurozone countries. First, the distance to default probability shows high explanatory power in hazard rate models. This reveals that

Table 3: Non-nested frailty model specifications with random effects

Dependent variable: Time to event				
	Model 1	Model 2	Model 3	Model 4
Distance to default prob.	1.221***	0.929***		0.925***
	(0.231)	(0.255)		(0.255)
Stock return		-0.298**	-0.490***	-0.291**
		(0.143)	(0.140)	(0.144)
Market return		-0.559**	-0.482*	-0.501*
		(0.276)	(0.290)	(0.290)
ln(age)		-0.384***	-0.375***	-0.386***
		(0.082)	(0.081)	(0.082)
3 month T-bill rate			-3.814	-3.125
			(4.960)	(4.968)
LogLik.(Fitted)	-2572.592	-2557.045	-2588.069	-2556.648
LogLik.(Integrated)	-2586.790	-2571.050	-2602.738	-2570.852
Integrated LR test	80.140***	111.620***	103.110***	112.020***
	[0.000]	[0.000]	[0.000]	[0.000]
Penalized LR test	108.540***	139.630***	132.450***	140.430***
	[0.000]	[0.000]	[0.000]	[0.000]
Pseudo-deviance	5173.580	5142.100	5205.476	5141.704
AIC	5175.580	5150.100	5213.476	5151.704
AICC	5175.601	5150.315	5213.691	5152.028
BIC	5175.861	5151.224	5214.600	5153.109
Dependence	0.185	0.179	0.203	0.186

Notes: The efron approximation is used to control for ties in the event times of firms. Standard errors and p-values are in round and square brackets, respectively. ***, ** * denote significance at the 1%, 5%, and 10% level, respectively. LogLik.(Fitted) and LogLik.(Integrated) are the fitted and integrated likelihoods due to unobserved factors, respectively. The terms Integrated LR and Penalized LR denote the unobserved factors-adjusted integrated and penalized likelihood ratio tests, respectively. The pseudo-deviance is used to compare the overall model fit of nested models, while the Akaike information criterion (AIC), corrected Akaike information criterion (AICC), and Bayesian information criterion (BIC) measures are used to compare either nested or non-nested models.

firms that usually exhibit averagely higher distance to default probability are more prone to experience failure within the Eurozone. Second, a rise in stock return and market return increases the expected time to default. This outcome seems to suggest that firms listed within the Euro area with a consistent increase in their returns are less likely to move towards a failure point, and performing markets tend to enhance the survival rate of such firms, as compared to those of averagely decreasing stock returns. Third, the significance of age in our models reveals that older firms in Euro area are less likely to fail than the younger ones. This may be due to the liability of newness (see e.g Baum, 1996; Aldrich and Ruef, 2006; Wiklund et al., 2010), as older firms may have more business contacts, better understanding of the dynamics of the business environment and more robust organisational structure. Further, older firms generally satisfy various regulatory requirements (see e.g. Baum, 1996; Gong et al., 2004; George, 2005). Finally, the lack of significance of the 3 month T-bill rate in our models indicate that the monetary authorities do not play a significant role in influencing the hazard rate of firms in the Eurozone during the period under investigation.

The results for non-nested frailty models are obtained under the hypothesis that firms are only exposed to country level (internal) unobserved factors. This implies that the potential group level (external) risk factors induced by financial and debt crises are completely ignored. Further, since the crises hit the Euro countries in different ways, the external factors may play here a relevant role. As such, it is worth investigating how the external factors induced by the crises may have affected the default rates of firms in the four groups of countries previously mentioned. We do this by assuming that countries in a group share similar characteristics due to the prevailing macroeconomic and firm-specific factors. In particular, we assume similar trends in firms' distance to default probabilities and stock returns for each group of countries, and run two regressions for each of the four pair of countries, PIIGS vs. non-PIIGS; PIIGSB vs. non-PIIGSB; PIIGSF vs. non-PIIGSF, and PIIGSBF vs. non-PIIGSBF (for example D_{PG} and S_{PG} denote the regressions for the PIIGS countries based on similar trends in firms' distance to default probability and stock return, respectively). We use model 2 (see Table 3) as our benchmark model.

The empirical results are illustrated in Tables 4 and 5. In all models, almost all the regressors are significant with the expected signs. For example, in the D_{PG} model, the coefficient of distance to default probability is positive and significant, and those of the stock return, market return, and ln(age) are negative and significant. While these results do not differ from those in Table 3, measures of dependence have improved considerably, regardless of the specification of the model. This implies that when determining risk rates of listed firms in an economic bloc during unfavourable market and economic conditions, it is more appropriate to group countries according to similar macroeconomic structures. Failure to do so may lead to underestimation of risk rates, since non-nested models are based on the hypothesis that countries are independent to each other. As such, the economic and financial activities between member states have no significant impact on firms, and the macroeconomic conditions in one or more countries may be not transmitted to another. Therefore, the risk level of firms is restricted to only country level, and the potential group level exposure is ignored in the estimation parameters. On the contrary, nested models assume dependence among member states through the interaction of countries. Therefore, using these models more accurate estimates of the risk level can be gained.

We complete our analysis with an investigation of the impact of firms' membership to the diverse groups of the Euro area on riskiness. In Table 6, we report results related to the risk scores and the level of riskiness of firms within countries, and PIIGS and non PIIGS group of countries. In the event of failure clustering, the country (group) score shows how firms are likely to fail either faster or slower. As such, we use value 1 (expected value of frailty) as a threshold value for gauging riskiness. A risk-score large than 1 implies more riskiness, while a score lower than 1 is considered less riskiness. Examples of more risky countries are Austria, Finland, Greece, Ireland, Netherlands and Portugal, while the less risky countries are Belgium, France, Germany, Italy and Spain. For instance, the risk scores for Portugal and Belgium are 1.231 and 0.984 respectively, values that are shared by firms in these countries. The country

Table 4: Nested frailty models: PIIGS and PIIGSBF groups

	Dependent variable: Time to event			
	PIIGS		PIIGSBF	
	D_{PG}	S_{PG}	D_{BF}	S_{BF}
Distance to default prob	1.338***	0.861***	1.238***	0.891***
	(0.278)	(0.259)	(0.289)	(0.259)
Stock return	-0.266*	-0.311**	-0.242	-0.316**
	(0.145)	(0.147)	(0.148)	(0.148)
Market return	-0.725**	-0.744**	-0.731**	-0.763**
	(0.319)	(0.311)	(0.338)	(0.311)
ln(age)	-0.373***	-0.356***	-0.361***	-0.397***
	(0.084)	(0.096)	(0.089)	(0.096)
LogLik.(Fitted)	-1844.363	-2490.691	-1234.067	-2468.146
LogLik.(Integrated)	-2561.092	-2592.953	-2586.009	-2592.104
Integrated LR test	131.540***	67.82.060***	81.700***	69.510***
	[0.000]	[0.000]	[0.000]	[0.000]
Penalized LR test	1564.990***	272.340***	2785.590***	317.430***
	[0.000]	[0.000]	[0.000]	[0.000]
Psuedo-deviance	5122.184	5185.906	5172.018	5184.208
AIC	5130.184	5193.906	5180.018	5192.208
AICC	5130.399	5194.121	5180.233	5192.423
BIC	5131.308	5195.030	5181.142	5193.332
Dependence	1.919	0.283	4.325	0.348

Notes: Standard errors and p-values are in round and square brackets, respectively. D_{PG} and S_{PG} indicate the models for PIIGS with similar trends in terms of the distance to default and stock returns respectively, whereas D_{BF} and S_{BF} are the models for the PIIGSBF. ***, ** * denote significance at the 1%, 5%, and 10% level, respectively. For further details, see Table 3.

Table 5: Nested frailty models: PIIGSB and PIIGSF groups

Dependent variable: Time to event				
	PIIGSB		PIIGSF	
	D_B	S_B	D_F	S_F
Distance to default prob	1.331***	0.847***	1.349***	0.876***
	(0.277)	(0.258)	(0.277)	(0.258)
Stock return	-0.272*	-0.320**	-0.262***	-0.317**
	(0.145)	(0.147)	(0.145)	(0.149)
Market return	-0.712**	-0.732**	-0.757***	-0.752**
	(0.318)	(0.311)	(0.321)	(0.311)
ln(age)	-0.369***	-0.356**	-0.378***	-0.391***
	(0.084)	(0.096)	(0.084)	(0.097)
LogLik.(Fitted)	-1839.700	-2487.728	-1788.692	-2451.328
LogLik.(Integrated)	-2560.389	-2592.065	-2563.549	-2592.324
Integrated LR test	132.940***	69.590 ***	126.620***	69.070***
	[0.000]	[0.000]	[0.000]	[0.000]
Penalized LR test	1574.320***	278.260***	1676.340***	351.060***
	[0.000]	[0.000]	[0.000]	[0.000]
Psuedo-Deviance	5120.778	5184.130	5127.098	5184.648
AIC	5128.778	5192.130	5135.098	5192.648
AICC	5128.993	5192.345	5135.313	5192.863
BIC	5129.902	5193.254	5136.222	5193.772
Dependence	1.929	0.288	2.094	0.398

Notes: Standard errors and p-values are in round and square brackets, respectively. D_B and S_B indicate the models for PIIGSB with similar trends in terms of the distance to default and stock returns respectively, whereas D_F and S_F are the models for the PIIGSF. ***, ** * denote significance at the 1%, 5%, and 10% level, respectively. For further details, see Table 3.

riskiness for Portugal and Belgium, 0.231 and -0.016, respectively, are obtained by subtracting the threshold value 1 from their scores. These values imply that firms in Portugal are about 23% more risky as compared to Belgian firms which are about 2% less risky.

The results related to group risk scores are reported in columns 3 and 6 of Table 6, and the corresponding values of group riskiness are in columns 4 and 7. Firms in a group are exposed to the same level of risk. For example, firms in the PIIGS group are at least 9% more risky, while those in non-PIIGS countries are at least 9% less risky. We perform the group score extraction to help determine the total riskiness (the sum of country level riskiness and group-based riskiness) of listed firms in a given country. This tends to offer information on the effects of unobserved factors on firms listed in a country, which is a member of the Euro area, and also a member of a group created by the Euro crises. For example, the total riskiness of Belgium is approximately -0.101, whereas Portugal shows a total riskiness score of at least 0.323. These values imply that Belgian firms are at least 10% less risky, while Portuguese firms are at minimum 30% more risky. When comparing the risk levels within the PIIGS, Greek firms seem to be the riskiest, followed by Irish and Portuguese firms, while Spanish and Italian firms are the less risky ones. For the non-PIIGS countries, German firms are the least risky, while the Austrian are the riskiest.

All these results suggest that firms in the Euro area are exposed to both country level and group-based unobserved risk factors. In addition, some countries in the non PIIGS (PIIGS) group have higher (lower) risk scores and total riskiness. This outcome supports our argument that to measure risk levels of firms accurately, it is fundamental to consider both country and group level. This information may play an important role for financial decision-making process. For example, investors within the bloc should consider potential divergence in the economies of countries, especially during distress market conditions, when gauging the trade-off between risk and returns of their portfolios.

In Table 7, the results concerning the level of riskiness of firms for the PIISGBF and non-PIIGSBF countries are illustrated. Greece is still the country with the highest risk level,

Table 6: Scores and riskiness for nested frailty models: PIIGS versus non-PIIGS

	County level		D_{PG}			S_{PG}		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PIIGS	Country Score	Country Riskiness	Group Score	Group Riskiness	Total Riskiness	Group Score	Group Riskiness	Total Riskiness
Portugal	1.231	0.231	1.092	0.092	0.323	1.134	0.134	0.365
Ireland	1.423	0.423	1.092	0.092	0.515	1.134	0.134	0.557
Italy	0.729	-0.271	1.092	0.092	-0.179	1.134	0.134	-0.137
Greece	1.449	0.449	1.092	0.092	0.541	1.134	0.134	0.583
Spain	0.777	-0.223	1.092	0.092	-0.131	1.134	0.134	-0.089
non-PIIGS								
Austria	1.793	0.793	0.915	-0.085	0.708	0.882	-0.118	0.675
Belgium	0.984	-0.016	0.915	-0.085	-0.101	0.882	-0.118	-0.134
Finland	1.044	0.044	0.915	-0.085	-0.041	0.882	-0.118	-0.074
France	0.583	-0.417	0.915	-0.085	-0.502	0.882	-0.118	-0.535
Germany	0.536	-0.464	0.915	-0.085	-0.549	0.882	-0.118	-0.582
Netherlands	1.209	0.209	0.915	-0.085	0.124	0.882	-0.118	0.091

Notes: D_{PG} (columns 3 to 5) and S_{PG} (columns 6 through 8) refer to PIIGS countries which have similar behaviour of distance to default probability and stock return, respectively. Figures in columns (1) and (2) indicate country-level scores and riskiness, respectively. Figures in column (2) are obtained by subtracting value 1 (expected value of the unobserved factors) from numbers in column (1). Figures in columns (4) and (7) are obtained by subtracting value 1 from figures in columns (3) and (6). Total riskiness for D_{PG} and S_{PG} are constructed by adding figures in columns (2) to those in column (4), and numbers in column (2) to those in column (7), respectively.

followed by Ireland and Portugal, while France has the lowest risk level. For the non-PIIGSBF group, Germany is the least risky country, whereas Austria is the riskiest one.

When comparing the results in Table 6 and 7, the following emerges. First, the riskiness for PIIGS countries decreases when Belgium and France, with relatively lower risk levels, are included in this group. For example, Portugal's risk level decreases to about 23% in the PIIGSBF group. However, Belgium and France are now 2% and 23% less risky, respectively. This implies that Belgium and France are relatively risky in the PIIGS group than in the non-PIIGS one. Second, the risk levels of non-PIIGSBF group increased, as a result of the absence of Belgium and France in the group. In particular, Germany is between 55-58% less risky, and this range decreases to about 46%. The outcome shows that weak countries in the Euro area tend to benefit more, through economic and financial activities, as compared to those with stronger economies.

Table 7: Scores and riskiness for nested frailty models: PIIGSBF versus non-PIIGSBF

	County level		D_{BF}			S_{BF}		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PIIGSBF	Country Score	Country Riskiness	Group Score	Group Riskiness	Total Riskiness	Group Score	Group Riskiness	Total Riskiness
Portugal	1.231	0.231	0.999	-0.001	0.230	1.000	0.000	0.231
Ireland	1.423	0.423	0.999	-0.001	0.423	1.000	0.000	0.423
Italy	0.729	-0.271	0.999	-0.001	-0.272	1.000	0.000	-0.271
Greece	1.449	0.449	0.999	-0.001	0.448	1.000	0.000	0.449
Spain	0.777	-0.223	0.999	-0.001	-0.224	1.000	0.000	-0.223
Belgium	0.984	-0.016	0.999	-0.001	-0.017	1.000	0.000	-0.016
France	0.583	-0.417	0.999	-0.001	-0.418	1.000	0.000	-0.417
non-PIIGSBF								
Austria	1.793	0.793	1.001	0.001	0.794	1.000	0.000	0.793
Finland	1.044	0.044	1.001	0.001	0.045	1.000	0.000	0.044
Germany	0.536	-0.464	1.001	0.001	-0.463	1.000	0.000	-0.464
Netherlands	1.209	0.209	1.001	0.001	0.210	1.000	0.000	0.209

Notes: D_{BF} (columns 3 to 5) and S_{BF} (columns 6 through 8) refer to PIIGSBF countries which have similar behaviour of distance to default probability and stock return, respectively. For further details, see notes in Table 6.

In order to ascertain the impact of individual membership of Belgium and France to the PIIGS' group riskiness, we also extract the group score, and compute the riskiness for PIIGSB and PIIGSF groups. The following results are obtained. The risk score falls within the range (1.121, 1.153) and (1.000, 1.003) for PIIGSB and PIIGSF, respectively. Thus, the group riskiness, when Belgium is regarded as a member of the PIIGS, falls within the range 12.10%-15.30%, while that of France is bounded by -0.3% and 0.3%. This seems to suggest that Belgium behaved more like the PIIGS countries than France does, as a result of the crisis.

The above empirical results show that accounting for country level (internal) risk factors may add some explanatory power to default rate models within the Euro area for ranking individual countries in terms of riskiness. However, firms are externally exposed to extra risk induced by the economic and financial activities among the member states of the Euro area, and neglecting the potential impacts of group level (external) risk factors on firms' behaviour may likely lead to the underestimation of failure rates and related dependencies among firms. Further, firms listed within the periphery (weaker) group of countries experience higher risk level compared to those listed in the non-periphery (stronger) group in the Euro area.

4 Conclusion

The estimates of failure probability and its correlation play an important role in contemporary risk management for corporations, regulators, investors and academics (see Shumway, 2001; Duffie et al., 2007; Duan et al., 2012, among others).

In this paper we employ a mixed effects Cox model that accounts for nested unobserved factors to investigate the hazard rates and dependence structures of public listed firms of the stock exchanges in 11 Euro countries. The model embodies both country (internal) and group level (external) risk unobserved factors. For comparison purpose, we also consider a non-nested frailty model, which only incorporates unobserved factors at country level.

In the empirical analysis, we employ covariates largely used in the empirical literature, such as distance to default probability, one year trailing stock return, one year trailing market return, firm age, and 3 month T-bill rate, and consider four different groups of countries for the nested frailty model, namely PIIGS, PIIGSBF (Belgium and France are included in the

PIIGS group), PIIGSB (Belgium is included in the PIIGS), and PIIGSF (France is regarded as a PIIGS country).

The empirical analysis delivers three main results. First, when considering countries as separate entities, country level unobserved factors play an important role in explaining failure rates. In addition, a rise in the distance to default probability causes a decrease in the firms' expected time to default, stocks of firms tend to be better off when the overall economic performance improves, and older firms with high stock returns are less likely to fail. Second, the estimates of the covariates in nested models do not differ from those in non-nested models. However, nested frailty models are able to accurately capture correlations induced by both internal and external factors, and therefore are likely to estimate failure rates with smaller margin of error. This implies that the effect of the crises on firms' behaviour in the Euro area can be better explained by using nested models. Third, models that account for the distance to default probability covariate tend to outperform their counterparts, since this covariate has a higher explanatory power in default rate models.

The empirical results imply the following. First, economic and financial activities among the Euro members have significant impact on firms. Second, macroeconomic or market conditions in one or more countries are likely to be transmitted to the other countries, which implies the risk level of firms should not be limited to only country level, but also to the group level. As such, when measuring the trade-off between risk and returns of portfolios, investors should consider the macroeconomic and market trends across the entire bloc.

This paper makes three contributions to the empirical literature. First, it takes into account both country and group level unobserved factors when estimating hazard rates and dependence, while previous studies have mainly focused on the economy-wide or industry level within a country (see Duffie et al., 2009; Chava et al., 2011; Atsu and Costantini, 2015, among others). Second, the paper examines default clustering of public listed firms on 11 stock markets in the Euro area. To the best of our knowledge, this is the first study to examine default clustering at country and group levels within the Euro area. Lastly, the paper examines the impact of firms' membership to groups of the Euro area on riskiness.

Further research may consider a three level nested frailty model that accounts for sector level exposures in addition to country and group levels. This is because firms may be subjected to some industry level regulatory requirements and competition which may influence their risk levels.

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