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NON-LINEARITIES AND PERSISTENCE

IN US LONG-RUN INTEREST RATES

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Abstract

This note examines the stochastic behaviour of US monthly 10-year government bond yields. Specifically, it estimates a fractional integration model suitable to capture both persistence and non-linearities, these being two important properties of interest rates. Two series are analysed, one from Bloomberg including end-of-the-month values over the period January 1962-August 2020, the other from the ECB reporting average monthly values over the period January 1900-August 2020. The estimation results indicate that both are highly persistent and exhibit non-linearities, the latter being more pronounced in the case of the ECB series.

Keywords: Long-term interest rates; government bond yields; fractional integration; persistence; non-linearities

JEL Classification: C22, E43

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1. Introduction

Two important properties of interest rates are their high degree of persistence and the presence of non-linearities. The former has implications for the effectiveness of monetary policy and the empirical relevance of different theories such as consumption-based asset pricing models and the Fisher effect. The latter has become even more relevant in the new economic environment characterised by unconventional monetary policy and the so-called zero lower bound (ZLB) for interest rates. Therefore it is crucial to adopt a modelling approach that can capture both. Earlier studies were normally based on the I(0)/I(1) dichotomy and on a linear framework. For instance, Cox et al. (1985) characterised the short-term nominal interest rate as a stationary and mean-reverting I(0) process, whilst Campbell and Shiller (1987) concluded that it exhibits a unit root and therefore is an I(1) process, which implies lack of mean reversion (namely, shocks to interest rates have permanent effects).

Since unit root tests are well known to have very low power against fractional alternatives (Diebold and Rudebusch, 1991, Hassler and Wolters, 1993, Lee and Schmidt, 1996, etc.), a number of subsequent studies have used a fractional integration framework to analyse the behaviour of interest rates. For instance, Lai (1997), Phillips (1998) and Tsay (2000) found that it is appropriate for US real interest rates (see also Barkoulas and Baum, 1997; Meade and Maier, 2003; Gil-Alana, 2004a,b), and Couchman et al. (2006) presented similar evidence for sixteen countries. Caporale and Gil-Alana (2009) reported that in the case of the Federal Funds effective rate the fractional integration parameter is sensitive to the specification of the error term, whilst Caporale and Gil-Alana (2016, 2017) obtained evidence of long memory and fractional integration and cycles for the Euribor and the Fed Funds rate respectively.

The most recent literature has also argued that fractional integration is very much related to non-linearities (see Granger and Hyung, 2004, etc.), and that those should also be taken into account when modelling interest rates. Therefore, the present note estimates a fractional integration model allowing for both persistence and non-linearities to investigate the stochastic behaviour of US monthly 10-year government bond yields. Below, Section 2 discusses the data and the empirical analysis, and Section 3 offers some concluding remarks.

2. Data and Empirical Results

We examine two monthly series for US 10-year Government Benchmark bond yields. The first (.USGG10YR Index) includes end-of-the-month values and has been obtained from Bloomberg over the period January 1962 – August 2020. The second reports instead average monthly values; the data source in this case is the European Central Bank (ECB) database and the sample period is January 1900 – August 2020. These two series are plotted in Figure 1 and 2 respectively. In both cases it is apparent that long-term rates, initially relatively low, peaked in the 1980s before falling again.

[INSERT FIGURE 1 AND 2 ABOUT HERE]

The estimated non-linear model, as in Cuestas and Gil-Alana (2016), is the following:

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t, \quad (1-L)^d x_t = u_t, \qquad t = 1, 2, ..., \qquad (1)$$

where y_t is the observed time series, and P_{iT} are the Chebyshev time polynomials defined as:

$$P_{0,T}(t) = 1, P_{i,T}(t) = \sqrt{2}\cos\left(i\pi(t-0.5)/T\right), \qquad t = 1, 2, ... , T; \quad i = 1, 2, ... , \qquad (2)$$

where m indicates the degree of non-linearity. Thus, the higher m is, the less linear the approximated deterministic component is (see Hamming, 1973 and Smyth, 1998). Bierens (1997) and Tomasevic et al. (2009) argue that it is possible to approximate highly non-linear trends with rather low degree polynomials. In this context, if m = 0 the specification contains only an intercept; if m = 1 a linear time trend is also included, and if m > 1, nonlinearities are allowed. When estimating the model given by (1) we set m = 3, thus capturing non-linearities in the series if θ_2 and/or θ_3 are statistically significant. We also assume that the errors are autocorrelated to allow for some degree of weak dependence. However, instead of using a standard AutoRegressive Moving Average (ARMA) model we follow the non-parametric approach of Bloomfield (1973) that has been shown to work well in the context of fractional integration (see Gil-Alana, 2004c).

[INSERT TABLE 1 ABOUT HERE]

Table 1 displays the results. The fractional differencing parameter d is estimated to be equal to 0.84 and 0.83 for the Bloomberg and ECB monthly series respectively, and in both cases the values in the confidence intervals are strictly smaller than 1, which supports the hypothesis of mean reversion. This is consistent with the findings of other studies on interest rates (Gil-Alana and Moreno, 2012; Abbritti et al., 2016; etc.). In the case of the Bloomberg series the four deterministic coefficients are statistically significant, which implies non-linear behaviour. However, in the case of the ECB series with average values only θ_0 and θ_3 are found to be significant, which indicates a lower degree of non-linearities.

3. Conclusions

This note provides some evidence on the behaviour of US long-term interest rates, more specifically the 10-year government bond yields. The fractional integration framework used is more general than the standard models based on the I(0) vs. I(1) dichotomy and can capture both persistence and non-linearities.

The results show that indeed both these features are present in US long-term interest rates; also, the evidence of non-linearities is stronger for the Bloomberg series including end-of-the-month values than for the ECB one with average values. Since theory suggests that a variety of shocks such as preference, technology, fiscal and monetary shocks can generate persistence, it would be interesting to carry out additional research to investigate their relative importance. Further, non-linear dynamics also have implications for the term structure of interest rates, namely that the relationship between short and long rates is no longer linear as in a standard cointegration model. Future work should also analyse such issues.

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Table 1: Estimates of the Non-linear I(d) Model

Time Series	d	θ_0	θ_1	θ_2	θ_3
Bloomberg monthly 10-year bond yield data (1962-2020)	0.84 (0.76, 0.95)	**5.7635 (3.18)**	**1.7422 (1.66)**	**-1.9333 (-3.03)**	**-1.0016 (-2.20)**
ECB monthly 10-year bond yield data (1900-2020)	0.83 (0.78, 0.89)	**2.9648 (2.08)**	-1.0753 (-1.28)	-0.6461 (-1.27)	**1.741 (4.80)**

In bold, significant coefficients at the 5% level. In parenthesis, in columns 3 - 6, the associated t-values.

Figure 1: 10-year US Bond yield (.USGG10Y index). Source: Bloomberg

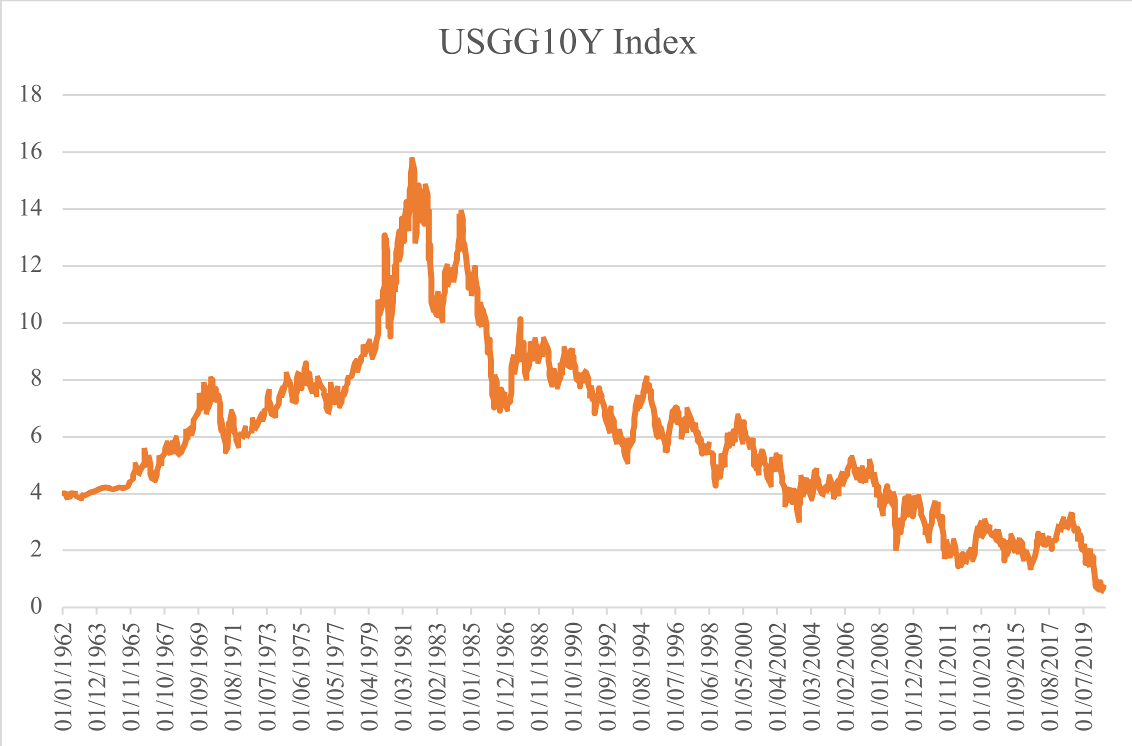

Figure 2: 10-Yr US Bond yield (average of monthly observations). Source: ECB