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PERSISTENCE IN THE REALIZED BETAS: SOME EVIDENCE FOR THE US STOCK MARKET

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Abstract

This paper examines the stochastic behaviour of the realized betas within the one-factor CAPM model for the largest 10 companies in terms of market capitalization included in the US Dow Jones stock market index. Fractional integration methods are applied to estimate their degree of persistence at the daily, weekly and monthly frequency over the period July 2000 – July 2020 using 1, 3 and 5-year samples. On the whole, the results indicate that the realized betas are highly persistent and do not exhibit weak mean-reverting behaviour at the weekly and daily frequencies, whilst there is some evidence of weak mean reversion at the monthly frequency. Our findings confirm the sensitivity of beta calculations to the choice of frequency and time span (number of observations).

Keywords: Realized beta; CAPM; persistence; mean reversion; long memory

JEL Classification: C22; G11

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1. Introduction

The one-factor capital asset pricing model (CAPM), initially introduced in the 1960s, is based on the idea that systematic risk is determined by the covariance between market and individual stock returns and is still the standard framework taught in finance courses and used by risk-averse investors for selecting optimal portfolios. Fama and MacBeth (1973) estimated this model to analyse the relationship between risk and return in NYSE stocks and documented a positive linkage between average return and market beta in the period 1926-1968; however, Fama and French (1992) found that this linear relationship had disappeared in the period 1963-1990.

The one-factor model has several limitations and is based on rather restrictive assumptions (see Fernandez, 2015, 2019); for instance, it requires investors to have homogeneous expectations (of returns, volatility and correlations for every security, over the same time horizon). In its standard formulation it is a linear regression, whose most critical parameter to be estimated is beta, which measures the risk arising from exposure to market-wide as opposed to idiosyncratic factors; polls are instead used to predict market risk, and the yield curve for the expected return of the risk-free asset.

Betas are normally predicted using historical data on the assumption that their future behaviour will be similar. Out of 150 finance textbooks we have reviewed 80 recommend some estimation method but differ in terms of the frequency (daily, weekly, monthly or annual) and the span of data (from 6 months to 25 years) used for this purpose. As in Campbell et al. (1997), we found that the most common estimation approach (in 64% of the cases) is to use monthly data over a 5-year period. However, more recently, higher frequency data have often been used as developments in IT have made computations easier. Table 1 summaries our findings concerning the frequency and the number of observations (time span) chosen for estimating the realized betas in the textbooks reviewed.

INSERT TABLE 1 ABOUT HERE

Among more recent studies focusing on higher frequency data, Andersen et al. (2003) and Bollerslev et al. (2009) analysed intraday trading with samples of 15 minutes. Damodaran[1] on his public portal for beta estimation selected different time periods (5 years and 2 years with weekly returns). Papageorgiou et al. (2016) analysed daily returns over a one-year period and showed that these results outperform those obtained using monthly data over a 5-year period as in Fama and MacBeth (1973). Cenesizoglu et al. (2016) evaluated the accuracy of one-month-ahead beta forecasts (at the monthly, daily and 30-minute frequency) and found that low (high) frequency returns produce the least (most) accurate estimates. Sharma (2016) analysed the conditional variance of various stock indices over 14 years. Bollerslev et al. (2016) investigated how individual stock prices respond to market price movements and jumps using data at the 5-minute intraday frequency with one-year samples, and found evidence that betas associated with intraday discontinuous and overnight returns entail significant risk premiums, while the intraday continuous betas do not. Cenesizoglu et al. (2018) used a realized beta estimator for daily returns over the previous year for 1, 3, and 6-month holding periods to explain momentum effects.

An appropriate estimation period and sampling frequency are clearly crucial for obtaining accurate beta forecasts. An important issue is the possibility of time variation in the betas (Andersen et al., 2003), which is not considered by the standard, one-factor CAPM. Multi-factor pricing models including additional empirically motivated factors, such as such firm size and book-to-market ratios (Fama and French, 1993), have been

[1] Damodaran online: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/variable.htm

shown to have better in-sample fit and to produce more accurate out-of-sample predictions, but are often criticized because of the difficulty in interpreting the expanded set of variables in terms of systematic risk.

An interesting question in this context is how persistent the realized betas are. Andersen et al. (2005) apply fractional integration methods to analyse data for 25 Dow Jones Industrial Average (DJIA) stocks over the period 1962-1999 and conclude that the corresponding betas are not very persistent and are best modelled as I(0) mean-reverting processes. The present paper uses a similar modelling framework but computes beta coefficients at the daily, weekly and monthly frequency over the period July 2000 – July 2020 using 1, 3 and 5-year samples for the 10 largest stocks in terms of capitalization included in the Dow Jones Index. In contrast to Andersen et al. (2005), we find evidence of high persistence at the daily and weekly frequency, though not at the monthly one, which confirms the sensitivity of beta calculations to the choice of frequency and time span (number of observations). The layout of the paper is as follows: Section 2 provides a brief literature review; Section 3 outlines the fractional integration model used for the analysis; Section 4 describes the data and discusses the empirical results; Section 5 offers some concluding remarks.

2. Literature Review

In this section we discuss in turn each of the three main approaches to modelling and forecasting the realized betas that have been adopted in the CAPM literature.

2.1 Realized variance and data filtering

A first group of studies focuses on realized variance, covariance, and data filtering. Ghysels and Jacquier (2006) proposed a mix of existing data-driven filters and parametric methods. Hooper et al. (2008) compared a series of competing models to forecast beta; specifically, the applied realized measures of asset return variances and covariances following the methodology proposed in Andersen et al. (2005). Christoffersen et al. (2008) used the information embedded in the prices of stock options and index options to compute the forward-looking market beta at the daily frequency, using option data for a single day. Chang et al. (2012) found that option-implied volatility was a good predictor of future realized betas and proposed a beta estimator based on this approach. Chen and Reeves (2012) estimated monthly realized betas with Hodrick–Prescott noise filters, while Reeves and Wu (2013) evaluated constant and autoregressive (AR) models of time-varying realized betas, showing that beta models with constant parameters generate more accurate quarterly forecasts.

2.2. Time-varying betas

A second group of studies are based on the idea that the betas may vary with the conditioning variables, which leads to the concept of "conditional CAPM", and therefore focus on time-varying betas. This approach was introduced by Dybvig and Ross (1985). Fama and French (1992) pointed out the inability of the static CAPM to explain the cross-section of average returns; more specifically, the robustness of the size effect and the absence of a relationship between beta and average returns are inconsistent with the CAPM. Fama and French (1993) examined common risk factors in the returns on stocks and bonds, namely factors related to markets, firm size and book-to-market ratio. Ferson et al. (1987) developed tests of asset-pricing empirical models allowing market betas to change over time. Ferson and Harvey (1991) analysed the predictable components of monthly common stock and bond portfolio returns. Jagannathan and Wang (1996) argued in favour of time-varying betas on the grounds that the relative risk of a firm's cash flow is likely to change with the business cycle. Wang (2003) used a non-parametric approach to incorporate the conditioning information. Ang and Chen (2007) proposed a conditional CAPM with time-varying betas and market risk premia.

In the last decade, additional factors have been considered. Garleanu and Pedersen (2011) introduced the margin-CAPM model where high-margin assets require higher returns. Ang and Kristensen (2011) estimated time-varying betas with non-parametric techniques, proposing a conditional CAPM and multifactor models for book-to-market and momentum decile portfolios. Engle and Rangel (2010) and Rangel and Engle (2012) provided evidence that models with volatility and correlation components outperform single component models. Patton and Verardo (2012) studied the information flow and its impact on the betas, finding that these increase on announcement days by a statistically significant amount. Buss and Vilkov (2012) used forward-looking information from option prices to estimate option-implied correlations. Boubaker and Sghaier (2013) analysed portfolio optimization in the presence of financial returns with long memory. Frazzini and Pedersen (2014) presented a leverage and margin constraint model that varies across investors and time. Jayasinghe et al. (2014) estimated the time-varying conditional variance of index returns, finding evidence of mean-reversion and long memory in the betas. More recently, Fama and French (2015) extended the standard CAPM model to include five additional factors representing size, value, profitability, and investment patterns in average stock returns. However, this five-factor model still fails to capture the low average returns on small stocks whose returns behave like those of firms that invest a lot despite low profitability.

Several more recent studies have proposed alternative beta estimation methods. Lu and Murray (2017) suggested a "bear beta" model, where time variation in the probability of future bear market states is priced. Pyun (2019) introduced a new out-of-sample forecasting method for monthly market returns using the Variance Risk Premium (VRP) defined in Bollerslev et al. (2009) as the difference between the objective and the risk-neutral expectations of the forward variance. Bai et al. (2019) proposed a general equilibrium model to quantify the consumption CAPM performance. Hollstein et al. (2019) proposed a link between conditional betas and high high-frequency data to explain asset pricing anomalies.

2.3 Long memory in asset pricing

A third approach introduced by Bollerslev et al. (1988) focuses on long-run dependence. Following the early contribution of Robinson (1991), many subsequent studies showed the empirical relevance of long memory for asset return volatility (e.g., Ding et al., 1993). Robinson (1995) developed a formal framework for testing long-run dependence in the logarithmic volatilities; the FIGARCH model was used by Baillie et al. (1996) to analyse exchange rates, and by Bollerslev and Mikkelsen (1996) to examine US stock market, in both cases long memory being detected, with the series being modelled as mean-reverting fractionally integrated processes, where the conditional variance decreases at a slow hyperbolic rate. Andersen and Bollerslev (1997) concluded that long memory is an intrinsic feature of returns. Bollerslev and Mikkelsen (1999) provided evidence of mean reversion in the volatility process using fractionally integrated models.

Cochran and DeFina (1995) found predictable periodicity in market cycles. Bollerslev and Mikkelsen (1996) concluded that long-run dependence in the US stock market is best modelled as a mean-reverting fractionally integrated process. However, Andersen and Bollerslev (1997) found that this process is very slow for most returns, and thus detecting mean reversion is not an easy task. Balvers et al. (2000) pointed out that, if it exists, it can only be detected over long horizons; nevertheless, investors try to discover mean-reverting patterns for forecasting purposes (Javasinghe, 2014).

Andersen et al. (2003) analysed the persistence and predictability of the realized betas as well as of the underlying market variances and covariances using intraday data over the period 1962-1999; the latter were found to be highly persistent and fractionally integrated processes, in contrast to the realized betas, which appear to be much less persistent and best modelled as a standard stationary I(0) process. Further, simple AR-type models were shown to outperform other parametric models in terms of their forecasting properties for the integrated volatility. Andersen et al. (2005) pointed out that it is possible for the betas to be only weakly persistent (short-memory, with $d \sim 0$), despite the widespread finding that realized variances and covariances exhibit long memory (fractionally integrated, with $d > 0$), in the case of fractional cointegration.

Regarding the sampling frequency, Bollerslev et al. (2006) found evidence of negative correlations between stock market movements and volatility at the intraday frequency. In particular, five-minute intervals appear to provide better results than one-day market sampling for assessing volatility asymmetries. Todorov and Bollerslev (2007) looked for a solution to the problem of modelling jumps in the betas using high-frequency data. Morana (2009) improved the realized beta estimator introduced by Andersen et al. (2005, 2006) by allowing for multiple non-orthogonal risk factors.

Bollerslev et al. (2011) explored alternative volatility measures to reduce the impact of the microstructure noise. Bollerslev et al. (2012) used intraday data for the S&P 500 and the VIX volatility indices and found further evidence that aggregate stock market volatility exhibits long-run dependence, while the volatility risk premium (VRP) is much less persistent. Bollerslev et al. (2013) concluded that market volatility is best described as a long-memory fractionally integrated process. Hansen et al. (2014) proposed a

GARCH model incorporating realized measures of variances and covariances. Engle (2016) put forward the Dynamic Conditional Beta (DCB) model to estimate regressions with time-varying parameters.

A brief comparison between the most popular market beta estimation techniques can be found in Hollstein and Prokopczuk (2016), who examined the performance of several time-series models and option-implied estimators, and suggested using the hybrid methodology of Buss and Vilkov (2012) since it consistently outperforms all other approaches.

3. Methodology

We analyse persistence in the realized betas by using fractional integration methods to estimate the degree of dependence in the data, which is measured by the differencing parameter d. For our purposes we define a covariance stationary process $\{x_t, t = 0, \pm 1, \ldots\}$ as integrated of order 0, and denote it by I(0), if the infinite sum of its autocovariances is finite. This type of processes, also known as short-memory ones, include not only the white noise but also the stationary and invertible ARMA-type of models. To generalise, we can define the process $\{y_t, t = 0, \pm 1, \ldots\}$ as integrated of order d, and denote it by I(d), if d-differences are required to make it I(0), i.e.,

$$(1 - B)^d x_t = u_t, \quad t = 0, \pm 1, \ldots, \qquad (1)$$

where B is the backshift operator, and d can be any integer or fractional value. Processes with d higher than 0 are known as long-memory ones because of the high degree of dependence between observations far apart in time, where the polynomial in B in equation (1) can be expressed in terms of its Binomial expansion, such that

$$(1 - B)^d = \sum_{j=0}^{\infty} \psi_j B^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j B^j = 1 - d\,B + \frac{d(d-1)}{2} B^2 - \ldots,$$

implying that

$$(1-B)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \ldots .$$

The parameter d plays a crucial role in this context, since it is a measure of the degree of persistence of the series: the higher is d, the higher is the degree of dependence between observations. More specifically, $d = 0$ implies short memory behaviour, while $0 < d < 0.5$ characterises a covariance stationary long-memory process; if $0.5 \leq d < 1$, the series is non-stationary but mean-reverting with shocks having long-lasting effects that disappear in the long run; finally, $d \geq 1$ implies non-stationarity and lack of mean reversion.

Although fractional integration was already proposed in the early 1980s by Granger (1980, 1981), Granger and Joyeux (1989) and Hosking (1981), it was not until the late 1990s and early 2000 that it become popular in economics and finance (Baillie, 1996; Gil-Alana and Robinson, 1997; Mayoral, 2006; Gil-Alana and Moreno, 2012; Abbritti et al., 2016; etc.). We estimate the differencing parameter using the Whittle function in the frequency domain (Dahlhaus, 1989) by using a version of the LM tests of Robinson (1994) which is computationally very attractive.

4. Data and Empirical Results

We have obtained data on daily, weekly and monthly returns from the Reuters Eikon database for the ten companies with the highest market capitalization included in the Dow Jones Industrial Average Index (.DJI) over the period 13 July 2000 – 14 July 2020. Specifically, we consider the following companies: Apple Inc (AAPL.O), Microsoft Corp (MSFT.O), Johnson & Johnson (JNJ), Procter & Gamble Co. (PG), Walmart Inc (WMT), Home Depot Inc. (HD), JPMorgan Chase & Co (JPM), Intel Corp (INTC.O), Verizon Communications Inc (VZ) and UnitedHealth Group Inc (UNH).

Using the raw data, we construct daily, weekly and monthly realized beta series by applying the formula $\frac{Covariance(Stock, Index)}{Variance(Index)}$ over 1, 3 and 5-year spans, thus obtaining 9 beta measures for each company. These are displayed in Figure 1.

INSERT FIGURE 1 AND TABLE 2 ABOUT HERE

Table 2 reports instead some descriptive statistics (standard deviation, average and scaled volatility calculated as the standard deviation over the average) for the series of interest. It can be seen that volatility is smaller at higher frequencies. For instance, over a 5-year span the average volatility coefficient is equal to 0.274 for the monthly series, 0.146 for the weekly one, and 0.135 for the daily one. Moreover, volatility tends to be higher over shorter spans – for example, at the monthly frequency its average is equal to 0.664 over a 1-year span, which is 2.4 times higher than for the 5-year span.

The estimated model used for analysing the stochastic properties of the constructed series is the following:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad (1 - L)^{d_o} x_t = u_t, \qquad t = 1, 2, \ldots, \qquad (1)$$

where y_t is the observed time series (the realized betas in our case), β_0 and β_1 are unknown coefficients on the intercept (constant) and the linear time trend, and x_t is I(d), where d is estimated from the data. We consider three model specifications, namely i) no deterministic terms, i.e., $\beta_0 = \beta_1 = 0$ in (1); ii) a constant only, i.e., $\beta_1 = 0$; and iii) a constant as well as a linear trend, i.e., β_0 and β_1 are estimated. Table 3 reports the estimated values of d along with their associated 95% confidence bands under the assumption of white noise errors for all three models; the coefficients in bold are in each case those from the preferred model, which has been selected on the basis of the statistical significance of the other parameters as indicated by the t-values; these are reported in Table 4 together with the corresponding estimates of d.

INSERT TABLE 3 AND 4 ABOUT HERE

As can be seen, the preferred specification includes an intercept only in 93% of the cases, whilst in two cases, both over a 5-year span and at the daily frequency, this also includes a linear time trend, and in four cases, all for the monthly series over a 1-year span, no deterministic trends are found to be significant. As for the estimated values of d, the unit root null cannot be rejected in most cases. In fact, the average value of d in all cases is very close to 1 (0.989).

INSERT TABLE 5 ABOUT HERE

Table 5 reports the estimated values of d with the corresponding volatility coefficients. When using the Fama and MacBeth (1973) "standard" beta measure (based on 5 years of monthly observations), the estimates of d are smaller than 1 in all cases (0.897 in average), which implies weak mean reversion. In general, d tends to be larger at higher frequencies and over longer time spans; in particular, for daily data (over 1y, 3y and 5y spans) or weekly ones (over 3y and 5y spans) the average value of d is 1.04, which implies lack of mean reversion. By contrast, volatility is smaller at higher frequencies and over longer time spans. The bottom rows of Table 5 report the average over the 10 stocks, for both the integration parameter d and the volatility coefficient, for each beta measure. Figure 2 provides a scatter diagram for these two parameters which confirms that d tends to be larger over longer spans and at higher frequencies whilst the opposite holds for volatility. Figure 3 provides the same type of information for each individual stock - the same broad picture emerges.

INSERT FIGURE 2 AND 3 ABOUT HERE

To summarise, we find evidence of non-stationary behaviour, with orders of integration equal to or higher than 1, in the daily and weekly series, whilst there is weak evidence of mean reversion ($d < 1$) at the monthly frequency, in contrast to Andersen et al. (2005), who conclude that it is possible that betas may be only weakly persistent (short-memory, I(d), with d close to 0). Our different results can be explained by the fact that we use much longer data spans (1y, 3y, 5y) than Andersen et al. (2005) whose estimates are based on 3-month periods with intraday frequencies.

5. Conclusions

In this study we have examined the statistical properties of the realized betas within the framework of the one-factor CAPM model using data on the 10 largest capitalized companies from the US Dow Jones index and applying fractional integration, long-memory techniques. In particular, we have estimated their degree of integration d to measure persistence.

Our results highlight the importance of the choice of frequency and time span (number of observations) for estimation purposes. In particular, we find that longer time spans and higher frequencies correspond to higher estimates of d. When using a monthly sampling 5-year span as in Fama and MacBeth (1973), the realized betas appear to be characterised by weak mean reversion, which implies that shocks do not have permanent effects; the use of different time spans is one of possible explanation for the differences between our results and those reported by Andersen et al. (2005).

On the whole, our analysis suggests that the standard practice of estimating the betas as in Fama and MacBeth (1973) using only a 5-year sample period is questionable given the lack of robustness of the results to the choice of frequency and time span (number of observations) as at higher frequencies volatility appears to be smaller but higher values of d are obtained and thus mean-reverting behaviour is not observed. Future work could also provide evidence for other developed stock markets to gain additional insights into the behavior of the realized betas.

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Table 1: Estimation of the realized betas: chosen frequency and number of observations (time span) in finance textbooks

	Daily	Weekly	Monthly	Quarterly	Annual
Number of textbooks	7	6	51	1	15
Average number of observations used	489	156	76	16	8
Most common number of observations used	765	26	60	16	10

It can be seen that, as the frequency increases, the selected time span decreases (on average, monthly estimates are based on a span of 6 years, weekly ones on a span of 3 years, and daily ones a span of 1.5 years), whilst the number of observations used for the analysis increases.

Table 2: Descriptive Statistics

	Monthly			Weekly			Daily		
Apple	5y	3y	1y	5y	3y	1y	5y	3y	1y
Stdev	0.306	0.449	0.843	0.093	0.206	0.370	0.128	0.183	0.290
Average	1.178	1.180	1.234	1.014	1.030	1.028	1.039	1.069	1.099
Stdev/average	0.259	0.380	0.683	0.092	0.200	0.360	0.123	0.172	0.264
Microsoft									
Stdev	0.108	0.253	0.596	0.093	0.121	0.216	0.083	0.115	0.163
Average	0.984	0.947	0.911	0.957	0.964	0.952	1.070	1.070	1.066
Stdev/average	0.109	0.267	0.655	0.097	0.125	0.227	0.077	0.107	0.153
Homedepot									
Stdev	0.173	0.197	0.429	0.117	0.123	0.182	0.097	0.110	0.169
Average	0.996	1.013	1.068	1.144	1.129	1.139	1.050	1.052	1.062
Stdev/average	0.174	0.194	0,401	0.102	0.109	0.160	0.093	0.104	0.159
Intel									
Stdev	0.387	0.440	0.638	0.157	0.194	0.269	0.190	0.204	0.251
Average	1,232	1.256	1.277	1.173	1.195	1.207	1.198	1.219	1.247
Stdev/average	0.314	0.351	0.499	0.134	0.162	0.223	0.158	0.168	0.202
Johnson & Johnson									
Stdev	0.150	0.215	0.334	0.093	0.120	0.181	0,084	0,097	0,162
Average	0.595	0.568	0.575	0.623	0.626	0.619	0,649	0,653	0,625
Stdev/average	0.252	0.378	0.581	0.150	0.191	0.292	0,129	0,149	0,260
JPmorgan									
Stdev	0.206	0.344	0.539	0.219	0.256	0.351	0,210	0,249	0,308
Average	1.402	1.371	1.364	1.487	1.427	1.375	1,523	1,453	1,391
Stdev/average	0.147	0.251	0.395	0.147	0.179	0.255	0,138	0,171	0,221
Procter and gamble									
Stdev	0.148	0.248	0.397	0.053	0.076	0.154	0,052	0,086	0,134
Average	0.489	0.483	0.508	0.551	0.539	0.515	0,620	0,617	0,601
Stdev/average	0.302	0.513	0.781	0.097	0.141	0.298	0,084	0,139	0,222
United Health									
Stdev	0.303	0.332	0.426	0.268	0.298	0,365	0,192	0,203	0,234
Average	0.692	0.623	0.601	0.890	0.829	0,776	0,909	0,875	0,825
Stdev/average	0.437	0.534	0.709	0.301	0.360	0,471	0,211	0,232	0,284
Verizon									
Stdev	0.279	0.326	0.505	0.116	0.153	0,241	0,120	0.140	0.178
Average	0.678	0.689	0.710	0.737	0.741	0.724	0.769	0.757	0.725
Stdev/average	0.411	0.474	0.710	0.158	0.206	0.333	0.156	0.185	0.246
Walmart									
Stdev	0.133	0.208	0.549	0.115	0.138	0.196	0.122	0.138	0.180
Average	0.400	0.401	0.448	0.646	0.661	0.675	0.669	0.689	0.695
Stdev/average	0.332	0.519	1.225	0.179	0.209	0.290	0.183	0.200	0.258
Average (all)									
Stdev	0.219	0.301	0.526	0.133	0.168	0.252	0.128	0.153	0.207
Average	0.865	0.853	0.870	0.922	0.914	0.901	0.950	0.945	0.934
Stdev/average	0.274	0.386	0.664	0.146	0.188	0.291	0.135	0.163	0.227

Table 3: Estimates of the differencing parameter d

	No terms	Constant	Constant + Time trend
APPLE INC.			
DAILY. 1 YEAR	1.01 (0.99. 1.03)	**1.03 (1.01. 1.05)**	1.03 (1.01. 1.05)
DAILY. 3 YEAR	1.00 (0.98. 1.04)	**1.06 (1.04. 1.08)**	1.06 (1.04. 1.08)
DAILY. 5 YEAR	1.00 (0.97. 1.03)	**1.07 (1.04. 1.09)**	1.07 (1.04. 1.09)
WEEKLY. 1 YEAR	0.96 (0.91. 1.01)	**0.96 (0.92. 1.01)**	0.96 (0.92. 1.01)
WEEKLY. 3 YEAR	0.94 (0.90. 0.99)	**0.90 (0.87. 0.94)**	0.90 (0.87. 0.94)
WEEKLY. 5 YEAR	0.96 (0.91. 1.01)	**0.88 (0.83. 0.93)**	0.88 (0.83. 0.93)
MONTHLY. 1 YEAR	0.86 (0.76. 0.98)	**0.84 (0.74. 0.97)**	0.84 (0.74. 0.97)
MONTHLY. 3 YEAR	0.91 (0.83. 1.02)	**0.90 (0.82. 1.02)**	0.90 (0.82. 1.02)
MONTHLY. 5 YEAR	0.92 (0.84. 1.03)	**0.91 (0.83. 1.02)**	0.91 (0.83. 1.02)
HOMEDEPOT			
DAILY. 1 YEAR	1.02 (0.98. 1.03)	**1.04 (1.01. 1.07)**	1.04 (1.01. 1.07)
DAILY. 3 YEAR	1.00 (0.98. 1.03)	**1.02 (1.00. 1.05)**	1.02 (1.00. 1.05)
DAILY. 5 YEAR	1.00 (0.97. 1.03)	**1.03 (1.01. 1.06)**	1.03 (1.01. 1.06)
WEEKLY. 1 YEAR	0.96 (0.91. 1.01)	**0.93 (0.88. 0.98)**	0.93 (0.88. 0.98)
WEEKLY. 3 YEAR	0.99 (0.94. 1.05)	**0.96 (0.91. 1.03)**	0.96 (0.91. 1.03)
WEEKLY. 5 YEAR	0.99 (0.94. 1.05)	**0.97 (0.91. 1.03)**	0.97 (0.91. 1.03)
MONTHLY. 1 YEAR	1.02 (0.98. 1.03)	**0.82 (0.75. 0.92)**	0.82 (0.75. 0.92)
MONTHLY. 3 YEAR	0.89 (0.78. 1.03)	**0.85 (0.73. 0.99)**	0.85 (0.73. 0.99)
MONTHLY. 5 YEAR	0.96 (0.87. 1.08)	**0.75 (0.67. 0.87)**	0.75 (0.67. 0.87)
INTEL			
DAILY. 1 YEAR	1.00 (0.98. 1.03)	**1.02 (1.00. 1.05)**	1.02 (1.00. 1.05)
DAILY. 3 YEAR	1.00 (0.97. 1.03)	**1.08 (1.05. 1.11)**	1.08 (1.05. 1.10)
DAILY. 5 YEAR	1.00 (0.98. 1.03)	**1.13 (1.10. 1.16)**	1.13 (1.10. 1.16)
WEEKLY. 1 YEAR	0.97 (0.92. 1.02)	**0.94 (0.89. 1.00)**	0.94 (0.89. 1.00)
WEEKLY. 3 YEAR	0.99 (0.94. 1.04)	**0.98 (0.94. 1.04)**	0.98 (0.94. 1.04)
WEEKLY. 5 YEAR	0.99 (0.94. 1.05)	**1.02 (0.96. 1.08)**	1.02 (0.96. 1.08)
MONTHLY. 1 YEAR	0.87 (0.78. 1.01)	**0.78 (0.67. 0.93)**	0.78 (0.65. 0.93)
MONTHLY. 3 YEAR	0.94 (0.85. 1.05)	0.73 (0.64. 0.87)	**0.76 (0.67. 0.88)**
MONTHLY. 5 YEAR	0.94 (0.85. 1.07)	**0.80 (0.71. 0.93)**	0.83 (0.75. 0.94)

(cont)

JOHNSON AND JOHNSON			
DAILY. 1 YEAR	1.03 (1.01. 1.05)	**1.03 (1.01. 1.05)**	1.03 (1.01. 1.05)
DAILY. 3 YEAR	1.01 (0.99. 1.03)	**1.05 (1.03. 1.07)**	1.05 (1.03. 1.07)
DAILY. 5 YEAR	1.01 (0.98. 1.03)	**1.06 (1.04. 1.08)**	1.06 (1.04. 1.08)
WEEKLY. 1 YEAR	1.05 (1.00. 1.11)	**1.05 (1.00. 1.11)**	1.05 (1.00. 1.11)
WEEKLY. 3 YEAR	1.05 (0.99. 1.11)	**1.21 (1.14. 1.28)**	1.21 (1.14. 1.28)
WEEKLY. 5 YEAR	1.03 (0.97. 1.09)	**1.23 (1.16. 1.32)**	1.23 (1.16. 1.32)
MONTHLY. 1 YEAR	0.81 (0.71. 0.94)	**0.81 (0.71. 0.94)**	0.81 (0.71. 0.94)
MONTHLY. 3 YEAR	0.89 (0.80. 1.00)	**0.88 (0.79. 0.99)**	0.88 (0.79. 0.99)
MONTHLY. 5 YEAR	0.85 (0.74. 0.97)	0.79 (0.70. 0.90)	**0.80 (0.72. 0.91)**
JP MORGAN			
DAILY. 1 YEAR	1.03 (1.00. 1.05)	**1.11 (1.09. 1.13)**	1.11 (1.09. 1.13)
DAILY. 3 YEAR	1.01 (0.99. 1.04)	**1.12 (1.10. 1.14)**	1.12 (1.10. 1.14)
DAILY. 5 YEAR	1.01 (0.98. 1.03)	**1.12 (1.10. 1.14)**	1.12 (1.10. 1.14)
WEEKLY. 1 YEAR	1.04 (0.99. 1.10)	**1.08 (1.03. 1.15)**	1.08 (1.03. 1.15)
WEEKLY. 3 YEAR	1.03 (0.98. 1.08)	**1.13 (1.08. 1.20)**	1.13 (1.08. 1.20)
WEEKLY. 5 YEAR	1.02 (0.97. 1.08)	**1.15 (1.08. 1.22)**	1.15 (1.08. 1.22)
MONTHLY. 1 YEAR	0.89 (0.79. 1.01)	**0.92 (0.81. 1.05)**	0.92 (0.81. 1.05)
MONTHLY. 3 YEAR	0.96 (0.87. 1.08)	**0.99 (0.90. 1.10)**	0.99 (0.91. 1.10)
MONTHLY. 5 YEAR	0.94 (0.85. 1.07)	**0.90 (0.83. 1.00)**	0.90 (0.83. 1.00)
MICROSOFT			
DAILY. 1 YEAR	1.00 (0.98. 1.03)	**1.02 (1.00. 1.04)**	1.02 (1.00. 1.04)
DAILY. 3 YEAR	1.00 (0.97. 1.02)	**1.01 (0.99. 1.04)**	1.01 (0.99. 1.04)
DAILY. 5 YEAR	1.00 (0.97. 1.02)	**1.02 (1.00. 1.04)**	1.02 (1.00. 1.04)
WEEKLY. 1 YEAR	0.98 (0.94. 1.03)	**0.98 (0.94. 1.03)**	0.98 (0.94. 1.03)
WEEKLY. 3 YEAR	0.98 (0.94. 1.04)	**0.98 (0.94. 1.04)**	0.98 (0.94. 1.04)
WEEKLY. 5 YEAR	0.98 (0.93. 1.04)	**1.05 (1.00. 1.10)**	1.05 (1.00. 1.10)
MONTHLY. 1 YEAR	1.00 (0.88. 1.13)	**1.03 (0.92. 1.16)**	1.03 (0.92. 1.16)
MONTHLY. 3 YEAR	0.99 (0.89. 1.12)	**1.03 (0.94. 1.14)**	1.03 (0.94. 1.14)
MONTHLY. 5 YEAR	0.94 (0.82. 1.09)	**0.93 (0.82. 1.07)**	0.93 (0.83. 1.07)
VERIZON			
DAILY. 1 YEAR	1.03 (1.01. 1.05)	**1.06 (1.03. 1.09)**	1.06 (1.03. 1.09)
DAILY. 3 YEAR	1.01 (0.98. 1.03)	**1.02 (1.00. 1.04)**	1.02 (1.00. 1.04)
DAILY. 5 YEAR	1.00 (0.98. 1.03)	0.99 (0.97. 1.01)	**0.99 (0.97. 1.01)**
WEEKLY. 1 YEAR	1.03 (0.99. 1.08)	**1.04 (0.99. 1.09)**	1.04 (0.99. 1.09)
WEEKLY. 3 YEAR	1.02 (0.97. 1.07)	**1.04 (0.99. 1.09)**	1.04 (0.99. 1.09)
WEEKLY. 5 YEAR	1.02 (0.97. 1.07)	**1.05 (1.01. 1.10)**	1.05 (1.01. 1.10)
MONTHLY. 1 YEAR	**0.90 (0.81. 1.01)**	0.90 (0.81. 1.01)	0.90 (0.81. 1.01)
MONTHLY. 3 YEAR	1.00 (0.92. 1.11)	**0.96 (0.87. 1.07)**	0.96 (0.87. 1.07)
MONTHLY. 5 YEAR	1.00 (0.92. 1.12)	**0.93 (0.85. 1.04)**	0.93 (0.84. 1.04)

(cont.)

WALMART			
DAILY. 1 YEAR	1.00 (0.98. 1.03)	**1.02 (0.99. 1.05)**	1.02 (0.99. 1.05)
DAILY. 3 YEAR	1.01 (0.97. 1.03)	**1.02 (1.00. 1.05)**	1.02 (1.00. 1.05)
DAILY. 5 YEAR	1.00 (0.97. 1.03)	1.03 (1.01. 1.06)	**1.03 (1.01. 1.06)**
WEEKLY. 1 YEAR	1.01 (0.95. 1.05)	**1.01 (0.95. 1.06)**	1.00 (0.95. 1.06)
WEEKLY. 3 YEAR	1.00 (0.96. 1.07)	**1.07 (1.02. 1.13)**	1.07 (1.02. 1.13)
WEEKLY. 5 YEAR	1.01 (0.96. 1.07)	**1.12 (1.07. 1.19)**	1.12 (1.07. 1.19)
MONTHLY. 1 YEAR	**0.99 (0.89. 1.12)**	0.99 (0.88. 1.11)	0.99 (0.88. 1.11)
MONTHLY. 3 YEAR	0.96 (0.86. 1.08)	**0.92 (0.83. 1.04)**	0.92 (0.83. 1.04)
MONTHLY. 5 YEAR	1.01 (0.92. 1.13)	**0.96 (0.87. 1.07)**	0.96 (0.87. 1.07)
UNITED HEALTH			
DAILY. 1 YEAR	1.01 (1.03. 1.06)	**1.03 (1.05. 1.07)**	1.03 (1.05. 1.07)
DAILY. 3 YEAR	1.04 (1.02. 1.07)	**1.08 (1.05. 1.10)**	1.08 (1.05. 1.10)
DAILY. 5 YEAR	1.03 (1.01. 1.06)	**1.09 (1.07. 1.12)**	1.09 (1.07. 1.12)
WEEKLY. 1 YEAR	1.01 (0.96. 1.06)	**1.02 (0.97. 1.07)**	1.02 (0.97. 1.07)
WEEKLY. 3 YEAR	1.08 (1.03. 1.14)	**1.12 (1.07. 1.18)**	1.12 (1.07. 1.18)
WEEKLY. 5 YEAR	1.06 (1.01. 1.12)	**1.13 (1.08. 1.20)**	1.13 (1.08. 1.20)
MONTHLY. 1 YEAR	**0.71 (0.62. 0.84)**	0.71 (0.61. 0.83)	0.71 (0.61. 0.83)
MONTHLY. 3 YEAR	0.86 (0.77. 0.97)	**0.87 (0.78. 0.98)**	0.87 (0.78. 0.98)
MONTHLY. 5 YEAR	0.93 (0.86. 1.01)	**0.94 (0.87. 1.02)**	0.94 (0.87. 1.02)
PROCTER			
DAILY. 1 YEAR	1.04 (1.01. 1.06)	**1.04 (1.02. 1.06)**	1.04 (1.02. 1.06)
DAILY. 3 YEAR	1.01 (0.99. 1.04)	**1.05 (1.03. 1.08)**	1.05 (1.03. 1.08)
DAILY. 5 YEAR	1.00 (0.98. 1.03)	**1.05 (1.02. 1.07)**	1.05 (1.02. 1.07)
WEEKLY. 1 YEAR	0.99 (0.94. 1.05)	**0.99 (0.94. 1.05)**	0.99 (0.94. 1.05)
WEEKLY. 3 YEAR	0.98 (0.94. 1.04)	**0.94 (0.89. 0.99)**	0.94 (0.89. 0.99)
WEEKLY. 5 YEAR	0.99 (0.94. 1.05)	**0.96 (0.90. 1.02)**	0.96 (0.90. 1.02)
MONTHLY. 1 YEAR	**0.81 (0.73. 0.90)**	0.80 (0.73. 0.90)	0.80 (0.73. 0.90)
MONTHLY. 3 YEAR	1.02 (0.94. 1.12)	**1.02 (0.94. 1.13)**	1.02 (0.94. 1.13)
MONTHLY. 5 YEAR	0.99 (0.91. 1.09)	**0.99 (0.91. 1.10)**	0.99 (0.92. 1.10)

Note: In bold. the selected specifications on the basis of the significance of the deterministic terms. In parenthesis the 95% confidence bands for the values of d. 1y. 3y and 5y stand for 1. 3 and 5 year time spans respectively. Red values indicate statistical evidence of mean reversion.

Table 4: Estimates of the differencing parameter d, the constant and the trend coefficient from the selected model specifications

APPLE INC.			
Series	d	Constant (t-value)	Time trend (t-value)
DAILY. 1 YEAR	1.03 (1.01. 1.05)	1.6077 (108.68)	---
DAILY. 3 YEAR	1.06 (1.04. 1.08)	1.1725 (219.08)	---
DAILY. 5 YEAR	1.07 (1.04. 1.09)	1.2341 (346.27)	---
WEEKLY. 1 YEAR	0.96 (0.92. 1.01)	1.2450 (17.41)	---
WEEKLY. 3 YEAR	0.90 (0.87. 0.94)	0.9460 (33.72)	---
WEEKLY. 5 YEAR	0.88 (0.83. 0.93)	1.0351 (52.88)	---
MONTHLY. 1 YEAR	0.84 (0.74. 0.97)	0.9699 (2.63)	---
MONTHLY. 3 YEAR	0.90 (0.82. 1.02)	1.2838 (9.74)	---
MONTHLY. 5 YEAR	0.91 (0.83. 1.02)	1.3565 (16.92)	---
HOMEDEPOT			
DAILY. 1 YEAR	1.04 (1.01. 1.07)	1.7660 (192.13)	---
DAILY. 3 YEAR	1.02 (1.00. 1.05)	1.3610 (365.57)	---
DAILY. 5 YEAR	1.03 (1.01. 1.06)	1.3395 (484.54)	---
WEEKLY. 1 YEAR	0.93 (0.88. 0.98)	1.8297 (39.84)	---
WEEKLY. 3 YEAR	0.96 (0.91. 1.03)	1.3601 (71.35)	---
WEEKLY. 5 YEAR	0.97 (0.91. 1.03)	1.3468 (95.50)	---
MONTHLY. 1 YEAR	0.85 (0.73. 0.99)	1.2262 (23.46)	---
MONTHLY. 3 YEAR	0.75 (0.67. 0.87)	1.5201 (6.50)	---
MONTHLY. 5 YEAR	0.82 (0.75. 0.92)	1.2262 (23.46)	---
INTEL			
DAILY. 1 YEAR	1.02 (1.00. 1.05)	1.7379 (153.35)	---
DAILY. 3 YEAR	1.08 (1.05. 1.11)	1.7014 (395.14)	---
DAILY. 5 YEAR	1.13 (1.10. 1.16)	1.6635 (549.21)	---
WEEKLY. 1 YEAR	0.94 (0.89. 1.00)	1.4212 (25.26)	---
WEEKLY. 3 YEAR	0.98 (0.94. 1.04)	1.4904 (69.62)	---
WEEKLY. 5 YEAR	1.02 (0.96. 1.08)	1.4810 (98.42)	---
MONTHLY. 1 YEAR	0.78 (0.67. 0.93)	1.9179 (7.21)	---
MONTHLY. 3 YEAR	0.76 (0.67. 0.88)	2.2423 (26.87)	-0.0077 (-3.90)
MONTHLY. 5 YEAR	0.80 (0.71. 0.93)	2.1440 (35.01)	---
JOHNSON AND JOHNSON			
DAILY. 1 YEAR	1.03 (1.01. 1.05)	0.1824 (23.79)	---
DAILY. 3 YEAR	1.05 (1.03. 1.07)	0.5251 (172.07)	---
DAILY. 5 YEAR	1.06 (1.04. 1.08)	0.5316 (234.23)	---
WEEKLY. 1 YEAR	1.05 (1.00. 1.11)	0.3276 (10.56)	---
WEEKLY. 3 YEAR	1.21 (1.14. 1.28)	0.5870 (51.09)	---
WEEKLY. 5 YEAR	1.23 (1.16. 1.32)	0.5590 (67.11)	---
MONTHLY. 1 YEAR	0.81 (0.71. 0.94)	0.3028 (1.97)	---
MONTHLY. 3 YEAR	0.88 (0.79. 0.99)	0.2723 (5.00)	---
MONTHLY. 5 YEAR	0.80 (0.72. 0.91)	0.2694 (8.44)	0.0025 (2.57)

(cont.)

JP MORGAN			
DAILY. 1 YEAR	1.11 (1.09. 1.13)	1.7388 (133.52)	---
DAILY. 3 YEAR	1.12 (1.10. 1.14)	1.5851 (239.56)	---
DAILY. 5 YEAR	1.12 (1.10. 1.14)	1.5182 (287.83)	---
WEEKLY. 1 YEAR	1.08 (1.03. 1.15)	1.6091 (26.63)	---
WEEKLY. 3 YEAR	1.13 (1.08. 1.20)	1.4782 (25.16)	---
WEEKLY. 5 YEAR	1.15 (1.08. 1.22)	1.4901 (53.59)	---
MONTHLY. 1 YEAR	0.92 (0.81. 1.05)	1.7416 (7.13)	---
MONTHLY. 3 YEAR	0.99 (0.90. 1.10)	1.8103 (20.28)	---
MONTHLY. 5 YEAR	0.90 (0.83. 1.00)	1.4200 (64.74)	---
MICROSOFT			
DAILY. 1 YEAR	1.02 (1.00. 1.04)	1.3670 (152.43)	---
DAILY. 3 YEAR	1.01 (0.99. 1.04)	1.2866 (356-95)	---
DAILY. 5 YEAR	1.02 (1.00. 1.04)	1.2453 (478.18)	---
WEEKLY. 1 YEAR	0.98 (0.94. 1.03)	0.7315 (16.31)	---
WEEKLY. 3 YEAR	0.98 (0.94. 1.04)	0.9853 (62.74)	---
WEEKLY. 5 YEAR	1.05 (1.00. 1.10)	0.9787 (91.63)	---
MONTHLY. 1 YEAR	1.03 (0.92. 1.16)	1.6761 (6.77)	---
MONTHLY. 3 YEAR	1.03 (0.94. 1.14)	1.2954 (16.51)	---
MONTHLY. 5 YEAR	0.93 (0.82. 1.07)	1.1091 (22.95)	---
VERIZON			
DAILY. 1 YEAR	1.06 (1.03. 1.09)	0.6732 (83.90)	---
DAILY. 3 YEAR	1.02 (1.00. 1.04)	0.8549 (268.43)	---
DAILY. 5 YEAR	0.99 (0.97. 1.01)	0.8368 (356.58)	-0.00008 (-2.38)
WEEKLY. 1 YEAR	1.04 (0.99. 1.09)	0.3860 (9.79)	---
WEEKLY. 3 YEAR	1.04 (0.99. 1.09)	0.4852 (33.43)	---
WEEKLY. 5 YEAR	1.05 (1.01. 1.10)	0.5321 (51.19)	---
MONTHLY. 1 YEAR	0.90 (0.81. 1.01)	---	---
MONTHLY. 3 YEAR	0.96 (0.87. 1.07)	0.8531 (10.84)	---
MONTHLY. 5 YEAR	0.93 (0.85. 1.04)	0.8421 (14.60)	---
WALMART			
DAILY. 1 YEAR	1.02 (0.99. 1.05)	1.0386 (116.79)	---
DAILY. 3 YEAR	1.02 (1.00. 1.05)	0.9207 (268.68)	---
DAILY. 5 YEAR	1.03 (1.01. 1.06)	0.9026 (370.02)	-0.00019 (-1.98)
WEEKLY. 1 YEAR	1.01 (0.95. 1.06)	0.7702 (19.72)	---
WEEKLY. 3 YEAR	1.07 (1.02. 1.13)	0.8185 (54.33)	---
WEEKLY. 5 YEAR	1.12 (1.07. 1.19)	0.7927 (78.57)	---
MONTHLY. 1 YEAR	0.99 (0.89. 1.12)	---	---
MONTHLY. 3 YEAR	0.92 (0.83. 1.04)	0.4108 (5.39)	---
MONTHLY. 5 YEAR	0.96 (0.87. 1.07)	0.3719 (7.98)	---

(cont.)

UNITED HEALTH			
DAILY. 1 YEAR	1.05 (1.03. 1.07)	0.5975 (46.12)	---
DAILY. 3 YEAR	1.08 (1.05. 1.10)	0.4647 (67.82)	---
DAILY. 5 YEAR	1.09 (1.07. 1.12)	0.5028 (92.59)	---
WEEKLY. 1 YEAR	1.02 (0.97. 1.07)	0.8156 (15.19)	---
WEEKLY. 3 YEAR	1.12 (1.07. 1.18)	0.4929 (21.42)	---
WEEKLY. 5 YEAR	1.13 (1.08. 1.20)	0.5399 (311.51)	---
MONTHLY. 1 YEAR	0.71 (0.62. 0.84)	---	---
MONTHLY. 3 YEAR	0.87 (0.78. 0.98)	0.1953 (2.0.8)	---
MONTHLY. 5 YEAR	0.94 (0.87. 1.02)	0.2112 (3.53)	---
PROCTER			
DAILY. 1 YEAR	1.04 (1.02. 1.06)	0.2120 (28.05)	---
DAILY. 3 YEAR	1.05 (1.03. 1.08)	0.4088 (140.18)	---
DAILY. 5 YEAR	1.05 (1.02. 1.07)	0.4494 (216.88)	---
WEEKLY. 1 YEAR	0.99 (0.94. 1.05)	0.4353 (12.41)	---
WEEKLY. 3 YEAR	0.94 (0.89. 0.99)	0.5084 (39.93)	---
WEEKLY. 5 YEAR	0.96 (0.90. 1.02)	0.4800 (50.68)	---
MONTHLY. 1 YEAR	0.81 (0.73. 0.90)	---	---
MONTHLY. 3 YEAR	1.02 (0.94. 1.13)	0.1168 (1.99)	---
MONTHLY. 5 YEAR	0.99 (0.91. 1.10)	0.1893 (5.68)	---

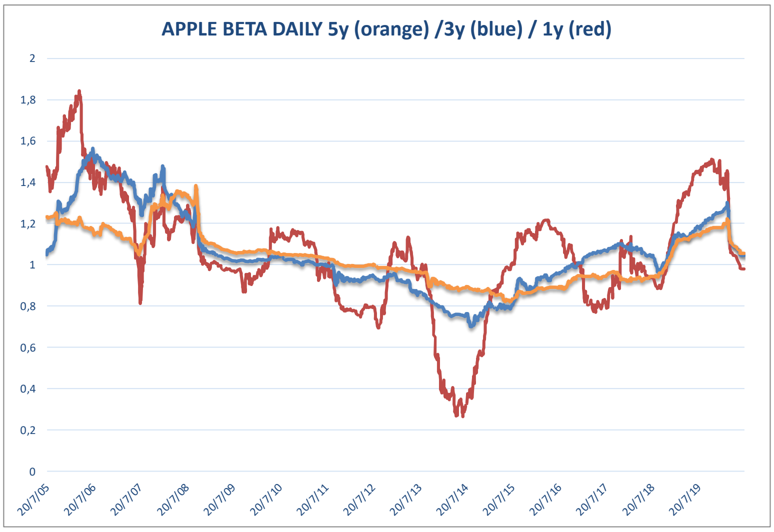
The values in parenthesis in the second column correspond to the 95% confidence bands. Those in columns 3 and 4 are t-values. Red values indicate statistical evidence of mean reversion.

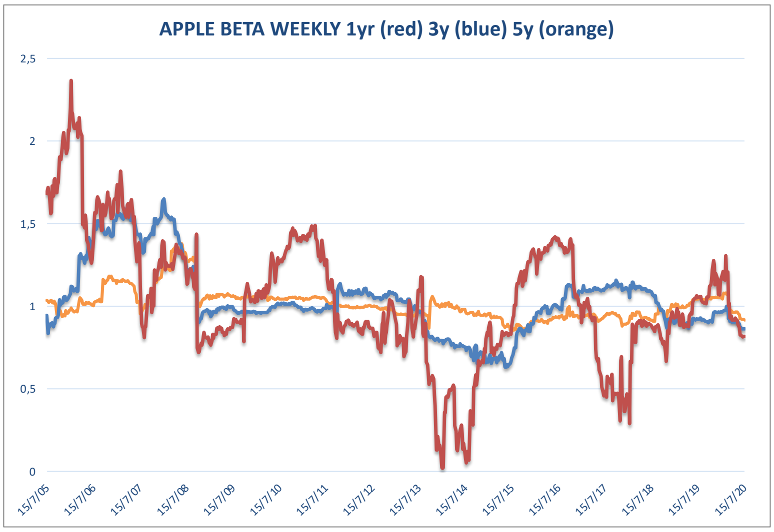
Table 5: Estimates of the d parameter with the corresponding volatility coefficient for each beta series

	Monthly			Weekly			Daily			
APPLE	5Y	3Y	1Y	5Y	3Y	1Y	5Y	3Y	1Y	Average
d	0.910	0.900	0.840	0.880	0.900	0.960	1.070	1.060	1.030	0.950
Beta Stdev/average	0.259	0.380	0.683	0.092	0.200	0.360	0.123	0.172	0.264	0.282
MICROSOFT										
d	0.930	1.030	1.030	1.050	0.980	0.980	1.020	1.010	1.020	1.006
Beta Stdev/average	0.109	0.267	0.655	0.097	0.125	0.227	0.077	0.107	0.153	0.202
HOMEDEPOT										
d	0.820	0.850	0.750	0.970	0.960	0.930	1.030	1.020	1.040	0.930
Beta Stdev/average	0.174	0.194	0.401	0.102	0.109	0.160	0.093	0.104	0.159	0.166
INTEL										
d	0.800	0.730	0.780	1.020	0.980	0.940	1.130	1.080	1.020	0.942
Beta Stdev/average	0.314	0.351	0.499	0.134	0.162	0.223	0.158	0.168	0.202	0.246
JOHNSON & JOHNSON										
d	0.790	0.880	0.810	1.230	1.210	1.050	1.060	1.050	1.030	1.012
Beta Stdev/average	0.252	0.378	0.581	0.150	0.191	0.292	0.129	0.149	0.260	0.265
JPMORGAN										
d	0.900	0.990	0.920	1.150	1.130	1.080	1.120	1.120	1.110	1.058
Beta Stdev/average	0.147	0.251	0.395	0.147	0.179	0.255	0.138	0.171	0.221	0.212
PROCTER AND GAMBLE										
d	0.990	1.020	0.800	0.960	0.940	0.990	1.050	1.050	1.040	0.982
Beta Stdev/average	0.302	0.513	0.781	0.097	0.141	0.298	0.084	0.139	0.222	0.286
UNITED HEALTH										
d	0.940	0.870	0.710	1.130	1.120	1.020	1.090	1.080	1.030	0.999
Beta Stdev/average	0.437	0.534	0.709	0.301	0.360	0.471	0.211	0.232	0.284	0.393
VERIZON										
d	0.930	0.960	0.900	1.050	1.040	1.040	0.990	1.020	1.060	0.999
Beta Stdev/average	0.411	0.474	0.710	0.158	0.206	0.333	0.156	0.185	0.246	0.320
WALMART										
d	0.960	0.920	0.990	1.120	1.070	1.010	1.030	1.020	1.020	1.016
Beta Stdev/average	0.332	0.519	1.225	0.179	0.209	0.290	0.183	0.200	0.258	0.377
d analysis										
MAX	0.990	1.030	1.030	1.230	1.210	1.080	1.130	1.120	1.110	
MIN	0.790	0.730	0.710	0.880	0.900	0.930	0.990	1.010	1.020	
Average	**0.897**	**0.915**	**0.853**	**1.056**	**1.033**	**1.000**	**1.059**	**1.051**	**1.040**	0.989
Beta Stdev/average										
MAX	0.437	0.534	1.225	0.301	0.360	0.471	0.211	0.232	0.284	
MIN	0.109	0.194	0.395	0.092	0.109	0.160	0.077	0.104	0.153	
Average	0.274	0.386	0.664	0.146	0.188	0.291	0.135	0.163	0.227	0.275

FIGURE 1. **Calculated Beta time series**

FIGURE 1a. APPLE.





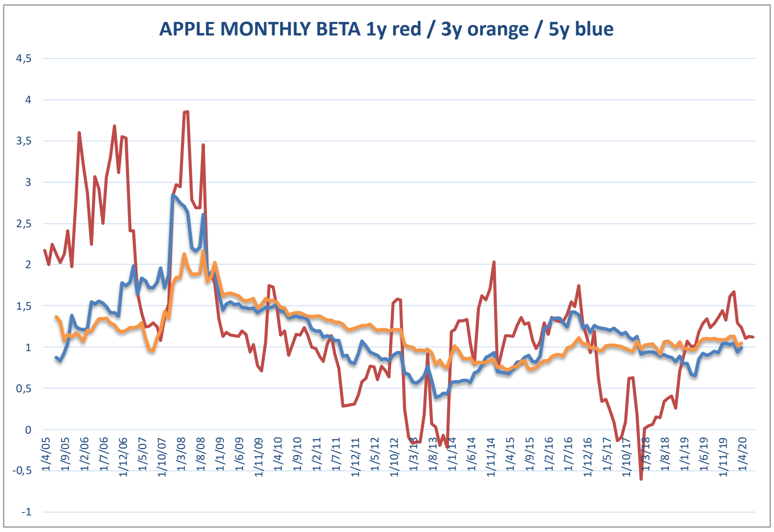
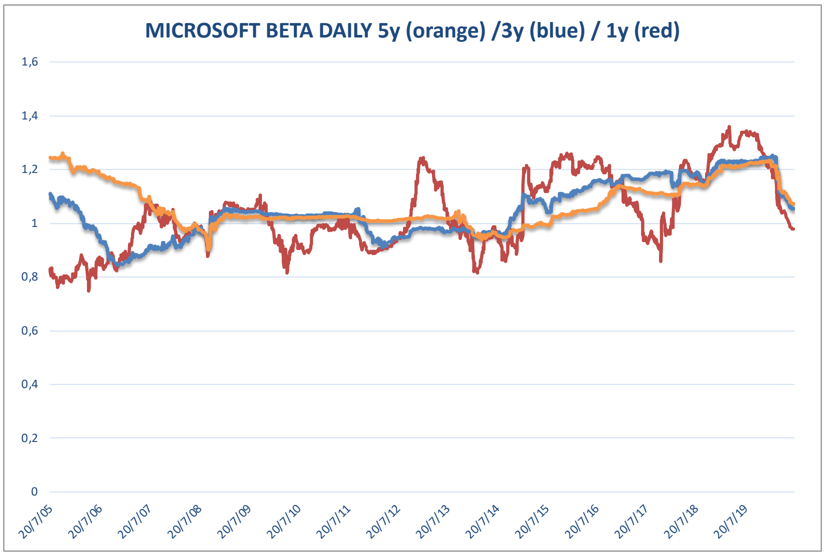
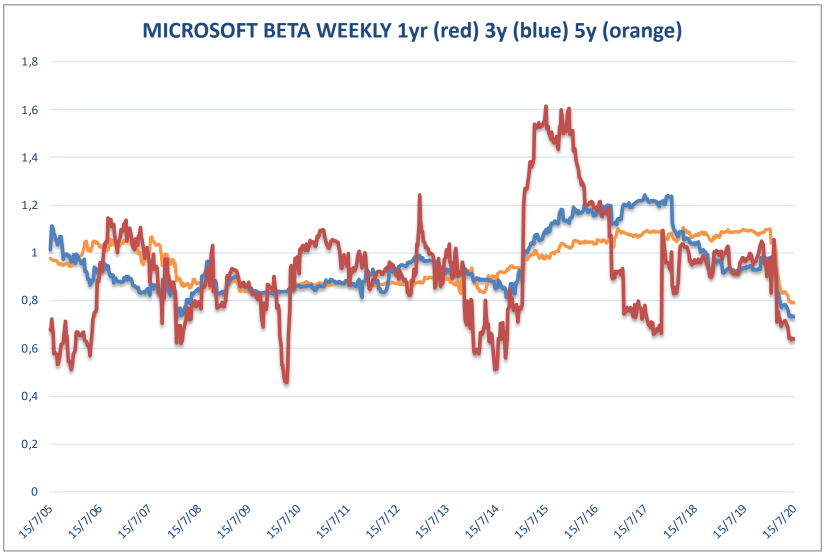


FIGURE 1b. MICROSOFT





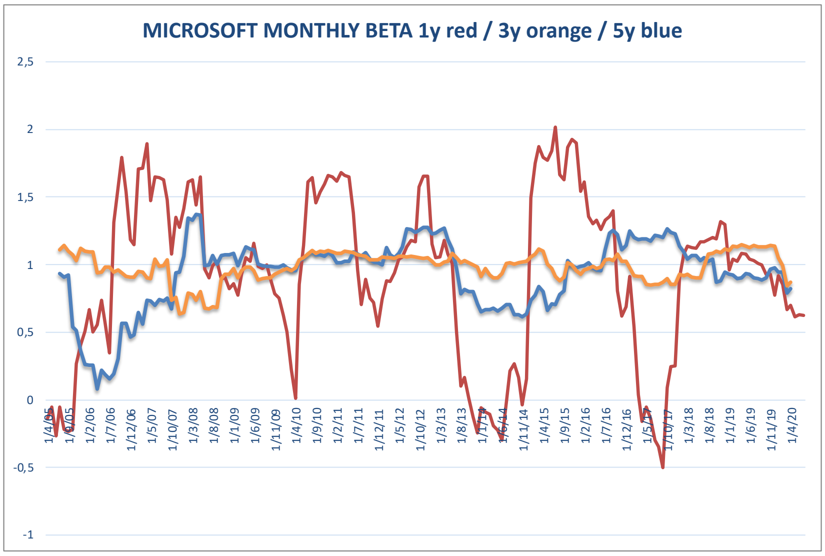
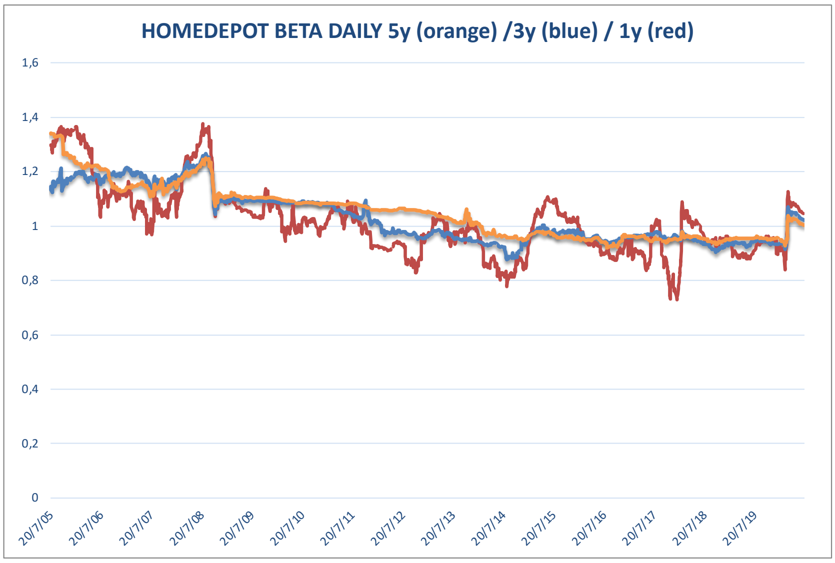
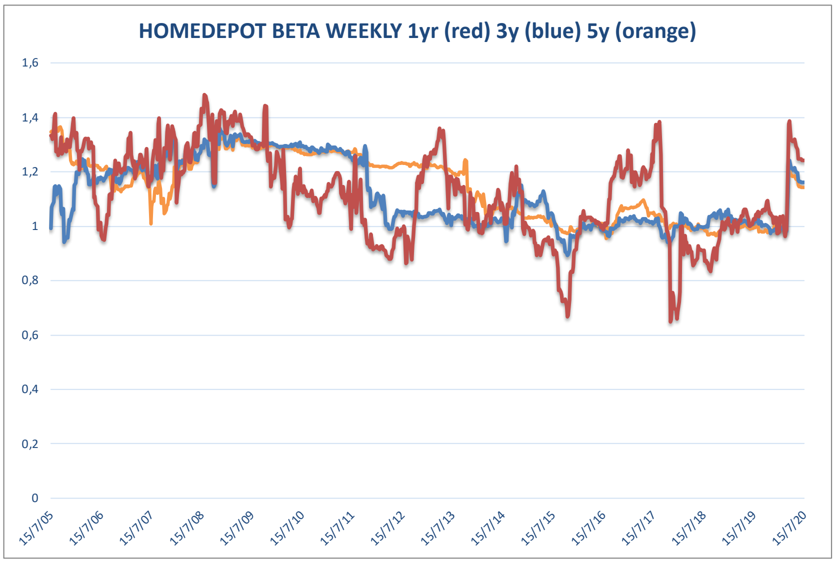


FIGURE 1c. HOMEDEPOT





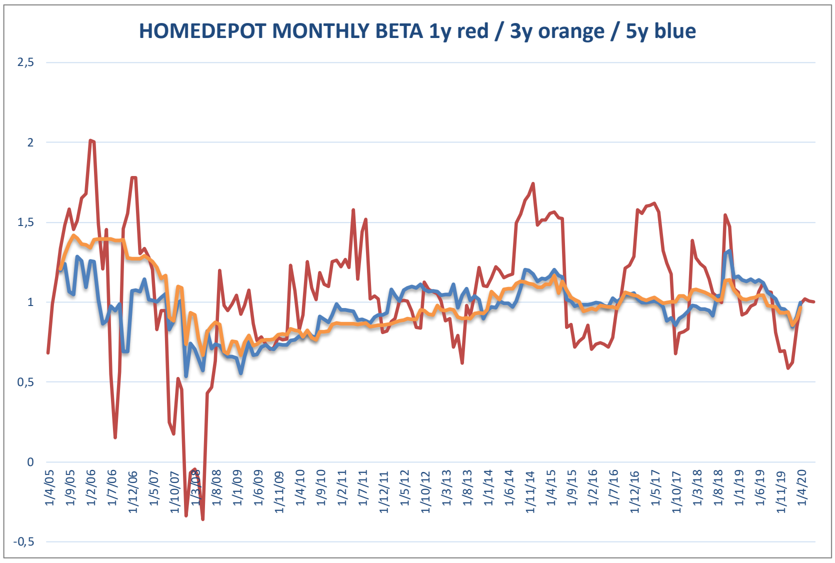
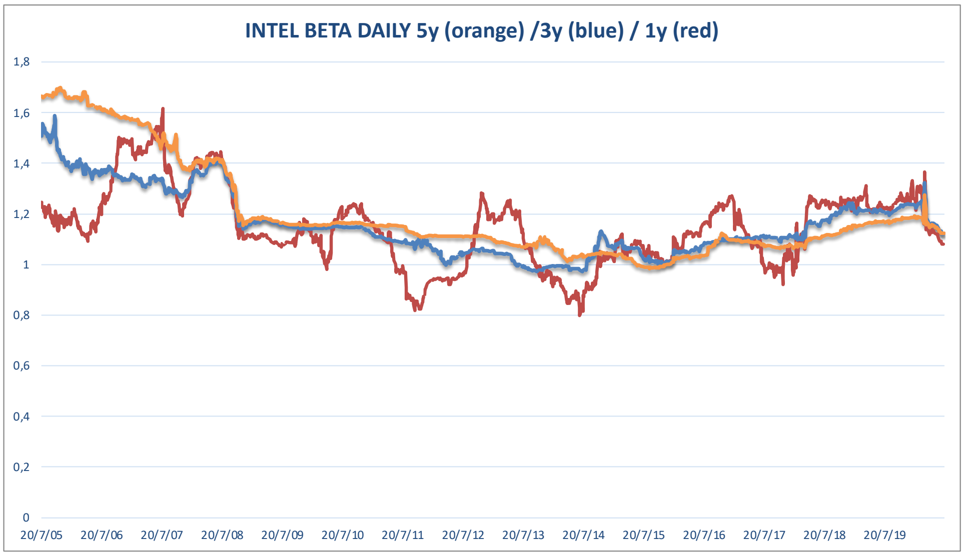
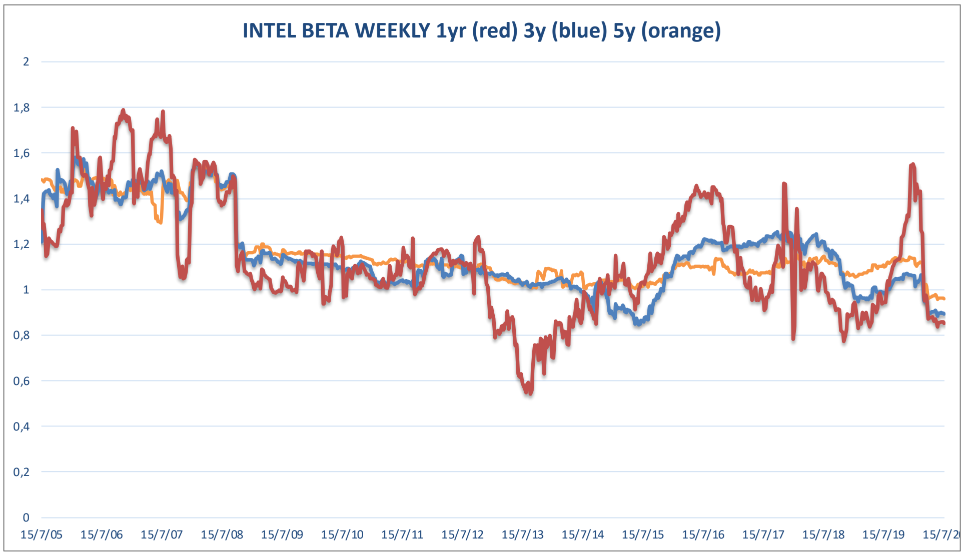


FIGURE 1d. INTEL





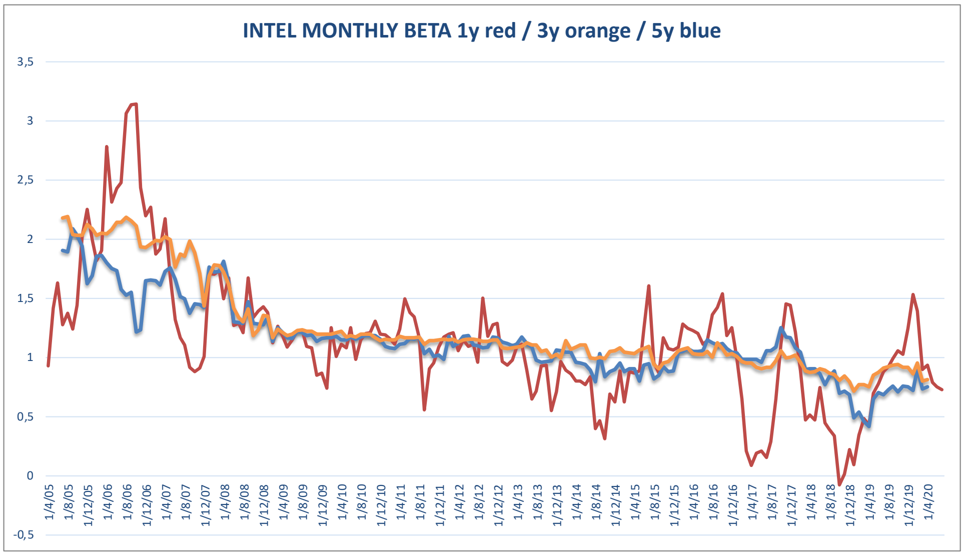
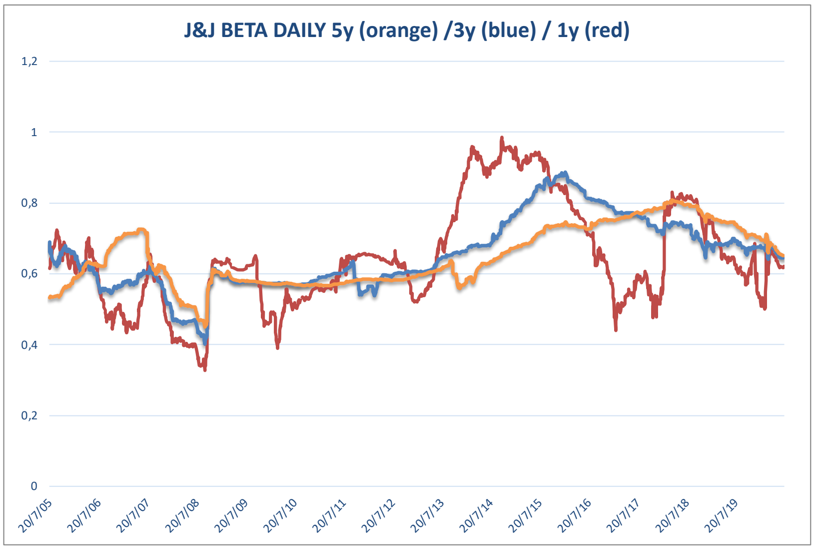
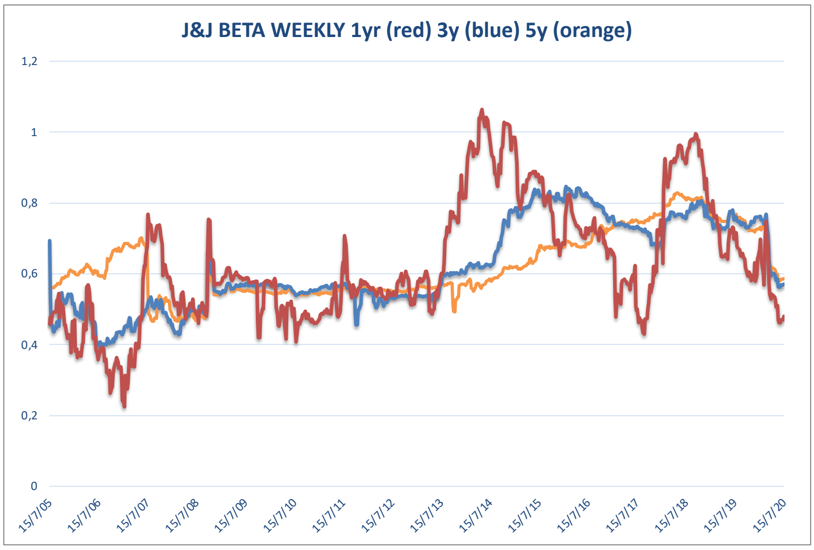


FIGURE 1e. JOHNSON AND JOHNSON





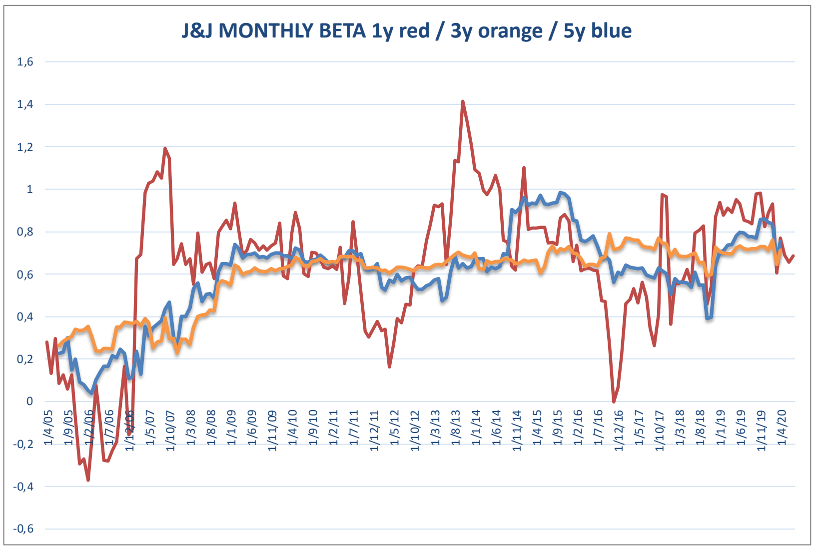
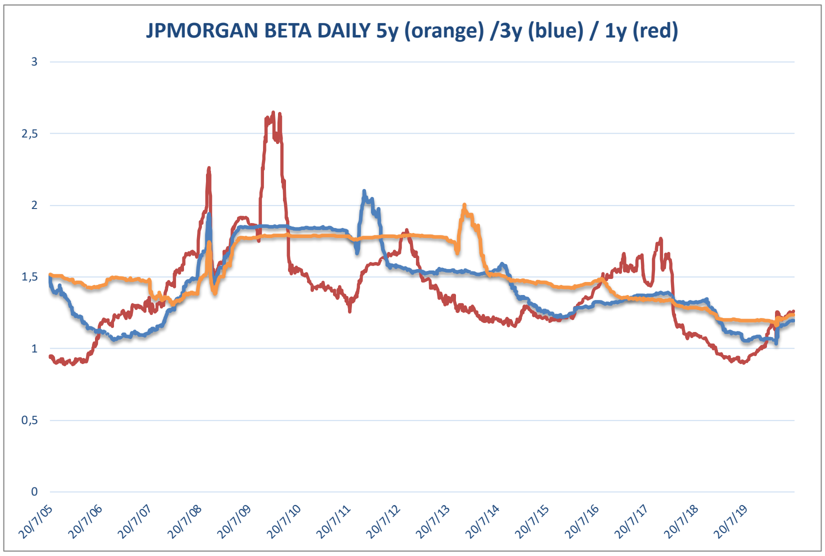
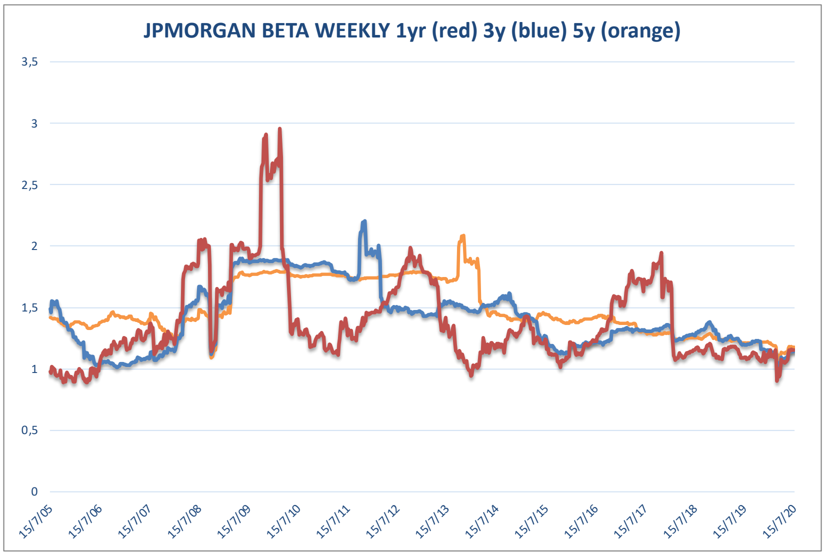


FIGURE 1f. JPMORGAN





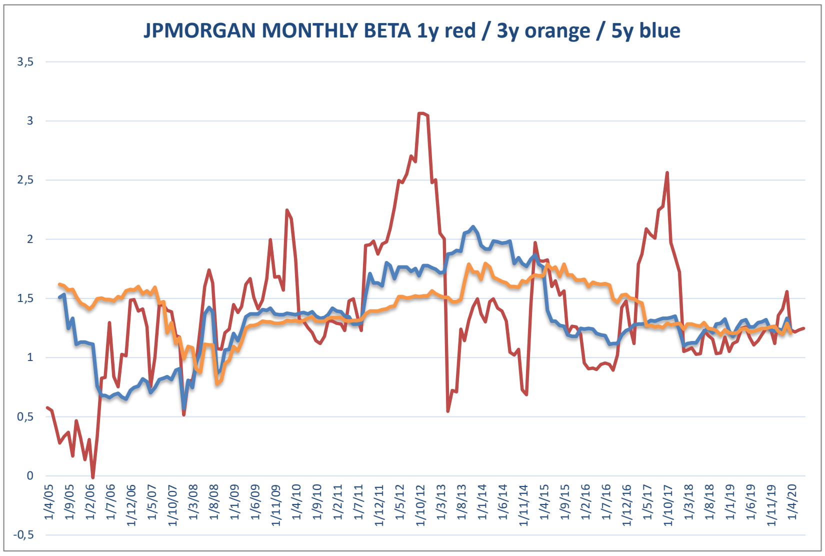
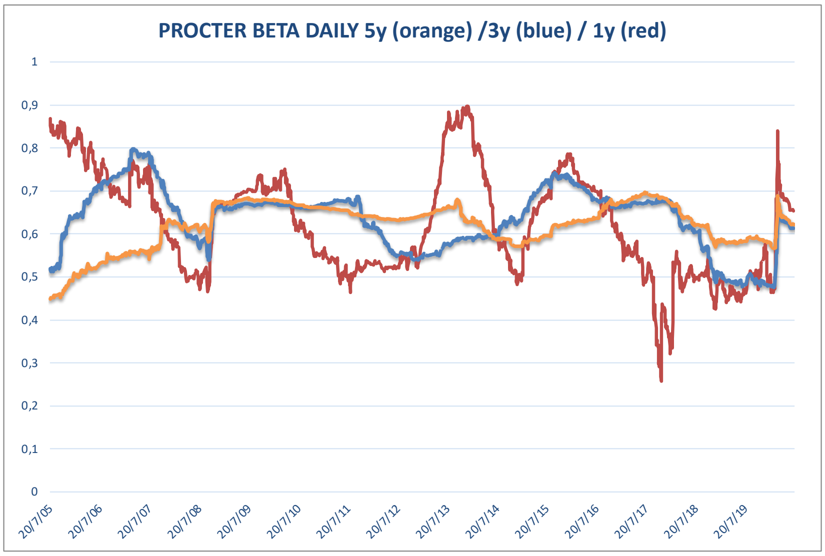
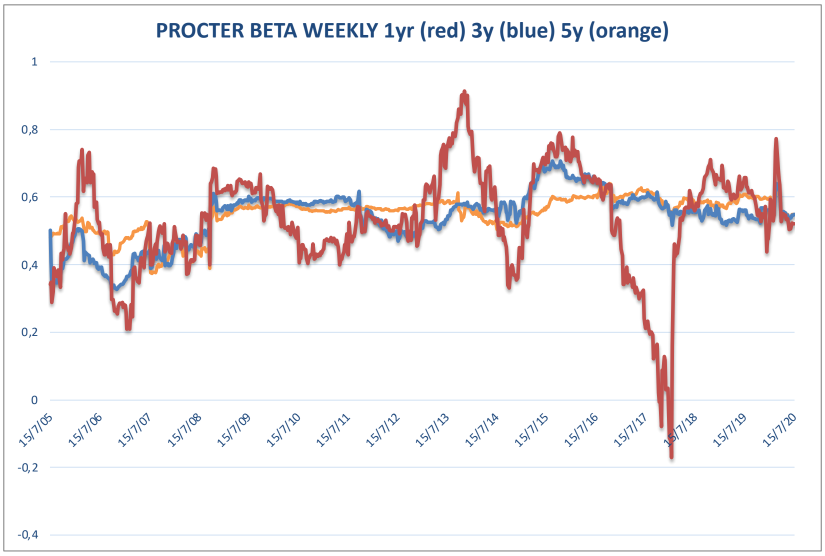


FIGURE 1g. PROCTER





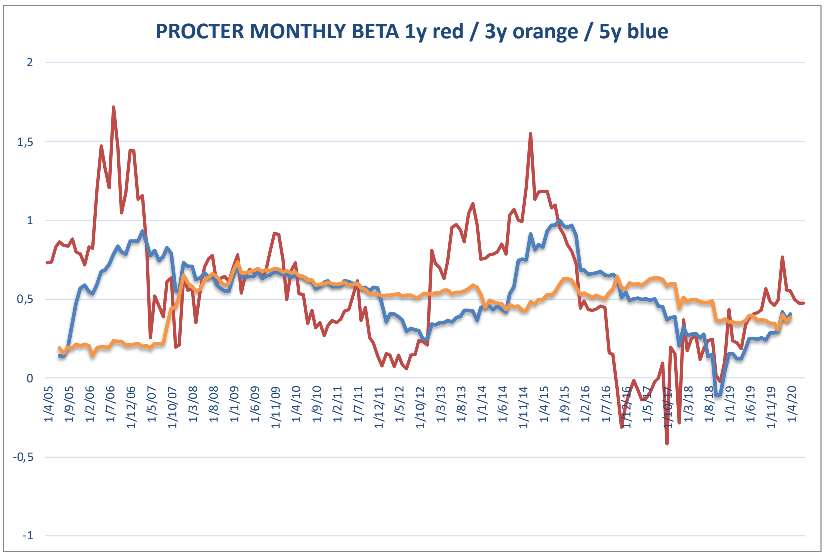
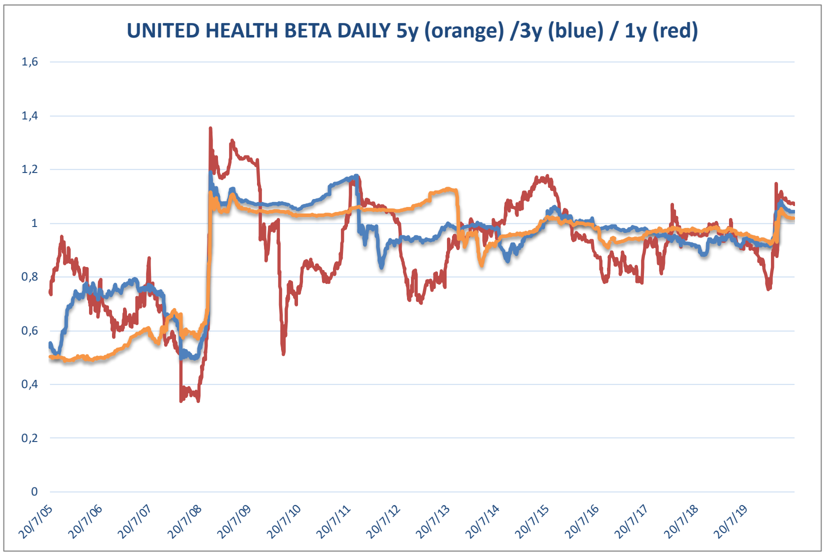
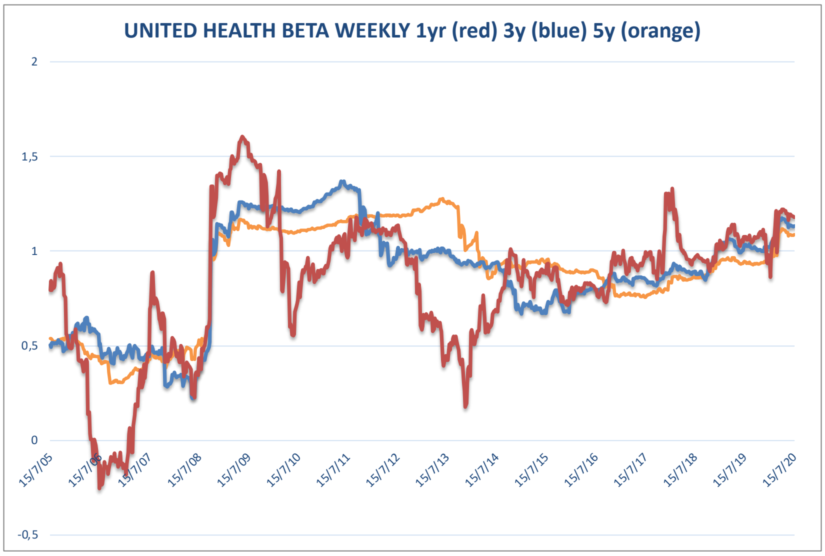


FIGURE 1h. UNITED HEALTH





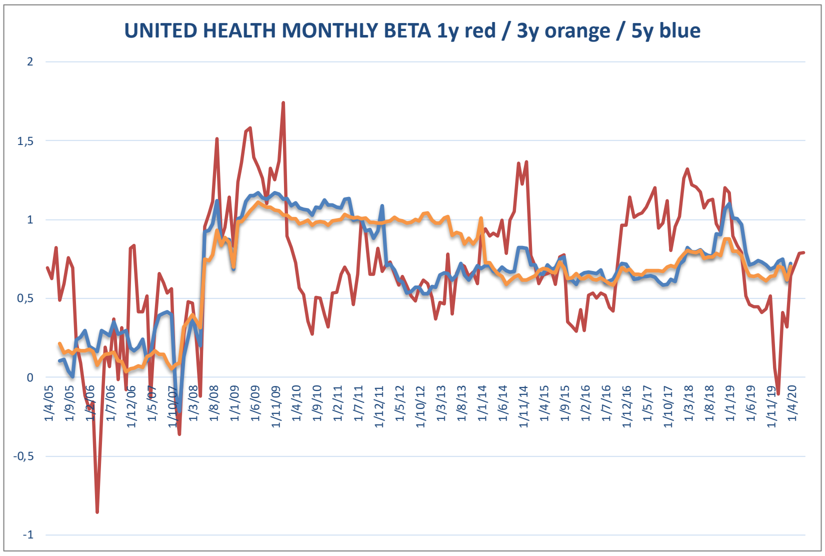
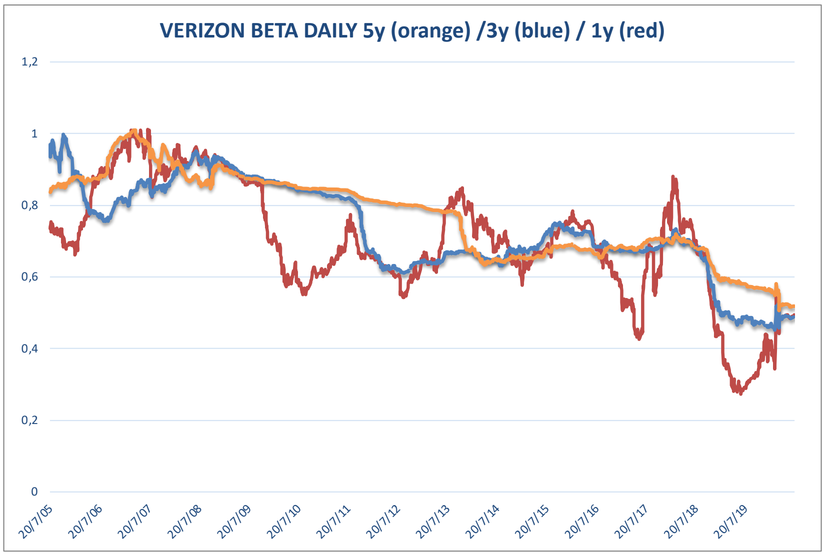
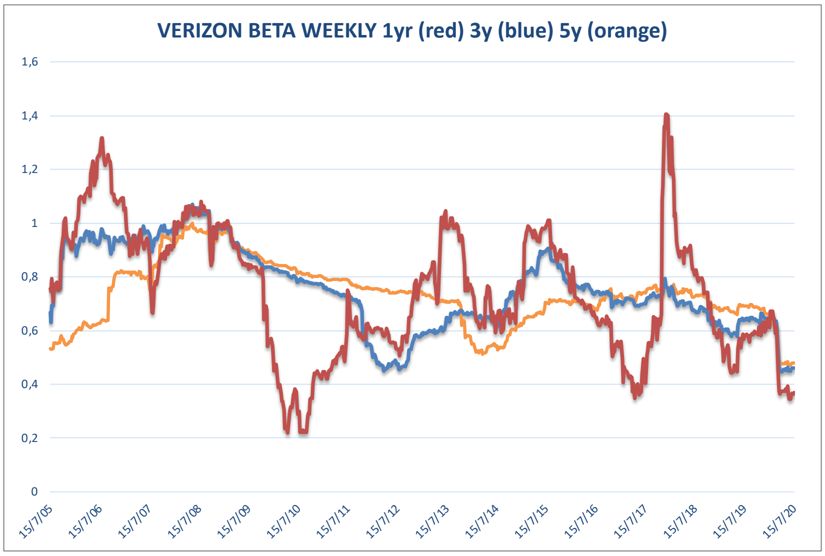


FIGURE 1i. VERIZON





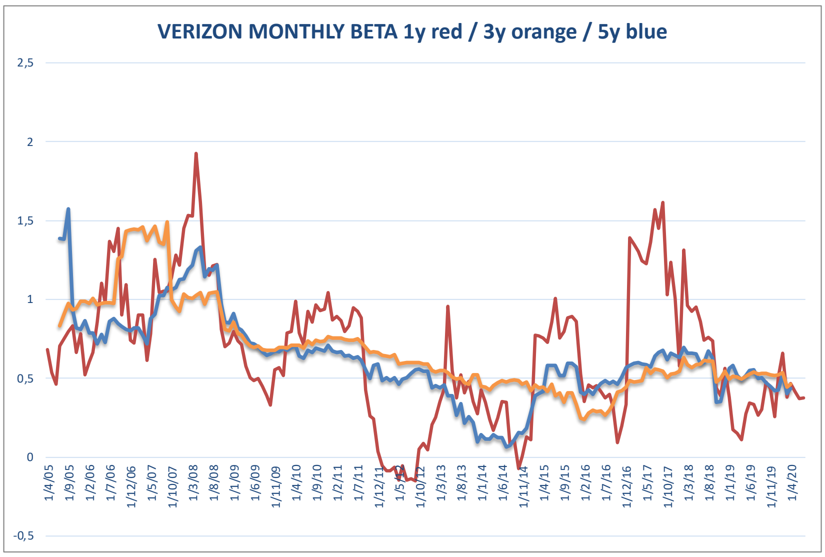
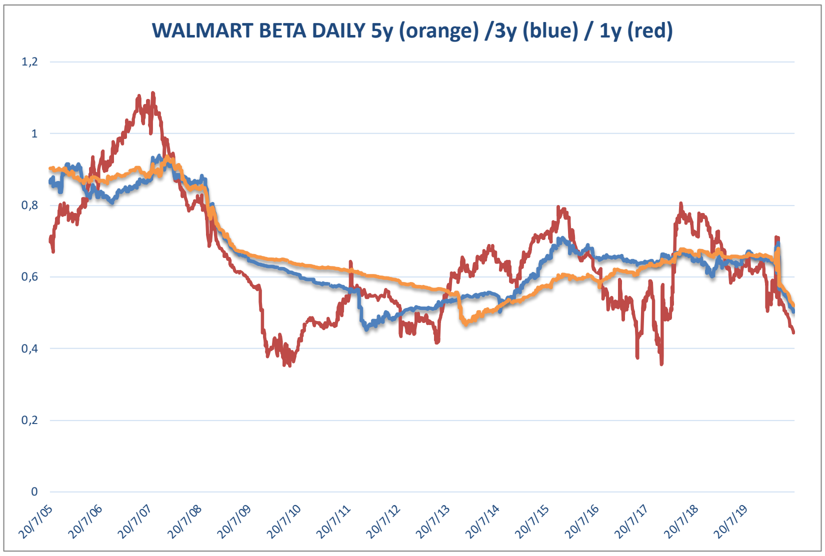
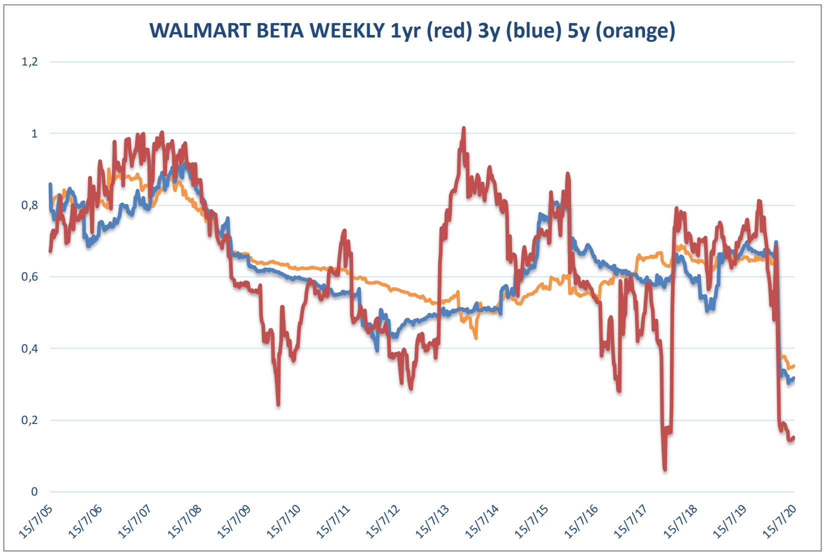


FIGURE 1j. WALMART





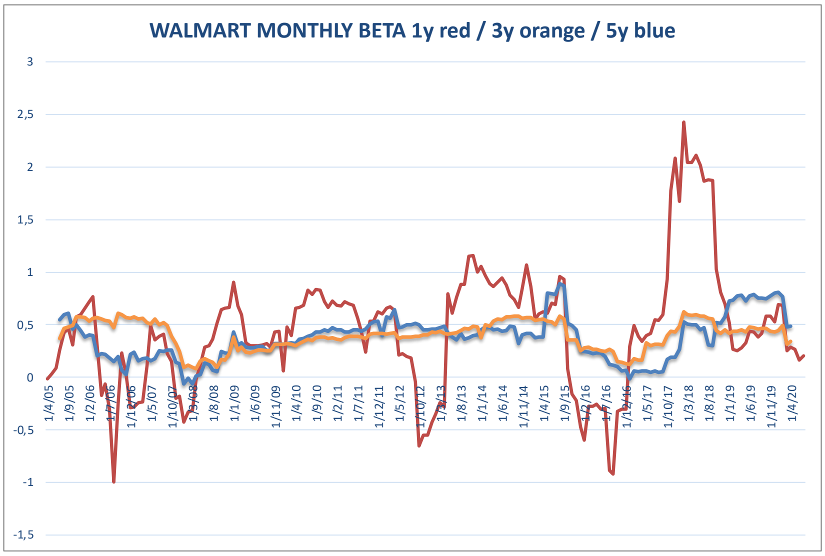


FIGURE 2. Scatter diagram showing the relationship between the average fractional integration parameter d (x axis) and volatility coefficient calculated as stdev/average (y axis) for each frequency/sample length

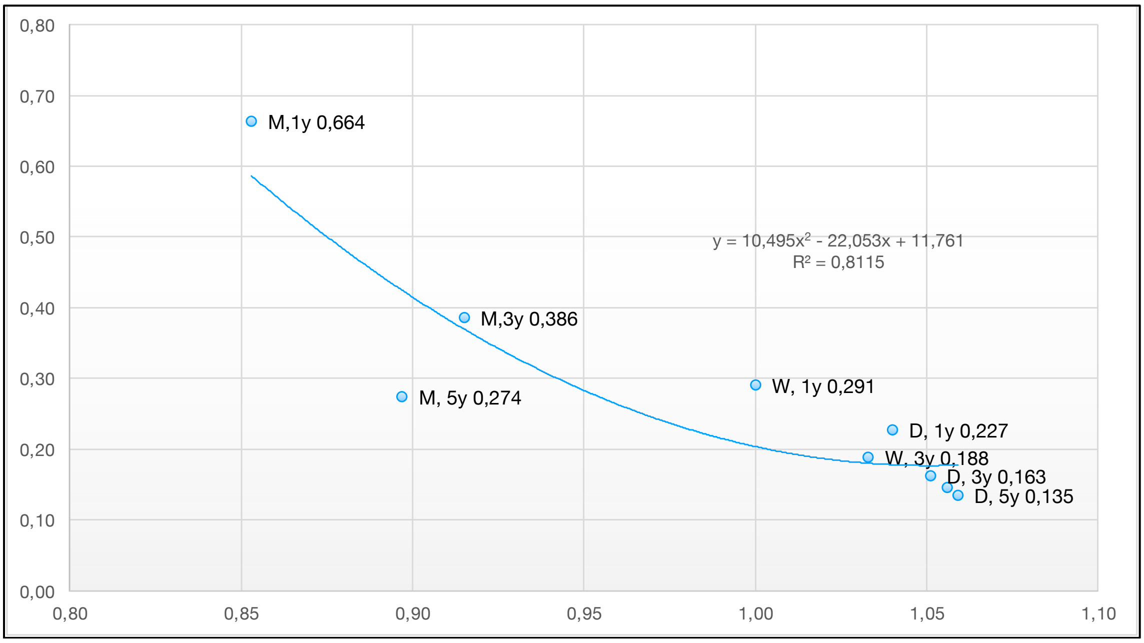


FIGURE 3. Scatter diagram showing the relationship between the fractional integration parameter d (x axis) and the volatility coefficient calculated as stdev/average (y axis) for each frequency/sample length and each stock