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Long-Run Linkages Between US Stock Prices and Cryptocurrencies: A Fractional Cointegration Analysis

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LONG-RUN LINKAGES

BETWEEN US STOCK PRICES AND CRYPTOCURRENCIES:

A FRACTIONAL COINTEGRATION ANALYSIS

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Abstract

This paper applies fractional integration and cointegration methods to examine respectively the univariate properties of the four main cryptocurrencies in terms of market capitalization (BTC, ETH, USDT, BNB) and of four US stock market indices (S&P500, NASDAQ, Dow Jones and MSCI for emerging markets) as well as the possible existence of long-run linkages between them. Daily data from 9 November 2017 to 28 June 2002 are used for the analysis. The results provide evidence of market efficiency in the case of the cryptocurrencies but not of the stock market indices considered. They also indicate that in most cases there are no long-run equilibrium relationships linking the assets in question, which implies that cryptocurrencies can be a useful tool for investors to diversify and hedge when required in the case of the US markets.

Keywords: Stock market prices; cryptocurrencies; persistence; fractional integration and cointegration

JEL Classification: C22; C58; G11; G15

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1. Introduction

Since the creation of Bitcoin (Nakamoto, 2008) cryptocurrencies have rapidly become a global phenomenon and generated new investment opportunities with important implications for portfolio diversification and hedging decisions. In the last decade numerous papers have analysed them from various perspectives, both theoretical and empirical. Examples include studies using applying extreme value theory (Gkillas and Katsiampla, 2018), modelling their volatility linkages with other markets (Carrick, 2016), examining their predictive power (Watorek et al., 2020), their degree of persistence (Caporale et al., 2018) and other characteristics such as sustainability (Giudici et al., 2019), and carrying out tests of the efficient market hypothesis (Gil-Alana et al., 2020), etc.

Another strand of the literature focuses on whether or not cryptocurrencies are linked to other types of assets, which has implications for whether or not they are suitable for diversification and hedging purposes. For instance, Corbet et al. (2018) followed the connectedness approach of Diebold and Yilmaz (2012) and found only short-run spillovers between cryptocurrencies and more traditional assets. Kurka (2019) used the same framework as well as the Spillover Asymmetry Measure (SAM) of Barunik (2016) and found evidence of asymmetries in the transmission of shocks between Bitcoin and other assets; moreover, there are spillovers over some of the sub-samples, and hence diversification/hedging strategies can only work at times. Stensas et al. (2019) estimated GARCH models and concluded that Bitcoin is useful for diversification purposes in the case of developed (but not developing) countries. Further (though weaker) evidence consistent with these findings was provided by Klein et al. (2018). Caferra and Tomas-Vidal (2021) used instead a wavelet coherence approach and also estimated a Markov

Switching autoregressive model; their results are more supportive of a possible hedging role for cryptocurrencies.

The present study examines linkages between the four main cryptocurrencies in terms of market capitalisation (Bitcoin, Ethereum, Tether and Binance Coin, for which the corresponding figures as of July 2022 are $439.39bn, $196.77bn, $65.90bn and $46.53bn respectively) and four US stock market indices (S&P500, Nasdaq, Dow Jones and MSCI emerging markets) using a fractional integration/cointegration approach which is more general and flexible than the standard framework based on the I(0) versus I(1) (stationary versus non-stationary) dichotomy used in most previous studies since it allows for fractional values of the differencing (cointegration) parameter and thus it encompasses a much wider range of stochastic processes. The data are daily and cover the period from 9 November 2017 to 28 June 2022. The empirical results provide useful information to investors for portfolio choices and diversification/hedging strategies (Urquhart, 2016). The paper is organised as follows: Section 2 describes the data; Section 3 presents the empirical analysis; Section offers some concluding remarks.

2. Data

We analyse daily data from Yahoo Finance on four cryptocurrencies (Bitcoin- BTC; Ethereum – ETH; Tether- USDT; and Binance Coin - BNB) and four US stock market indices (S&P500, NASDAQ, Dow Jones and MSCI for emerging markets) covering the period from 9 November 2017 (since some cryptocurrencies were not being traded before then) to 28 June 2022, for a total of 1,165 trading days. Note that data for holidays or weekends are available for cryptocurrencies but not for stock market indices, therefore when analysing the relationships between the former and the latter series only weekdays are considered in order to match them. Table 1 reports some descriptive statistics for all series, whilst Figure 1 displays their correlation coefficients. It can be seen that the Dow Jones has the highest value and MSCI the lowest one whilst the Nasdaq has the highest mean and standard deviation in the case of the stock market indices; as for the cryptocurrencies, USDT has the highest mean and standard deviation and BNB the lowest ones. Concerning the correlations, they are generally high between the stock market indices but not between them and the four cryptocurrencies considered; as for the latter, there appear to be strong linkages only BTC, ETH and BNB.

INSERT FIGURE 1 AND TABLE 1 ABOUT HERE

3. Empirical results

3a Univariate analysis

As a first step, we carry out univariate analysis using fractional integration methods. The estimated model is the following:

$$y_t = \alpha + \beta t + x_t, \quad (1-B)^d x_t = u_t, \quad t = 1.2, ... \qquad (1)$$

where y_t stands for the series of interest (the log of stock market indices and cryptocurrencies respectively); α and β are unknown parameters to be estimated, specifically a constant and a (linear) time trend, x_t is assumed to be I(d) (where d is a real value estimated from data), B is the backshift operator, i.e, $Bx_t = x_{t-1}$, and u_t is I(0) by assumption. Note that the model above can be re-written as:

$$\tilde{y}_t = \alpha \tilde{1}_t + \beta \tilde{t}_t + u_t, \quad t = 1.2, ... \qquad (2)$$

where

$$\tilde{y}_t = (1-B)^d y_t; \quad \tilde{1}_t = (1-B)^d 1; \quad \tilde{t}_t = (1-B)^d t,$$

and u_t is I(0) by assumption, which implies that standard t-tests remain valid. Following (Robinson, 1994) the estimation is carried out using a Whittle function in the frequency domain as in many other long-memory studies, and the series are logged to smooth them.

Tables 2 - 5 display the estimates of d along with the 95% confidence bands for the differencing parameter for three different specifications, namely i) without deterministic terms, i.e. setting $\alpha = \beta = 0$ in (1); ii) with a constant only, i.e. setting $\beta = 0$ in (1); and iii) with a constant and a linear time trend. The coefficients in bold are those from the model selected in each case on the basis of the statistical significance of the deterministic terms. Table 2 reports the estimates of d when assuming that u_t in (1) is a white noise process, whilst Table 4 presents those for the case of autocorrelated disturbances using the non-parametric approach of Bloomfield (1973) rather than a classical ARMA structure. Tables 3 and 5 display instead the estimated coefficients of the selected models.

Under the assumption of white noise residuals (see Tables 2 and 3) both the constant and the time trend are found to be significant in the case of Bitcoin, S&P500 and Nasdaq. In all other cases only the constant is significant. Concerning the estimates of d, in the case of stock market indices they are quite large and close to 1; however, the confidence intervals are quite wide, such that all values are strictly below 1 and the I(1) hypothesis is rejected in favour of some degree of mean reversion ($d < 1$), which implies that shocks only have transitory effects. By contrast, the I(1) hypothesis (no mean reversion) cannot be rejected for any of the four cryptocurrencies, the lowest value of d (0.46) being estimated for USDT. This evidence suggests that the Efficient Market Hypothesis (EMH), which in its weak form requires prices to be random, holds for the cryptocurrencies but not for the stock market indices under examination.

When allowing instead for autocorrelated residuals (see Tables 4 and 5) the estimated values of d are slightly higher than in the previous case, and evidence of unit roots (or lack or mean reversion) is found for all four stock market indices and three out of the four cryptocurrencies, USDT being the only exception.

3b Bivariate analysis

Next we test for fractional cointegration between each series and all others on a pairwise basis, thus examining all 16 possible pairings. Specifically, we use the two-step method proposed by Engle and Granger (1987); this involves running regressions between each pair of series in the first step, and then in the second step estimating the value of the differencing parameter d as in Equation (1) for the residuals from those regressions. Note, that the confidence intervals for the purpose of statistical inference are obtained using Monte Carlo simulations since the residuals from the regression are estimated and not observed values, which produces a bias (see, e.g., Gil-Alana, 2003). The results are shown in Table 6 for the case of autocorrelated disturbances (similar results, not reported to save space, were obtained under the assumption of white noise errors). As can be seen, in most cases the unit root null hypothesis ($d = 1$) cannot be rejected, the only exceptions being the pairings of the Nasdaq and the S&P500 respectively with USDT – in other words, in most cases there is no evidence of a long-run equilibrium relationship linking the assets in question. Consequently, it would normally be possible for investors to use cryptocurrencies for diversification or hedging purposes in the case of the US markets.

4. Conclusions

This paper has applied fractional integration and cointegration methods to examine respectively the univariate properties of the four main cryptocurrencies in terms of market capitalization (BTC, ETH, USDT, BNB) and of four US stock market indices (S&P500, NASDAQ, Dow Jones and MSCI for emerging markets) as well as the possible existence of long-run linkages between them. Daily data from 9 November 2017 to 28 June 2002 are used for the analysis. The results provide evidence of market efficiency in the case of the cryptocurrencies but not of the stock market indices examined. They also indicate that in most cases there are no long-run equilibrium relationships linking the assets in question, which implies that cryptocurrencies can be a useful tool for investors to diversify and hedge when required in the case of the US markets.

Future work could carry out some robustness checks using other (semi-parametric) methods (Geweke & Porter-Hudak, 1984) (Shimotsu & Phillips, 2006) for the univariate analysis and the FCVAR approach of Johansen and Nielsen (2010, 2012) for the multivariate one, as well as allowing for nonlinearities in the long memory framework (Gil-Alana and Cuestas, 2016; Yaya et al., 2021).

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Figure 1: Correlations of Cryptocurrencies and Stock Market Indices

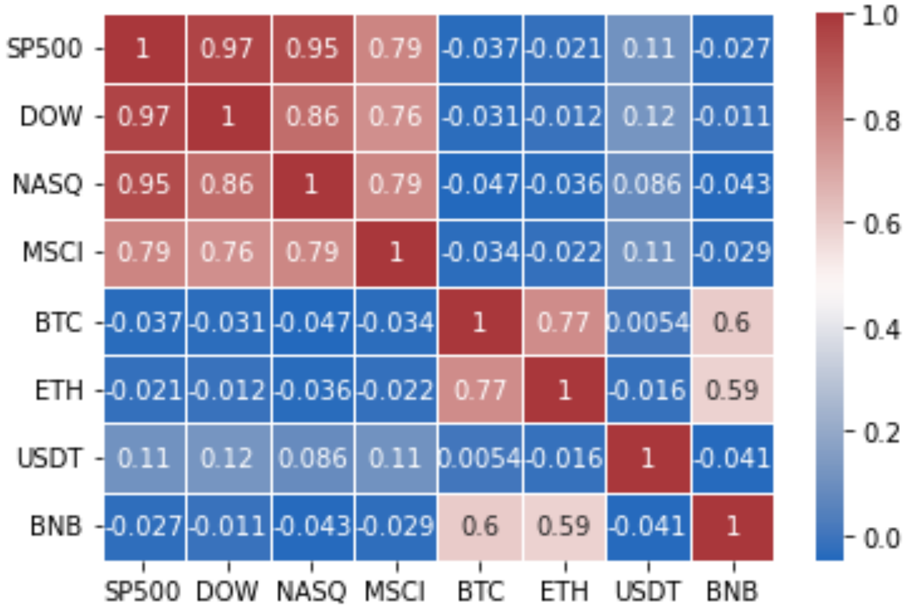

Table 1: Descriptive statistics

i) Stock Market Prices				
Series	Maximum	Minimum	Mean	Std. Dev.
S&P 500	9.383	-11.984	0.043	1.350
DOW JONES	11.365	-12.927	0.034	1.371
NASDAQ	9.346	-12.321	0.056	1.570
MSCI	8.053	-12.479	-0.001	1.473
ii) Cryptocurrencies				
Series	Maximum	Minimum	Mean	Std. Dev.
ETH	29.788	-42.347	0.295	6.045
BNB	102.483	-4.8987	0.001	0.516
USDT	5.824	-41.905	0.688	7.875
BTC	27.467	-3.717	0.209	4.860

Table 2: Estimates of d: White noise errors

Series	No deterministic terms	With an intercept	With an intercept and a time trend
i) Stock Market Prices			
S&P 500	0.99 (0.95, 1.04)	0.91 (0.87, 0.95)	**0.91 (0.87, 0.95)**
DOW JONES	1.00 (0.96, 1.04)	**0.91 (0.87, 0.95)**	0.91 (0.87, 0.95)
NASDAQ	0.99 (0.95, 1.04)	0.91 (0.88, 0.95)	**0.91 (0.88, 0.95)**
MSCI	0.99 (0.95, 1.04)	**0.91 (0.88, 0.95)**	0.91 (0.88, 0.95)
i) Cryptocurrencies			
ETH	1.00 (0.96, 1.04)	**1.04 (1.00, 1.08)**	1.04 (1.00, 1.08)
BNB	1.01 (0.97, 1.05)	1.02 (0.99, 1.06)	**1.02 (0.99, 1.06)**
USDT	0.46 (0.43, 0.51)	**0.46 (0.42, 0.50)**	0.46 (0.42, 0.51)
BTC	0.99 (0.95, 1.04)	**1.03 (0.99, 1.07)**	1.03 (0.99, 1.07)

The values are the estimates of the differencing parameter. Those in parenthesis are the 95% confidence bands for the estimates of d. In bold, the selected specification in relation with the deterministic terms.

Table 3: Estimated coefficients for the selected models in Table 2

Series	d	Intercept (t-value)	Time trend (t-value)
i) Stock Market Prices			
S&P 500	0.91 (0.87, 0.95)	7.856 (587.62)	0.00036 (1.68)
DOW JONES	0.91 (0.87, 0.95)	10.0623 (740.96)	----
NASDAQ	0.91 (0.88, 0.95)	8.81733 (566.87)	0.00048 (1.89)
MSCI	0.91 (0.88, 0.95)	3.83490 (262.46)	----
i) Cryptocurrencies			
ETH	1.04 (1.00, 1.08)	5.7697 (94.34)	----
BNB	1.02 (0.99, 1.06)	0.6888 (9.34)	0.00407 (1.65)
USDT	0.46 (0.42, 0.50)	0.00414 (1.98)	----
BTC	1.03 (0.99, 1.07)	8.8754 (181.37)	----

The values in column 2 are the estimates of the d parameter (and in brackets the 95% confidence intervals) from the selected models. The values in parenthesis in columns 3 and 4 are the t-values of the coefficients on the deterministic terms.

Table 4: Estimates of d: Autocorrelated errors

Series	No deterministic terms	With an intercept	With an intercept and a time trend
i) Stock Market Prices			
S&P 500	0.99 (0.93, 1.06)	**1.09 (1.00, 1.17)**	1.09 (1.00, 1.17)
DOW JONES	0.99 (0.93, 1.07)	**1.09 (1.00, 1.19)**	1.09 (1.00, 1.19)
NASDAQ	0.99 (0.93, 1.06)	**1.06 (0.99, 1.14)**	1.06 (0.99, 1.14)
MSCI	1.00 (0.92, 1.06)	**1.07 (0.99, 1.14)**	1.07 (0.99, 1.14)
i) Cryptocurrencies			
ETH	1.01 (0.95, 1.09)	**1.10 (1.03, 1.19)**	1.10 (1.03, 1.19)
BNB	1.10 (1.04, 1.17)	**1.09 (1.03, 1.17)**	1.09 (1.03, 1.17)
USDT	0.51 (0.44, 0.59)	**0.51 (0.43, 0.58)**	0.52 (0.44, 0.58)
BTC	1.00 (0.94, 1.08)	**1.08 (1.02, 1.16)**	1.08 (1.02, 1.16)

The values are the estimates of the differencing parameter. Those in parenthesis are the 95% confidence bands for the estimates of d. In bold, the selected specification in relation with the deterministic terms.

Table 6: Estimated coefficients on the selected models in Table 5

Series	d	Intercept (t-value)	Time trend (t-value)
i) Stock Market Prices			
S&P 500	1.09 (1.00, 1.17)	7.8563 (594.70)	----
DOW JONES	1.09 (1.00, 1.19)	10.0612 (748.37)	----
NASDAQ	1.06 (0.99, 1.14)	8.8172 (570.29)	----
MSCI	1.07 (0.99, 1.14)	3.8350 (264.03)	----
i) Cryptocurrencies			
BTC	1.10 (1.03, 1.19)	5.7691 (95.13)	----
ETH	1.09 (1.03, 1.17)	0.7040 (9.64)	----
USDT	0.51 (0.43, 0.58)	0.00515 (1.95)	----
BNB	1.08 (1.02, 1.16)	8.8792 (181.23)	----

The values in column 2 are the estimates of the d parameter (and in brackets the 95% confidence intervals) from the selected models. The values in parenthesis in columns 3 and 4 are the t-values of the coefficients on the deterministic terms.

Table 6: Estimates of the fractional cointegration parameter in the bivariate regressions

	No terms	An intercept	An intercept with a linear time trend
Dow Jones / BTC	1.02 (0.94, 1.11)	**1.01 (0.93, 1.09)**	1.01 (0.93, 1.09)
Dow Jones / USDT	0.95 (0.88, 1.06)	**0.93 (0.86, 1.03)**	0.93 (0.86, 1.03)
Dow Jones / BNB	1.03 (0.94, 1.12)	**1.02 (0.95, 1.13)**	1.02 (0.95, 1.13)
Dow Jones / ETH	1.03 (0.95, 1.12)	**1.01 (0.95, 1.11)**	1.01 (0.95, 1.11)
MSCI / BTC	1.04 (0.97, 1.12)	**1.05 (0.97, 1.13)**	1.05 (0.97, 1.13)
MSCI / USDT	1.07 (0.99, 1.16)	**1.07 (1.00, 1.16)**	1.07 (1.00, 1.16)
MSCI / BNB	1.04 (0.96, 1.14)	**1.05 (0.98, 1.13)**	1.05 (0.98, 1.13)
MSCI/ ETH	1.03 (0.96, 1.11)	**1.03 (0.95, 1.11)**	1.03 (0.95, 1.11)
Nasdaq / BTC	1.04 (0.97, 1.14)	**1.02 (0.95, 1.11)**	1.02 (0.95, 1.11)
Nasdaq / USDT	0.85 (0.80, 0.92)	**0.79 (0.73, 0.84)***	0.79 (0.72, 0.84)
Nasdaq / BNB	1.03 (0.96, 1.11)	**1.05 (0.98, 1.11)**	1.05 (0.98, 1.11)
Nasdaq / ETH	1.09 (0.97, 1.11)	**1.01 (0.94, 1.08)**	1.01 (0.94, 1.08)
S&P500 / BTC	1.04 (0.96, 1.12)	**1.02 (0.94, 1.10)**	1.02 (0.94, 1.10)
S&P500 / USDT	0.89 (0.83, 0.97)	**0.85 (0.78, 0.92)***	0.85 (0.78, 0.92)
S&P500 / BNB	1.01 (0.94, 1.11)	**1.03 (0.95, 1.10)**	1.03 (0.95, 1.10)
S&P500 / ETH	1.02 (0.97, 1.12)	**1.01 (0.95, 1.08)**	1.01 (0.93, 1.08)

The values in parenthesis are the 95% confidence intervals for the estimates of d. In bold, the selected specification on the basis of the statistical significance of the deterministic terms. * indicates evidence of mean reversion.