Brunel University London

Department of Economics, Finance and Accounting

Working Paper No. 2510

Economics and Finance Working Paper Series

Guglielmo Maria Caporale and Luis Alberiko Gil-Alana

Persistence and Nonlinearities in the US Federal Funds Rate

May 2025

https://www.brunel.ac.uk/economics-and-finance/research-and-phd-programmes/research-papers

Persistence and Nonlinearities in the US Federal Funds Rate

Guglielmo Maria Caporale, Brunel University of London, London, UK

Luis Alberiko Gil-Alana, University of Navarra, NCID, DATAI, Pamplona, Spain and Universidad Francisco de Vitoria, Madrid, Spain

May 2025

Abstract

This paper examines persistence and nonlinearities in the US Federal Funds rate over the period from July 1954 to April 2025 by using fractional integration methods. More precisely, a general model including both deterministic and stochastic components is estimated under alternative assumptions concerning the error term (white noise and autocorrelation), and both linear and a nonlinear specification (the latter based on Chebyshev polynomials) are considered. The empirical results provide evidence of mean reversion but also of high persistence when allowing for autocorrelation in the errors. Moreover, they point towards significant nonlinearities in the stochastic behaviour of the series. Both are important properties of the Federal Funds rate, mainly reflecting underlying inflation persistence and policy shifts respectively.

Keywords: US Federal Funds rate; fractional integration persistence; nonlinearities

JEL classification: C22, E43

Corresponding author: Professor Guglielmo Maria Caporale, Department of Economics, Finance and Accounting, Brunel University of London, Uxbridge, UB8 3PH, UK. Email: Guglielmo-Maria.Caporale@brunel.ac.uk; https://orcid.org/0000-0002-0144-4135

Luis A. Gil-Alana gratefully acknowledge financial support from the Grant D2023-149516NB-I00 funded by MCIN/AEI/ 10.13039/501100011033, from 'Ministerio de Ciencia e Innovación' (MICIN), `Agencia Estatal de Investigación' (AEI) Spain and `Fondo Europeo de Desarrollo Regional' (FEDER).

1. Introduction

The Federal Funds rate is the interest rate that banks in the US charge each other to borrow or lend excess reserves overnight, its weighted average across all transactions being known as the Federal Funds effective rate. Depository institutions are required by US law to meet the reserve requirements set by the Federal Reserve System (the Fed) through making deposits known as Federal Funds at the Fed. Therefore, banks with excess reserve requirements typically lend funds overnight to other banks with a reserve deficit. The target rate is set by the Federal Open Market Committee (FOMC) eighty times a year on the basis of key economic indicators. It affects short-term borrowing rates and sentiment in the stock market, and eventually the real economy through various transmission channels. It is therefore a key policy rate and as such it attracts a lot of attention from market participants as well as academics.

As a result, a huge literature has developed to examine its behaviour and understand its properties. For instance, Bernanke and Blinder (1988, 1992) proposed a model of monetary policy transmission and showed the usefulness of the Federal Funds rate to forecast real macroeconomic variables, whilst Bartolini et al. (2002) analysed the interbank money market to explain its observed volatility in the presence of central bank interventions and reserve requirements. Some studies examine the role of the Taylor rule in determining policy rates (see, e.g., Taylor, 1993 and Clarida et al., 2000), whilst others focus on the interbank market (e.g., Hamilton, 1996, and Taylor, 2001). Another set of papers instead assess the forecasting properties of different models of the Federal Funds rate – in particular, an extensive investigation by Sarno et al. (2005) finds that the best forecasting model is a univariate one using the difference between the effective and the target rate to forecast the future effective rate.

It is noteworthy that most of the existing literature is based on the classical dichotomy between stationary (or integrated of order 0, denoted as I(0)) and non-stationary (or I(1)) series.

However, it is now well known that standard unit roots tests have very low power against fractional alternatives integration (see, e.g., Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996; etc.). By contrast, a fractional integration framework (see Granger, 1980; Granger and Joyeux, 1980, and Hosking, 1981) is much more general, since the differencing parameter d is allowed to take any real value, including fractional ones. Consequently, it is suitable to model a much wider range of stochastic behaviours, including the unit root case; moreover, it sheds light on whether or not the series of interest is mean-reverting (and thus on whether exogenous shocks have permanent or transitory effects) and on its persistence. This is particularly important in the case of the Federal Funds rate, given the available evidence of the high degree of persistence of inflation, one of the key variables determining interest rate decisions (see, e.g., Pivetta and Reis, 2007, and Caporale et al., 2022a).

For these reasons, in a previous study Caporale and Gil-Alana (2017) estimated a fractional integration model incorporating cyclical components and showed that the Federal Funds rate is a mean-reverting, but highly persistent series; moreover, the chosen specification appears to outperform rival ones in terms of its forecasting ability and thus it is most informative for monetary authorities. However, a number of papers provide evidence that nonlinearities are another important property of the Federal Funds rate, especially in the current century, when the 2008 financial crisis led to a period characterised by unconventional monetary policy and the so-called zero lower bound (ZLB) for interest rates, but subsequently rates were increased significantly to tackle an inflation resurgence (see Caporale et al., 2022b, for some evidence concerning nonlinearities in US long-term rates). For example, Shively (2005) shows that the evolution over time of the Federal Funds rate can be captured by a nonlinear, three-regime threshold process in which the two outer (middle) regimes are driven by temporary (permanent) innovations. Petersen (2007) employs a smooth transition regression model to provide evidence that the Fed follows a non-linear Taylor rule, namely that it reacts more strongly to inflation when it approaches a certain threshold (see also, Dolado et al., 2004, and Hayat and Mishra, 2010, for additional evidence on nonlinearities in the Taylor rule estimated for the US). More recently, Richter and Throckmorton (2016) show the importance of using nonlinear methods specifically to account for the zero lower bound (ZLB) on the Fed's policy rate. Specifically, they estimate a Vector Autoregression (VAR) with regime switching and variation in transition probabilities across regimes which depends on the level of output growth; within this framework, changes in the monetary policy stance affect the economy in a nonlinear way.

In view of the above evidence, the present study re-examines the behaviour of the Federal Funds rate by estimating both a linear and a nonlinear fractional integration model to allow for possible nonlinearities in addition to persistence. The main difference compared to Caporale and Gil-Alana (2017), which also uses fractional integration methods, concerns the modelling of the cyclical structure; specifically, in the earlier paper it is assumed for this purpose that the spectral density function of the series contains a pole or singularity at a non-zero frequency, whilst in the current one the assumption is that the frequency corresponding to the pole in the spectrum is zero and cyclical patterns are captured by means of Chebyshev polynomials in time. The results confirm that, when autocorrelation in the disturbances is taken into account, the series under investigation appears to be characterised by a high degree of persistence. Moreover, the findings provide evidence of nonlinear behaviour, which is consistent with the fact that the Fed has opted for significant policy changes at different points in time. For instance, following the 2008 financial crisis, as already mentioned, it kept rates near zero for several years to stimulate economic recovery. By contrast, it increased them during periods of high inflation to keep it under control, most recently following the onset of the Russia-Ukraine war and the resulting upsurge in energy prices.

The remainder of the paper is organised as follows: Section 2 outlines the econometric approach used for the analysis; Section 3 describes the data and presents the empirical results; Section 4 offers some concluding remarks.

2. Econometric Framework

The estimated model includes two components, of a deterministic and stochastic nature respectively, and is specified as follows:

$$y(t) = f(z(t); \theta) + x(t), \quad t = 1, 2, \ldots, \tag{1}$$

where y(t) is the observed time series (in our case, the US Federal Funds rate), f is a linear or non-linear function that depends on a vector of deterministic terms z(t) and an unknown vector of coefficients θ; x(t) is the stochastic error term which is assumed to be integrated of order d, where d is a real scalar value, i.e.,

$$(1 - L)^d x(t) = u(t), \tag{2}$$

L being the lag operator, i.e., $L^k x(t) = x(t-k)$, and u(t) a short memory or integrated of order 0 process, denoted as I(0). The latter is initially treated as a white noise, using the fractional differencing polynomial to capture time dependence; however, given the evidence of remaining weak autocorrelation, the nonparametric approach of Bloomfield (1973), which approximates autoregressive structures, is then implemented.

For the deterministic component we first assume that f is linear and that $z(t) = (1, t)^T$, and estimate the following regression including a linear time trend (Bhargava, 1986; Phillips and Schmidt, 1992; etc.):

$$y(t) = \theta_1 + \theta_2 t + x(t), \quad t = 1, 2, \ldots . \tag{3}$$

However, this assumption, which is common in the context of non-stationarity and unit root tests, might be implausible for the Federal Funds rate (see Figure 1 below), and thus we also allow for a nonlinear trend in the form of Chebyshev polynomials in time, i.e.,

$$y(t) = \sum_{i=1}^{m} \theta_i \; P_i(t) + x(t), \quad t = 1, 2, \ldots . \qquad (4)$$

where *m* denotes the number of coefficients of the Chebyshev polynomials, and $P_{i,T}(t)$ stands for the polynomials defined as:

$$P_{0,T}(t) = 1,$$

$$P_{i,T}(t) = \sqrt{2}\cos\left(i\pi(t-0.5)/T\right), \qquad t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots \qquad (5)$$

Clearly, when $m = 0$ the model includes only an intercept, while $m > 0$ implies the presence of nonlinearities – the higher *m* is, the less linear the approximated deterministic component becomes. A full description of this approach can be found in Hamming (1973) and Smyth (1998), and Tomasevic et al. (2009), who proposed it to approximate highly non-linear trends with polynomials of a relatively low order.

As for the stochastic component of the model, given by equation (2), we use a fractional integration framework, such that the polynomial in L, $(1 - L)^d$ can be expressed as:

$$(1 - L)^d = \sum_{j=1}^{\infty} \frac{\Gamma(d-1)\,(-L)^j}{\Gamma(d-j+1)\,\Gamma(j+1)},$$

where Γ is the gamma function, which is defined as:

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t}\, d(t).$$

An alternative way of expressing such polynomials is the following:

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \ldots$$

which is valid for any real d. Thus, equation (2) can be written as:

$$x(t) = d\,x(t-1) - \frac{d(d-1)}{2}\, x(t-2) + \ldots + u(t),$$

which is the infinite AR representation of the model. Similarly, x(t) can be expressed as an infinite MA process, with the coefficients decaying towards zero at a hyperbolic rate if d is smaller than 1. Robinson (1978) had originally shown that processes with such properties could result from the aggregation of heterogenous series. They were then proposed for modelling purposes by Granger (1980), Granger and Joyeux (1980) and Hosking (1981), became very popular in the late 90s (see, e.g., Baillie, 1996) and Gil-Alana and Robinson, 1997), and are now widely used for capturing the behaviour of economic and financial time series (see, e.g., Abbritti et al., 2016; Martin-Valmayor et al., 2024; etc.).

More specifically, the two approaches we follow for the estimation and testing are the linear one put forward by Robinson (1994), and the non-linear one based on Chebyshev polynomials which was developed by Cuestas and Gil-Alana (2016). Both use a testing procedure based on the Lagrange Multiplier principle, which involves testing the null hypothesis H_o: $d = d_o$ using equation (2) together with (3) (Robinson, 1994) and (4) (Cuestas and Gil-Alana, 2016).

3. Data Description and Empirical Results

We use monthly data on the Federal Funds Effective Rate (FEDFUNDS) over the period from July 1954 to April 2025, obtained from the Federal Reserve Bank of St. Louis Database (https://fred.stlouisfed.org/series/FEDFUNDS). This series is plotted in Figure 1. It can be seen that initially it exhibits an upward trend, with a peak in the early 1980s, and then follows a downward trend, staying near zero for various years in the current century before an uptick in the most recent period. Its evolution over time reflects well-known policy shifts in US monetary policy in response to changing economic conditions. For instance, rates were set at very high levels during the 1980s in the presence of spiralling inflation, they were kept at record lows following the 2008 financial crisis, and increased in the most recent period to tackle an upswing in inflation mainly reflecting higher energy prices as a result of the Russia-Ukraine conflict.

[Insert Figure 1 about here]

To capture the behaviour of this series, first we consider the linear specification based on equations (3) and (2), i.e.,

$$y(t) \;=\; \theta_1 + \theta_2\, t \;+\; x(t), \quad (1 - L)^d\, x(t) \;=\; u(t), \qquad t \;=\; 1, 2, \ldots . \qquad (6)$$

under two different assumptions concerning the error term. Specifically, we assume that u(t) in (6) is a white noise process, or alternatively that it is weakly autocorrelated as in the exponential spectral model of Bloomfield (1973). The latter is a non-parametric approach, in the sense that is only expressed in terms of its spectral density (f(λ)), which is given by:

$$f(\lambda) \;=\; \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{r=1}^{m} \tau_r \cos \lambda\, r\right),$$

where σ^2 is the variance of the error term, and m is the number of parameters in the model (in our case we set m = 1). An advantage over the AR specifications is that it is stationary for the entire range of its values; moreover, it can be conveniently implemented in the context of the tests used here (Robinson, 1994, and Cuestas and Gil-Alana, 2016), and it produces autocorrelation coefficients decaying at an exponential rate as in the AR case.

Table 1 displays the estimates of d in equation (6) under the two alternative assumptions for the error term and for three different specifications. Specifically, column 2 reports the results obtained when both coefficients, θ_1 and θ_2 are set equal to zero, i.e., no deterministic terms are included; column 3 displays the estimates for the model including only a constant, i.e., when setting $\theta_2 = 0$; finally, column 4 shows the results obtained when no restrictions are imposed. To select our preferred model we follow a "general to specific" approach, i.e., we start with the specification including both an intercept and a linear time trend; then, if the time trend coefficient is found to be statistically insignificant, we move on to the case with an intercept only; finally, if the intercept is also insignificant, we select the model without deterministic terms. Table 2 displays the full set of estimated coefficients from the preferred models.

[Insert Tables 1 and 2 about here]

It can be seen from Table 1 that, under the assumption of white noise errors, the chosen model is the one without deterministic terms, and the estimated value of d is 1.29, which is significantly higher than 1. This implies that the series is highly persistent, with shocks having permanent effects. However, standard tests suggest the presence of autocorrelation (Box and Pierce, 1970; Ljung and Box, 1978) and thus of time dependence in the data, therefore we re-estimate the model allowing for autocorrelation using Bloomfield's (1973) approach. In this case the intercept is found to be statistically significant, but not the coefficient on the time trend, and the estimate of d is substantially smaller compared to before, being equal to 0.87 with a 95% confidence interval of (0.80, 0.97). Thus, the unit root null hypothesis is now rejected in favour of mean reversion.

Table 3 reports the results for the nonlinear model. The estimates of d are very similar to the linear ones. More specifically, with white noise errors the estimated value of d is 1.28 and the unit rot null is rejected in favour of $d > 1$. However, when allowing for autocorrelated (Bloomfield) errors, the corresponding value is 0.84 and the I(1) hypothesis is rejected in favour of mean reversion. More importantly, under the latter assumption, the four Chebyshev coefficients are found to be statistically significant, which supports the nonlinear specification. The estimated nonlinear trend is shown in Figure 1 together with the series of interest; it can be seen that it fits the data well.

On the whole, our empirical analysis confirms previous evidence suggesting that the Federal Funds rate is a mean-reverting nut highly persistent series, but also indicates the presence of nonlinearities resulting from policy shifts.

4. Conclusions

This paper examines persistence and nonlinearities in the US Federal Funds rate over the period from July 1954 to April 2025 by using fractional integration methods. More precisely, a general model including both deterministic and stochastic components is estimated under alternative assumptions concerning the error term (white noise and autocorrelation), and both a linear and a nonlinear specification (the latter based on Chebyshev polynomials) are considered. The motivation for the latter is the existing body of evidence suggesting the presence of nonlinearities in the Taylor rule implicitly followed in the US, as implied, for instance, by the ZLB approach adopted in response to the 2008 financial crisis (see, e.g., Petersen, 2007, and Richter and Throckmorton, 2016). The empirical results provide evidence of mean reversion but also of high persistence when allowing for autocorrelation in the errors. Moreover, they point towards significant nonlinearities in the stochastic behaviour of the series. Both appear to be important properties of the Federal Funds rate, mainly reflecting underlying inflation persistence and regime changes respectively.

These findings confirm the importance of allowing for both (slow) mean reversion and nonlinearities to understand the behaviour of policy rates in the US. Whilst Caporale and Gil-Alana (2017) had already shown the suitability of a fractional integration framework to capture the former property, the present study highlights that the latter is also a key modelling issue given the various monetary policy shifts which have occurred over time in response to a changing economic environment. In particular, as already mentioned, in the last couple of decades the Fed first had to cut rates to zero or near zero to boost growth following the 2008 financial crisis and its severe negative impact on the world economy, and most recently had to raise them again to cool down inflation as energy price increases were generating an inflation spiral. The results are consistent with those in Caporale and Gil-Alana (2017), who also found high levels of persistence in the Federal Funds rate, at different frequencies, for a shorter time period and with a different long memory approach.

Future work could extend the analysis by (i) using alternative non-linear deterministic models, still in an I(d) framework, such as those based on Fourier functions in time (Gil-Alana and Yaya, 2021) or neural networks (Furuoka et al., 2024); (ii) assessing the forecasting performance of the proposed model relative to more standard ones such as AR(I)MA and ARDL specifications.

References

Abbritti, M., L.A. Gil-Alana, Y. Lovcha, and A. Moreno (2016), "Term Structure Persistence", Journal of Financial Econometrics 14 (2): 331–52.

Baillie, R.T. (1996), Long memory and fractional integration in econometrics, Journal of Econometrics 73, 1, 5-59.

Bartolini, L., Bertola, G. and Prati, A. (2002), "Day-to-day monetary policy and the volatility of the Federal Funds interest rate, "Journal of Money, Credit and Banking, 34, 1, 137-159.

Bernanke, B.S. and Blinder, A.S. (1988), "Credit, money and aggregate demand", American Economic Review Papers and Proceedings, 78, 435-439.

Bernanke, B.S. and Blinder, A.S. (1992), "The Federal Funds rate and the channels of monetary transmission", American Economic Review, 82, 4, 901-921.

Bhargava, A. (1986), On the theory of testing for unit roots in observed time series, Review of Economics Studies 53, 369-384.

Bierens, H.J. (1997), "Testing the unit root with drift hypothesis against nonlinear trend stationarity with an application to the us price level and interest rate", Journal of Econometrics 81(1), 29-64.

Box, G.E.P., Pierce, D.A. (1970), "Distribution of residual autocorrelations in autoregressive integrated moving average time series models". J. Am. Stat. Assoc. 65, 1509–1526. https://doi.org/10.1080/01621459.1970.10481180.

Bloomfield, P. (1973). "An exponential model in the spectrum of a scalar time series", Biometrika, 60(2), 217-226. https://doi.org/10.1093/biomet/60.2.217

Caporale, G.M. and L.A. Gil-Alana (2017), "Persistence and cycles in the US Federal Funds rate", International Review of Financial Analysis, 52, 1-8.

Caporale, G.M., Luis-Alana, L. and T. Trani (2022a), "On the persistence of UK inflation: a long-range dependence approach", International Journal of Finance and Economics, 27, 1, 439-454.

Caporale, G.M., Gil-Alana, L.A. and M.A. Martin-Valmayor (2022b), "Non-linearities and persistence in US long-run interest rates", Applied Economics Letters, 29:4, 366-370, DOI: 10.1080/13504851.2021.1897511

Clarida, R.H., Gali, J. and Gertler, M. (2000), "Monetary policy rules and macroeconomic stability: evidence and some theor", Quarterly Journal of Economics, 115, 147-180.

Cuestas, J.C. and L.A. Gil-Alana (2016), "Testing for long memory in the presence of non-linear Chebyshev polynomials in time", Studies in Nonlinear Dynamics and Econometrics 20(1), 57-74.

Diebold, F.X. and G.D. Rudebusch (1991), "On the power of Dickey-Fuller test against fractional alternatives", Economics Letters 35, 155-160.

Dolado, J., Pedrero, R.M.-D. and J.F. Ruge-Murcia (2004), "Nonlinear Monetary Policy Rules: Some New Evidence for the U.S.", Studies in Nonlinear Dynamics & Econometrics, 8, 3, 2.

Furuoka, F., Gil-Alana, L.A., Yaya, O.S. *et al.* (2024), "A new fractional integration approach based on neural network nonlinearity with an application to testing unemployment hysteresis". Empirical Economics 66, 2471–2499 (2024). https://doi.org/10.1007/s00181-023-02540-5

Gil-Alana, L. A., & Robinson, P. M. (1997). "Testing of unit roots and other nonstationary hypotheses in macroeconomic time series", Journal of Econometrics, 80(2), 241-268. https://doi.org/10.1016/S0304-4076(97)00038-9

Gil-Alana, L. A. and Yaya, O. (2021), "Testing fractional unit roots with non-linear smooth break approximations using Fourier functions", Journal of Applied Statistics 48(13-15), 2549-2559.

Granger, C.W. J. (1980), "Long memory relationships and the aggregation of dynamic models", Journal of Econometrics, 14(2), 227–238. https://doi.org/10.1016/0304-4076(80)90092-5

Granger, C. W. J., and Joyeux, R. (1980). "An introduction to long-memory time series models and fractional differencing", *Time Series With Long Memory*, 49–64. https://doi.org/10.1093/OSO/9780199257294.003.0004

Hamilton, J.D. (1996), "The daily market for Federal Funds. Journal of Political Economy", 5, 1135-1167.

Hamming, R. (1973), "Numerical methods for scientists and engineers", Courier Corporation, 1973.

Hassler, U. and J. Wolters (1994), "On the power of unit root tests against fractional alternatives", Economics Letters 45, 1-5.

Hayat, A. and S. Mishra (2010), "Federal reserve monetary policy and the non-linearity of the Taylor rule," Economic Modelling, 27, 5), 1292-1301.

Hosking, J. R. M. (1981). "Fractional differencing", Biometrika, 68, 165-76. https://doi.org/10.2307/2335817

Jackson, L.E., Owyang, M.T. and D. Soques (2016), "Nonlinearities, Smoothing and Countercyclical Monetary Policy," Working Papers 2016-8, Federal Reserve Bank of St. Louis.

Lee, D. and P. Schmidt (1996), "On the power of the KPSS test of stationarity against fractionally integrated alternatives", Journal of Econometrics 73, 285-302.

Ljung, G.M., Box, G.E.P., (1978), "On a Measure of Lack of Fit in Time Series Models", Biometrika 65 (2), 297–303. McMillan, D.G., Speight, A.E.H., 2001

Martin-Valmayor, M., N. Carmona-Gonzalez, M.P. Sanchez-Martin and L.A. Gi-Alana (2024), "Persistence in sovereign debt during the past two centuries: Evidence for the US and the largest European economies", Economic Analysis and Policy 83, 390-403.

Petersen, K. (2007), "Does the Federal Reserve Follow a Non-Linear Taylor Rule?", University of Connecticut, Department of Economics Working Paper 2007-37.

Ramirez, C.D., (2024), "The effect of economic policy uncertainty under fractional integration", Portuguese Economic Journal 23, 89-110.

Richter, A.W. and N.A. Throckmorton (2016), "Are Nonlinear Methods Necessary at the Zero Lower Bound?", Federal Reserve Bank of Dallas, Research Department Working Paper 1606.

Robinson, P.M. (1978), "Statistical Inference for a Random Coefficient Autoregressive Model", Scandinavian Journal of Statistics 5, 163–68.

Robinson, P.M. (1994), "Efficient tests of nonstationary hypotheses", Journal of the American Statistical Association, 89, No. 428, 1420-1437.

Sarno, L., Thornton, D.L. and Valente, G. (2005), "Federal Funds rate prediction", Journal of Money, Credit and Banking, 37, 3, 449-471.

Schmidt, P. and P.C.B. Phillips (1992), "LM Tests for a Unit Root in the Presence of Deterministic Trends", Oxford Bulletin of Economics and Statistics 54, 3, 256.287.

Shively, P.A. (2005), "Threshold nonlinear interest rates", Economics Letters, 88, 3, 313-317.

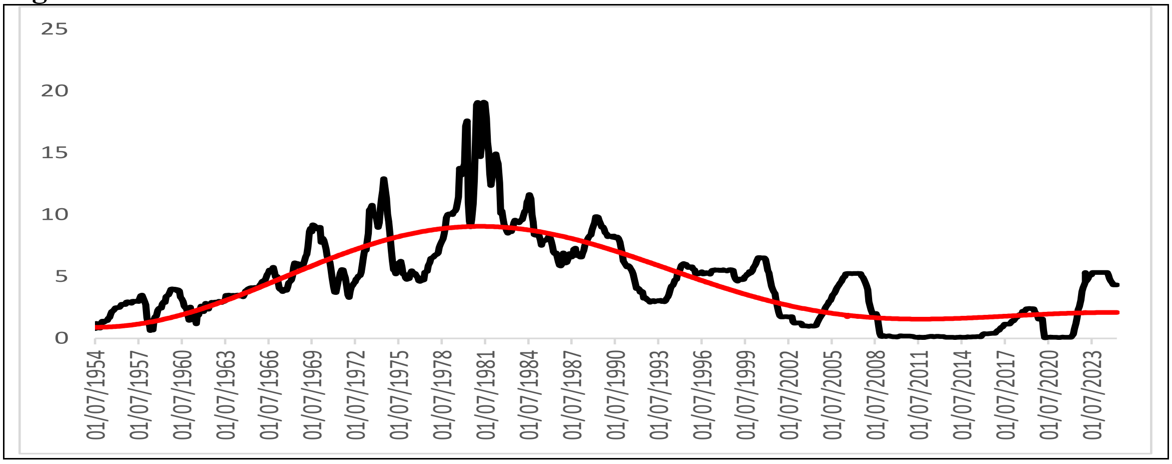
Smyth, G.K. (1998), "Polynomial approximation", Encyclopedia of Biostatistics, P. Armitage and T. Colton (eds.), Wiley, London, pp. 3425- 3433.

Taylor, J.B. (1993), "Discretion versus policy rules in practice", Carnegie Rochester Conference Series on Public Policy, 39, 195-214.

Taylor, J.B. (2001), "Expectations, open market operations, and changes in the Federal Funds rate", Federal Reserve Bank of St. Louis Review, 83, 33-47.

Tomasevic, N., M. Tomasevic, and T. Stanivuk (2009), "Regression analysis and approximation by means of Chebishev polynomials", Informatologia 42(3), 166-172.

Figure 1: The Federal Funds rate and the estimated non-linear trend



Note: The black line is the Federal Funds rate while the red one is the estimated non-linear trend. The sample period goes from July 1954 to April 2025.

Table 1: Estimates of d based on a linear model

Type of errors	No terms	An intercept	An intercept and a linear time trend
White noise	**1.29 (1.20, 1.38)**	1.29 (1.20, 1.38)	1.29 (1.20, 1.38)
Autocorrelation	0.88 (0.81, 0.96)	**0.87 (0.80, 0.97)**	0.87 (0.80, 0.97)

Note: The reported values are the estimates of the differencing parameter d. In brackets, the 95% confidence intervals. In bold the estimates of d from the model selected on the basis of the statistical significance of the regressors.

Table 2: Estimated coefficients from the selected models in Table 1

Type of errors	d (95% conf. band)	Intercept (t-value)	Time trend(t-value)
White noise	1.29 (1.20, 1.38)	-----	-----
Autocorrelation	0.87 (0.80, 0.97)	0.8981 (2.08)	-----

Note: The values in column 2 are the estimates of d (with their 95% confidence interval in brackets), whilst those in columns 3 and 4 are the estimates of the intercept and of the coefficient on a linear time trend (with t-values in brackets). ----- indicates lack of statistical significance.

Table 3: Estimated coefficients of the nonlinear model

Type of errors	d (95% c.i.)	θ_1 (t-value)	θ_2 (t-value)	θ_3 (t-value)	θ_4 (t-value)
White noise	1.28 (1.18, 1.37)	0.713 (0.293)	-0.028 (-0.01)	-0.256 (-0.45)	-0.071 (-0.30)
Autocorrelation	**0.84 (0.72, 0.98)**	**4.400 (1.96)**	**1.110 (1.69)**	**-2.052 (-1.93)**	**-1.536 (-2.02)**

Note: The values in column 2 are the estimates of d (with their 95% confidence intervals in brackets). The other columns report the estimates of the Chebyshev polynomials with their associated t-values in brackets. In bold, the estimated coefficients from the selected model specification.