Branes in Minimal String Theory from Matrix Models

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Overview and Aims

- Minimal string theory provides an exactly solvable model to study nonperturbative aspects of string theory and QG
- Despite exact solvability, brane spectrum ("FZZT" and "ZZ") only fully known at genus zero
- Unlike traditional worldsheet approach, matrix models provide a nonperturbative definition of the minimal string
- Exploit this to **compute higher genus corrections to the brane amplitudes from a matrix model**

Review: Minimal String Theory

- Minimal string theory = Liouville theory $\otimes$ conformal minimal model $\otimes$ ghosts $\quad c_L + c_M = 26$
- Theories labelled by coprime integers (p, q)
- Brane partition functions encoded in algebraic curve $\mathcal{M}_{p,q}$ [1]

$$T_q(x) - T_p(y) = 0, \qquad T_n(x)\text{: } n^{th} \text{ Chebyshev pol.}$$

$$Z(\mu_B) = \int^{\mu_B} y\mathrm{d}x, \quad \text{"FZZT brane"} \qquad Z_{r,s} = \oint_{B_{r,s}} y\mathrm{d}x \quad \text{"ZZ brane"}$$

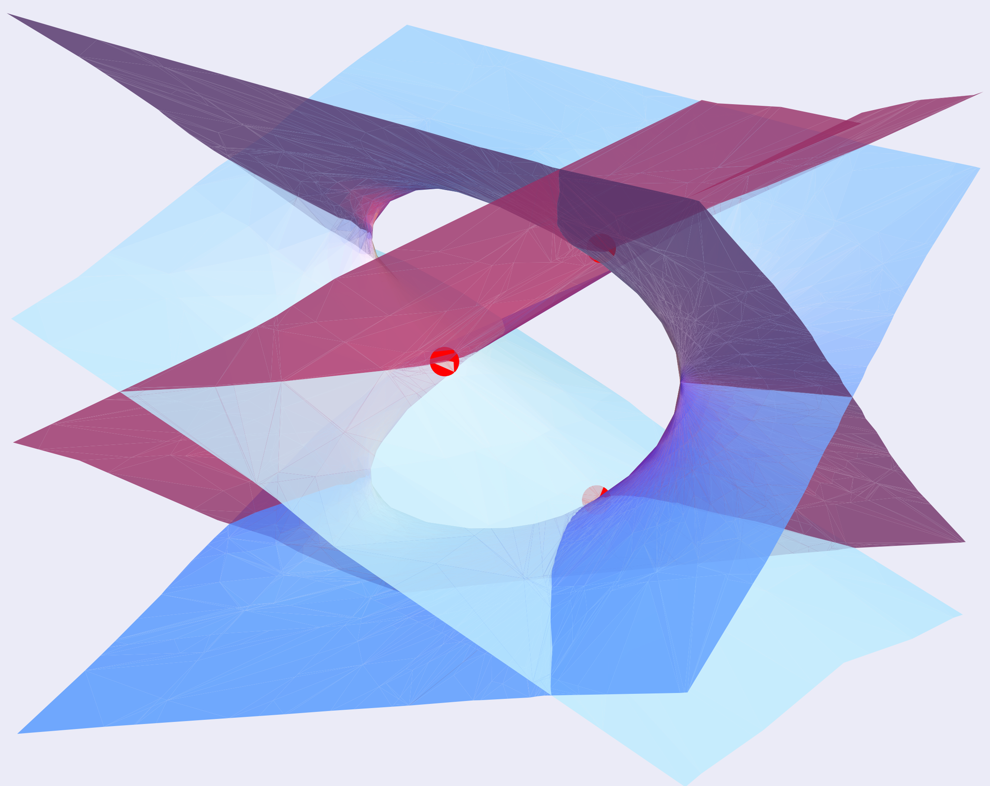

Figure: Riemann surface for $(p, q) = (3, 4)$

- One FZZT and one ZZ brane for each Cardy state (r, s) [1]. At genus zero:

$$x(\phi) = \cosh(p\phi), \quad y(\phi) = \cosh(q\phi) \qquad \phi\text{: uniformises } \mathcal{M}_{p,q}$$

$$y_{r,s}(\phi) = \frac{S_{(r,s)(1,1)}}{S_{(1,1)(1,1)}} y(\phi) = \sum_{m,n} \cosh(q\phi + i\pi(m + nq/p))$$

- Shows that in the planar limit, **all** (r, s) **branes are superpositions of** $(1, 1)$ **brane** [1]
- Not true at higher genus [2, 3] – branes with different Cardy labels quantum mechanically **distinct**
- General (r, s) brane amplitudes at higher genus not known

Review: Matrix Model Formulation

- World sheet discretisation via Hermitian two-matrix model

$$Z = \int \mathrm{d}X\mathrm{d}Y e^{-\frac{N}{g}\mathrm{tr}[V(X)+V(Y)-cXY]}$$

- Critical V with $\deg V = p$ gives p^{th} multicritical point [4]
- Approaching the crit. point in scaling limit, resolvent

$$y(x) \sim_x = N^{-1}\langle \mathrm{tr}(x - X)^{-1}\rangle$$

obeys algebraic equation defining $\mathcal{M}_{p,q}$ of the minimal string [5] $\to (r, s) = (1, 1)$ brane amplitude **only**
- Higher genus contributions from finite N corrections

FZZT Branes from a Matrix Model

- Atkin and Zohren enlarge operator content of matrix model [6],

$$Z = \int \mathrm{d}X\mathrm{d}Y\mathrm{d}M e^{-N\mathrm{tr}[U(X)+U(Y)+\frac{\lambda}{2}M^2-(X+Y)M-XY]}$$

$$\frac{1}{N}\left\langle \mathrm{tr}\frac{1}{z-(X+Y)}\right\rangle \quad \to \quad \frac{1}{N}\left\langle \mathrm{tr}\frac{1}{z-M}\right\rangle$$

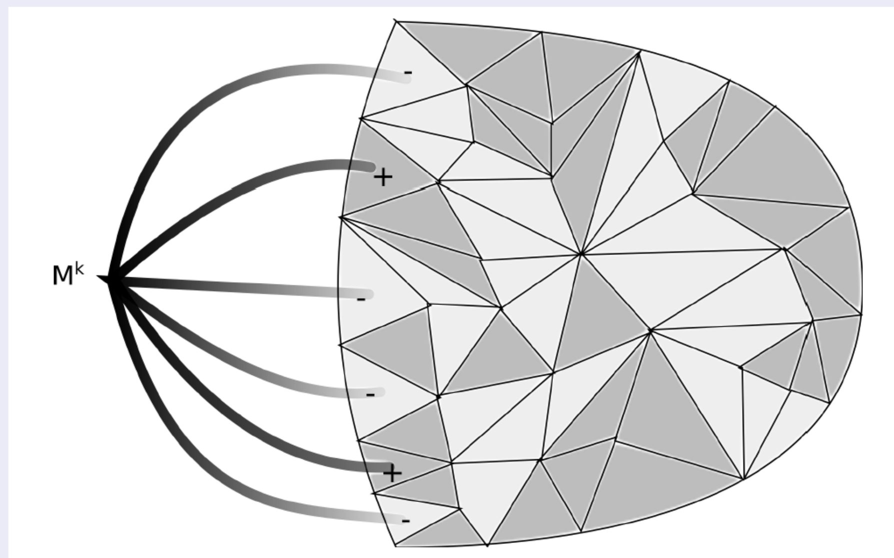

Figure: Free spin state in the two-matrix model, taken from [6]

- Implements free spin $(r, s) = (1, p - 1)$ b.c.
- First check: saddle point analysis yields partial agreement with known results [6]
- **Current work**: use Dyson-Schwinger ("loop") equations to derive algebraic curve directly

Summary and future directions

- Implementation of nontrivial boundary conditions in the matrix model formulation of the minimal string
- Allows for systematic investigation of the relations between FZZT and ZZ branes
- Starting point for a first-principles calculation of the full brane spectrum of the minimal string

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