Semiclassical approach to n-point spectral statistics

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It is widely believed [1] that the local spectral statistics of chaotic systems should be universal, and given by RMT. It is a long-standing problem to show this fact starting from a semiclassical approximation.

Using the Gutzwiller trace formula, which expresses the density of states as a sum over periodic orbits, this problem was solved for the 2-point spectral correlation function in [2], with only non-oscillatory contributions, and in [3], with the oscillatory ones.

In our work we have calculated semiclassically the non-oscillatory contributions to all n-point spectral correlation functions, for the unitary symmetry class. We go beyond the `diagonal` approach [4].

We have achieved this is three steps: First, we obtain semiclassical diagrammatic rules; Second, we map this diagrammatic theory into a matrix model; Third, we solve this matrix model.

Let $\{E_1, E_2, E_3, E_4, \cdots\}$ be the energy levels of a quantum system for which the corresponding classical dynamics is completely chaotic (hyperbolic and mixing). Let $\rho(E)$ denote the density of states.

$R_n(\vec{\epsilon}) = \frac{1}{[\rho(E)]^n}\langle \rho(E+\epsilon_1)\rho(E+\epsilon_2)\cdots\rho(E+\epsilon_n)\rangle$ is the n-point spectral correlation function, where the brackets denote average over E.

For GUE, $R_n(\vec{\epsilon}) = \det\left(\frac{\sin(\epsilon_i-\epsilon_j)}{\epsilon_i-\epsilon_j}\right)$

They can be divided into oscillatory and non-oscillatory parts,

$R_n(\vec{\epsilon}) = R_{n,osc}(\vec{\epsilon}) + R_{n,non}(\vec{\epsilon})$

We focus on the non-oscillatory part (short times).

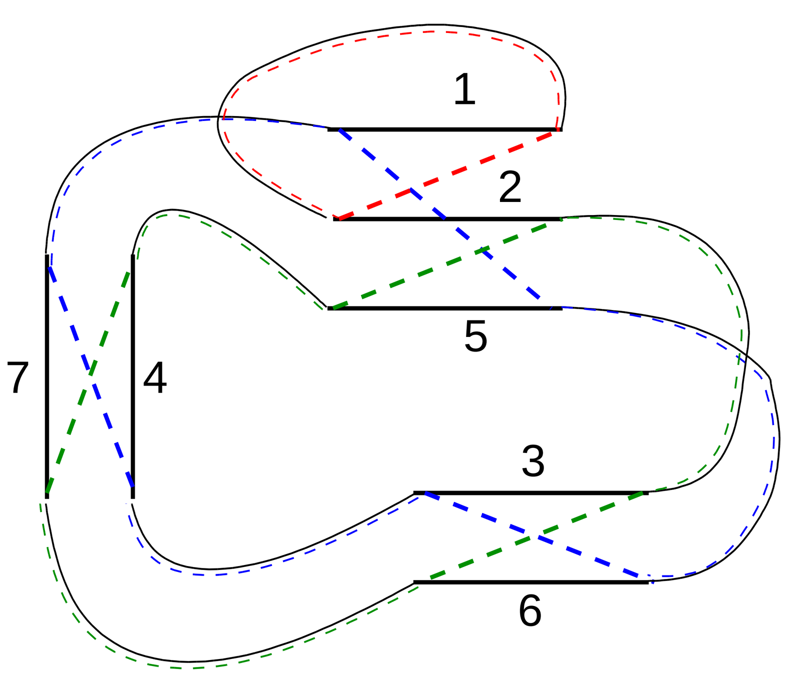
The Gutzwiller trace formula is $\rho(E) \approx \frac{1}{\pi\hbar}\sum_p A_p T_p \sin(S_p/\hbar)$

Where $S_p(E)$ and $T_p(E)$ are the action and period of periodic orbit p and $A_p(E)$ is related to its stability. Semiclassically we have

$$R_n(\vec{\epsilon}) = \frac{1}{[\rho(E)]^n}\left\langle \prod_j \sum_{p_j} A_{p_j} T_{p_j} \sin(S_{p_j}(E+\epsilon_j)/\hbar)\right\rangle$$

Performing the average under stationary phase requires action correlations. Simplest case is pairwise concidence among the orbits. This predicts the exact GUE values. It remains to show that there are no corrections. Off-diagonal correlations come from encounters [2,3,5]: There are two sets of orbits that must be approximately piecewise equal.

The next diagam depicts a contribution to the function R4 where 1 orbit is correlated with 3 others.



We have established the following diagrammatic rules for semiclassical calculation of spectral correlation functions:

$$R_n(\vec{\epsilon}) = \frac{\partial^n}{\partial\epsilon_1\cdots\partial\epsilon_n}\sum_{\text{diagrams}}\frac{(-1)^V}{V!}\prod_{jk}(\epsilon_j-\epsilon_k)^{-N_{jk}}$$

where V is the number of encounters and N_{jk} is the number of times orbits j and k run together, but are not the first ones to arrive at an encounter.

We do not deal with the combinatorics of encounters directly, instead, we show that the same diagrammatic rules hold for the matrix integral

$$F(X,Y) = \frac{1}{\mathcal{Z}}\int dZ e^{-\mathrm{Tr}[XZZ^\dagger - Z^\dagger ZY]} e^{-\sum_q \mathrm{Tr}[X(ZZ^\dagger)^q - (Z^\dagger Z)^q Y]}$$

where X and Y are diagonal matrices wiht the ϵs and dZ is the flat measure on the space of complex matrices. This matrix integral can be computed exactly as a determinant.

$$F(X,Y) = \det(1 - C^{-1}B)$$

where $C_{ij} = (\epsilon_i - \epsilon_j)^{-1}$ and $B_{ij} = \frac{e^{\epsilon_i-\epsilon_j}Ei(2N+1,\epsilon_i-\epsilon_j)}{\epsilon_i-\epsilon_j}$

The off-diagonal corrections to the spectral correlation function are obtained as derivatives of the function F(X,Y). They turn out to be zero, therfore implying equivalence between semiclassics and RMT.

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