Level statistics of the Gaussian beta ensemble across the many-body localization transition

Wouter Buijsman[1], Vadim Cheianov[2], and Vladimir Gritsev[1,3]

Level statistics provide a common diagnostic for ergodicity. We study the level statistics of a paradigmatic model in the field of many-body localization, and compare them with the level statistics of the Gaussian β ensemble. We report near-perfect agreement over the full crossover range from the thermal to the many-body localized phase.

Level statistics

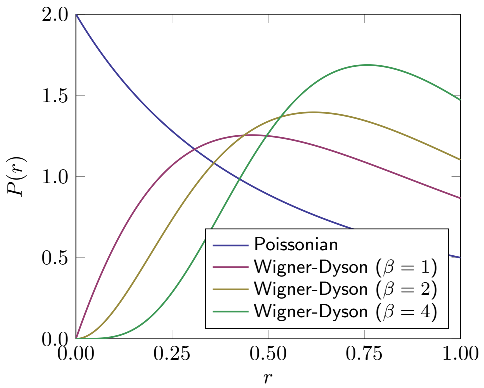
• Energy spectra show universal short-range statistics [1]. One finds Poissonian (Wigner-Dyson) level statistics for non-ergodic (ergodic) systems.

• Spectral statistics can be probed through the ratios of consecutive level spacings

$$r_i = \min\left(\frac{s_{i+1}}{s_i}, \frac{s_i}{s_{i+1}}\right) \in [0,1], \tag{1}$$

$$s_i = E_{i+1} - E_i. \tag{2}$$

• Plot: distributions of r for Poissonian and Wigner-Dyson level statistics [2]. Dyson index $\beta = 1, 2, 4$ if the Hamiltonian is diagonalized by respectively an orthogonal, unitary, or symplectic transformation.



Gaussian beta ensemble

• Wigner-Dyson level statistics are obtained from the level statistics of the Gaussian β ensemble [3] for $\beta = 1, 2, 4$.

• The Gaussian β ensemble has joint probability distribution

$$\rho(E_1, \dots, E_N) = c_{\beta,N} \prod_{i<j} |E_i - E_j|^\beta \prod_{i=1}^{N} e^{-\frac{\beta}{2} E_i^2}, \tag{3}$$

with $c_{\beta,N}$ a normalization constant.

Going beyond beta=1,2,4

• We propose to generalize Wigner-Dyson level statistics from $\beta = 1, 2, 4$ to $\beta \in (0, \infty)$.

• These statistics cover both Poissonian ($\beta \to 0$) and Wigner-Dyson ($\beta = 1, 2, 4$).

• These statistics smoothly interpolate between Poissonian and Wigner-Dyson.

Physical model

• Standard model in studies on many-body localization:

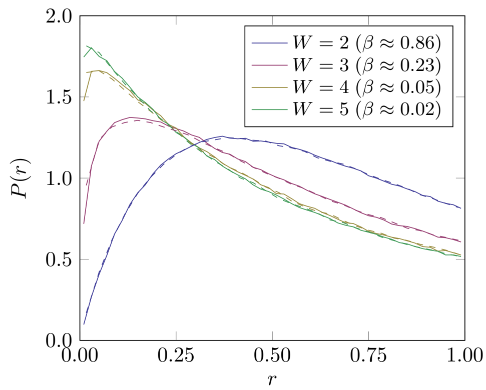
$$H = \frac{1}{2} \sum_{i=1}^{L} \left(c_i^\dagger c_{i+1} + c_i c_{i+1}^\dagger\right) + \sum_{i=1}^{L} h_i \left(n_i - \frac{1}{2}\right) + \sum_{i=1}^{L} \left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right). \tag{4}$$

• $L = 16$, h_i sampled from uniform distribution over $[-W, W]$, half-filling, focus restricted to the middle of the spectrum.

• Model shows intermediate level statistics between Wigner-Dyson ($\beta = 1$) and Poissonian for $1.7 \lesssim W \lesssim 4.0$. Finite-size scaling analysis shows a many-body localization transition at $W \approx 3.6$ [4].

Results

• Plot: distributions r for the Hamiltonian (solid lines) and the Gaussian β ensemble (dashed lines) with β a single fitting parameter.



• Agreement is near-perfect for all W: level statistics of the Gaussian β ensemble accurately describe level statistics for the Hamiltonian over the full crossover range.

References

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[1] University of Amsterdam, Amsterdam, The Netherlands
[2] Leiden University, Leiden, The Netherlands
[3] Russian Quantum Center, Moscow, Russia

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