Non-Gaussian Models for MIMO Communications in Enclosures
MIMO in a Box

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Introduction

The mutual information for an $N \times M$ MIMO system with uncorrelated noise is

$$C = \log_2 \det\left(I_N + \frac{\rho}{N}\Gamma_n\right),$$

where the Graham matrix is defined as

$$\Gamma_n = \mathcal{H}_t^{\dagger}\mathcal{H}_t.$$

In confined MIMO communications, near-field effects are relevant and the channel transfer matrix $\mathcal{H}_t$ is best characterised in terms of impedance matrix

$$\mathcal{H}_t = Z_{\mathrm{RT}}.$$

Random Coupling Model: A micro-primer

What is the impedance matrix Z_{RT} of an antenna array operating inside a complex enclosure with losses?

G. Gradoni, J.-H. Yeh, B. Xiao, T. M. Antonsen, S. M. Anlage, E. Ott *Wave Motion*, **51** 606, (2014)

The Random Coupling Model (RCM) provides an answer based on random matrix theory and semiclassical analysis

$$Z_{\mathrm{RT}} = (Z_{RR}^{rad})^{-1/2}\,\xi\,(Z_{TT}^{rad})^{-1/2},$$

with

- Z_{ii}^{rad} free-space radiation impedance of the antenna array - can be calculated with full-wave EM simulators.
- ξ normalised cavity impedance whose entry reads

$$\xi_{lq} = \frac{\mathrm{i}}{\pi}\sum_n \frac{\phi_l\phi_q}{\mathcal{K}_0 - \mathcal{K}_n + \mathrm{i}\alpha},$$

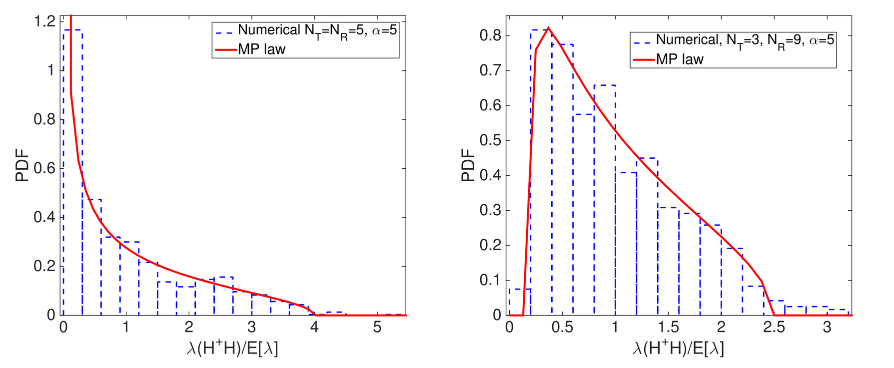
where $\mathcal{K}_n$ is a set of eigenvalues of a random matrix drawn from the GOE, $\phi_{l,q}$ zero mean unit width GRVs, and

$$\alpha = \frac{f_0}{2Q\Delta f}$$

the average resonance overlapping factor.

Gaussian Statistics for High Losses

In the regime where $\alpha > 1$, singular value PDF of $\mathcal{H}_t$ follows a Marchenko-Pastur distribution.



- Analysis: Mutual information PDF found in closed-form for $N \times 1$ MISO systems

$$f_C(c) = \frac{\ln 2}{(N-1)!}\left[\frac{N\pi}{\mu T}\right]^N 2^c\,(2^c - 1)^{N-1} \times \exp\left[-\frac{N\pi}{\mu T}(2^c - 1)\right],$$

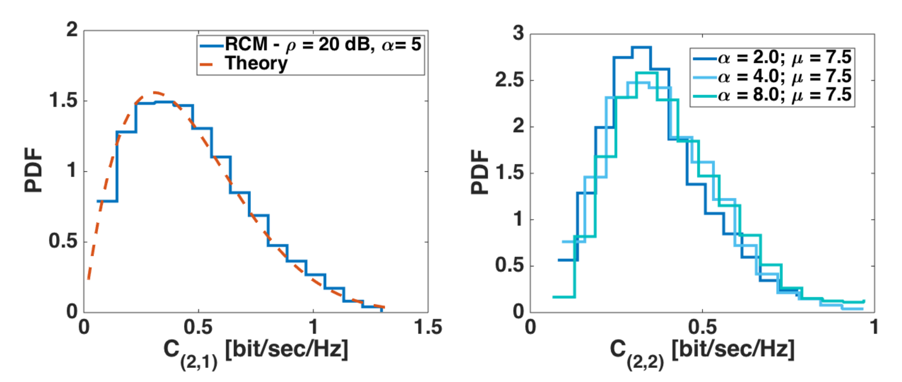
with T coupling factor defined in terms of radiation impedances of transmitting and receiving antenna arrays.

- Enclosure effect: Mutual information PDF of $N \times M$ MIMO systems depends on a single parameter given by the SNR-to-loss ratio

$$\mu = \frac{\rho}{\alpha},$$

representing the interplay between *enclosure losses* and *system noise*, whence the modal overlapping is a penalty for the SNR.

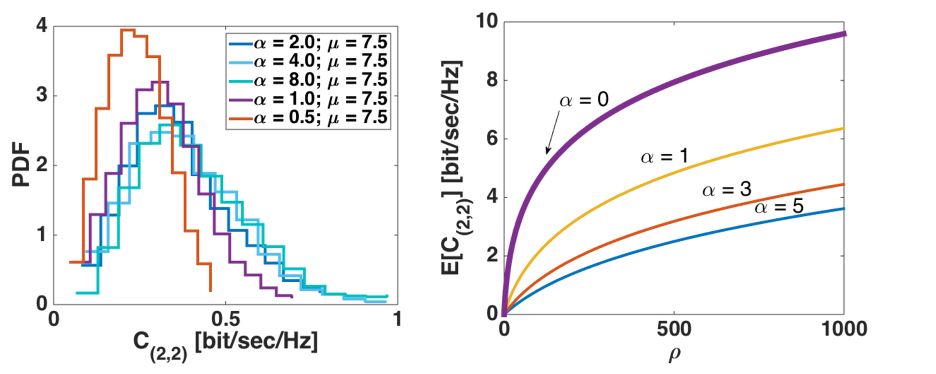
- Near-field effect: RCM includes inter-element coupling within antenna arrays through self- and mutual-impedances.
- Monte Carlo results using the RCM impedance statistics.



Low Losses

In the regime where $\alpha < 1$, singular value PDF of $\mathcal{H}_t$ follows a Fisher distribution.

- Enclosure effect: Mutual information PDF of $N \times M$ MIMO systems depends on SNR ρ and α individually. The scaling law involving a single parameter μ ceases to be valid.
- Ergodic channel capacity: Lower losses due to a reduced resonance density increases the average mutual information.



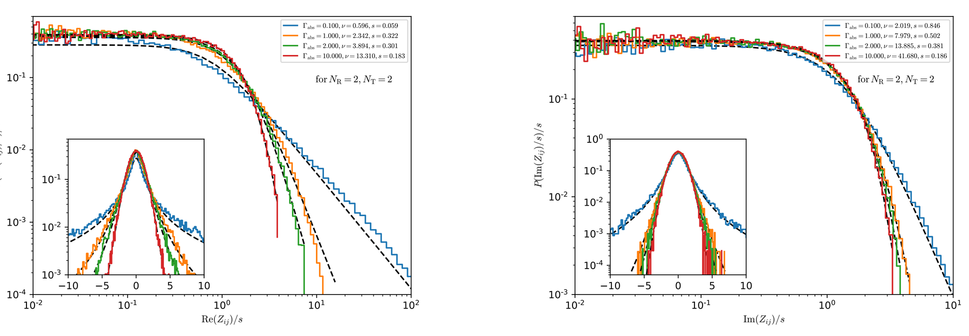
- Impedance statistics: Real and imaginary part follow unequal non-Gaussian PDFs.

Student's t-distribution ansatz.

Student's-t Distributions

- Minimal Model: 2×2 MIMO
- Start from GOE Ensemble, add uniform absorption, $\frac{\mathrm{i}}{2}\Gamma_{\mathrm{abs}}$, calculate Impedance

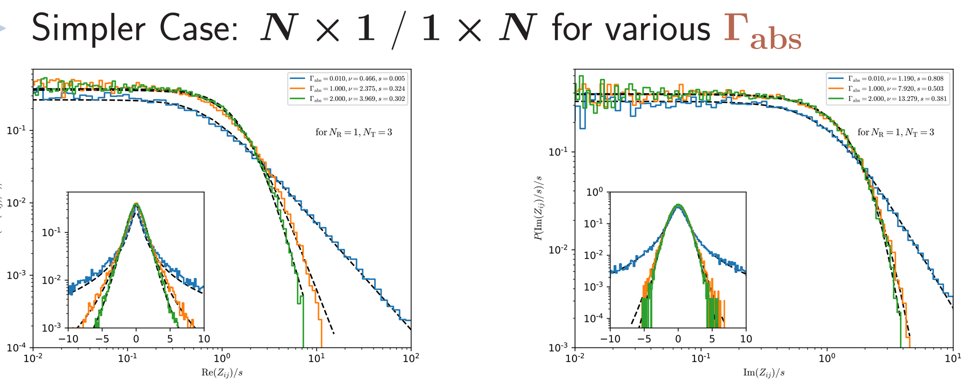
$$Z = \mathrm{i}K = \mathrm{i}W^T\left(\frac{\mathrm{i}}{2}\Gamma_{\mathrm{abs}} - H_{\mathrm{GOE}}\right)^{-1}W$$



- Aim: Find a simple (empirical?) parametrisation of $P(Z_{ij})$
- Try: Student's-t Distribution for *Real* and *Imaginary* part

G. Gradoni, T. M. Antonsen, E. Ott, *PRE* **86**, 046204, (2012)

- Simpler Case: $N \times 1$ / $1 \times N$ for various Γ_{abs}



- Assumption for *Transfer Function* $\mathcal{H}_t = Z_{\mathrm{RT}}$
- Allows for Analytical Treatment in $X := \mathcal{H}_t^{\dagger}\mathcal{H}_t$

$$\begin{aligned} X &= \sum_{j=1}^{N} |h_j^{\mathrm{re}}|^2 + |h_j^{\mathrm{im}}|^2 \\ &= s^{\mathrm{re}\,2}\sum_{j=1}^{N}|t_j^{\mathrm{re}}|^2 + s^{\mathrm{im}\,2}\sum_{j=1}^{N}|t_j^{\mathrm{im}}|^2 \\ &= s^{\mathrm{re}\,2}\sum_{j=1}^{N} f_j^{\mathrm{re}} + s^{\mathrm{im}\,2}\sum_{j=1}^{N} f_j^{\mathrm{im}}. \end{aligned}$$

where:
h_i – Gaussian
t_i – Student's-t
f_i – Fisher

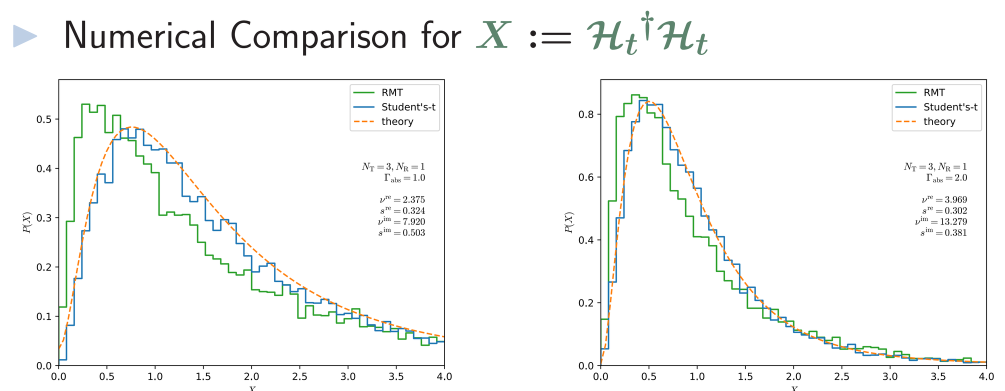
- Distribution of X follows via *characteristic function*

$$p_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}s\ \mathrm{e}^{-\mathrm{i}sx}\varphi_X(s)$$

$$\varphi_X(t) = \varphi_f(s^{\mathrm{re}\,2}\cdot t)^N \cdot \varphi_f(s^{\mathrm{im}\,2}\cdot t)^N$$

$$\varphi_f(s) = \frac{\Gamma\left(\frac{1+\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} U\left(\frac{1}{2}, 1 - \frac{\nu}{2}, -\mathrm{i}\nu s\right)$$

- Numerical Comparison for $X := \mathcal{H}_t^{\dagger}\mathcal{H}_t$



- *Mutual Information* follows

$$P(I) = \frac{2^I \ln 2}{\rho} p_X\left(\frac{2^I - 1}{\rho}\right)$$

Phase Rigidity for Low Losses

- *Phase Rigidity* $|\rho|$ of a Wave Function

$$\rho = |\rho|\mathrm{e}^{\mathrm{i}\varphi} = \frac{\langle\psi^2\rangle}{\langle|\psi|^2\rangle}$$

- Known: *Intensity Distribution* for Open Quantum Systems

$$P(I;\rho) = \frac{1}{\sqrt{1-|\rho|^2}}\mathrm{e}^{\left(-\frac{I}{1-|\rho|^2}\right)} I_0\left(\frac{|\rho| I}{1-|\rho|^2}\right)$$

And therefore

$$P(I) = \int_0^1 \mathrm{d}\rho P_\rho(\rho)\cdot P(I;|\rho|)$$

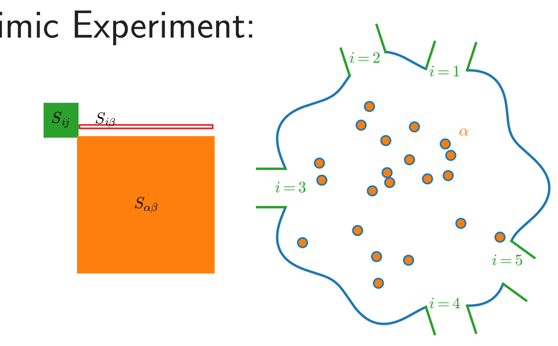
J. B. Gros, U. Kuhl, O. Legrand, F. Mortessagne, E. Richalot *IEEE MetroAeroSpace*, 225, (2015)

- Aim: Test whether the following holds:
Can we split $P_{\Gamma_{\mathrm{abs}}}(Z_{ij})$ into Γ_{abs}-independent contributions?

$$P_{\Gamma_{\mathrm{abs}}}(Z_{ij}) = \int_0^1 \mathrm{d}|\rho|, P(Z_{ij};|\rho|)\cdot P_{\Gamma_{\mathrm{abs}}}(|\rho|)$$

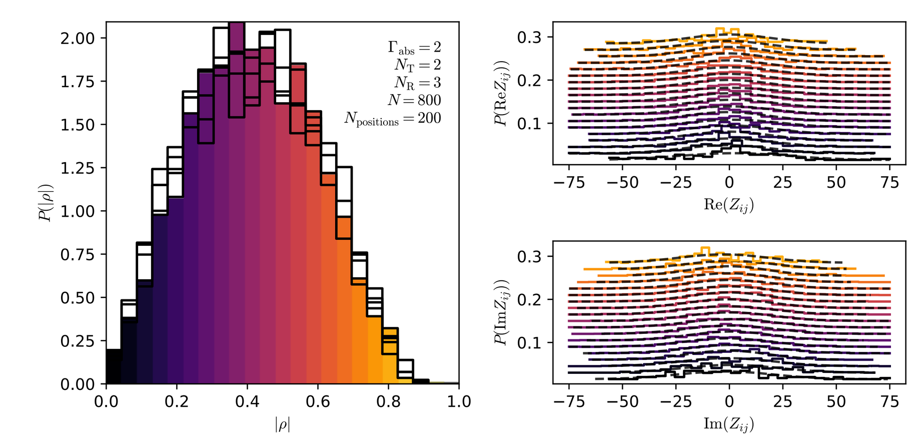
- How to define ρ for $P(Z)$? Mimic Experiment:

Introduce *weakly coupled* internal channels,
off-diagonal $S_{i\beta} \sim \psi$
Calculate ρ for each realization of S (and Z)

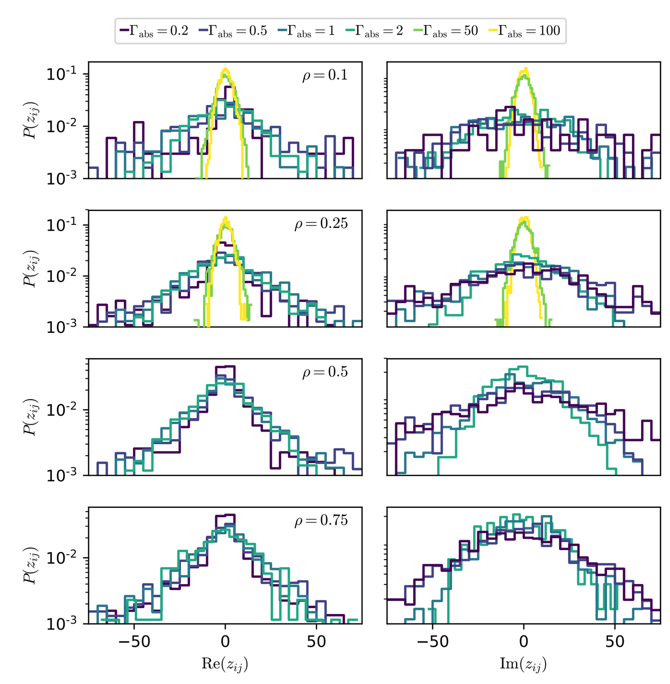


Phase Rigidities For Impedances

- Distribution of ρ for Scattering Systems:
Different Distributions of $P(Z_{ij};|\rho|)$

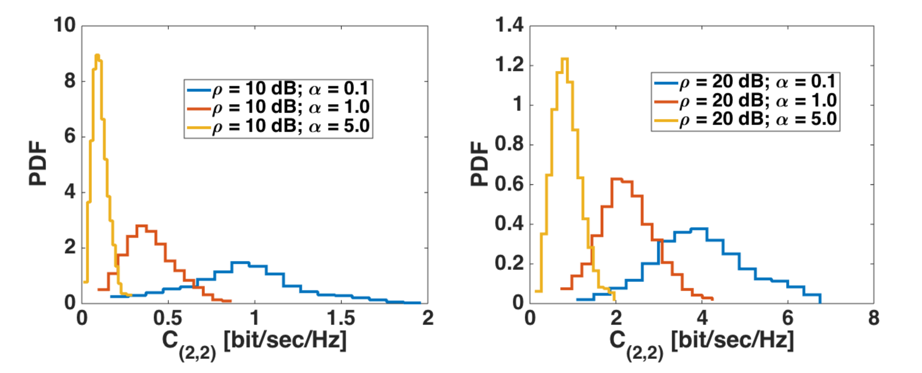


- Independence of $P(Z)$ from Γ_{abs} for fixed ρ?
Yes, **only** for $\Gamma_{\mathrm{abs}} < 1$



Outlook $N \times M$

- Enclosure with high-losses: Extension of know results of mutual information PDF for uncorrelated antenna arrays to include α. Closed form expressions combining antenna radiation, enclosure losses, and array correlation.
- Enclosure with low/intermediate losses: Judicious formulation of ansatz informed by Monte Carlo RCM/EH simulations of the mutual information PDF.



- Use *Empirical Ansatzes* and approaches known from experiments can shed light.