TRAPPED FERMIONS AND RMT MODELS

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NON-INTERACTING FERMIONS IN $d=1$ AT $T=0$

N non-interacting fermions: $\hat{\mathcal{H}}_N = \sum_{n=1}^{N} \hat{H}_n$

Single-particle wave functions

$$\hat{H}\phi_k(x) = -\frac{1}{2}\partial_x^2\phi_k(x) + V(x)\phi_k(x) = \epsilon_k\phi_k(x)\ ,\ \int dx\phi_k(x)\phi_l(x) = \delta_{k,l}$$

At $T=0$, only occupied levels $\epsilon_1 \le \epsilon_2 \le \cdots \epsilon_N = \mu$

$$|\Psi_0(x_1,\cdots,x_N)|^2 = \frac{1}{N!}\left|\det_{1\le i,j\le N}\phi_j(x_i)\right|^2 = \frac{1}{N!}\det_{1\le i,j\le N} K_\mu(x_i,x_j)$$

$\Rightarrow$ Determinantal Point Process (valid for any $d\ge 1$ at $T=0$) [1]

$$K_\mu(x,y) = \sum_{k=1}^{N}\phi_k^*(x)\phi_k(y) = \sum_k \phi_k^*(x)\phi_k(y)\Theta(\mu-\epsilon_k)$$

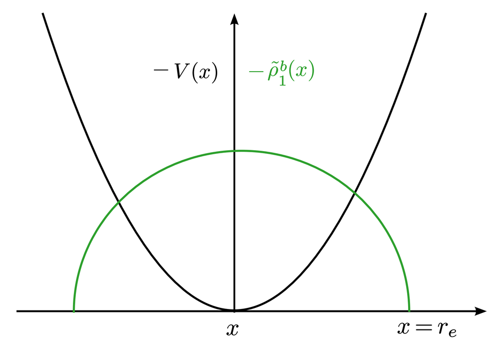
Mean density of fermions [1]

$$\tilde{\rho}(x) = \frac{1}{N}\langle\sum_k \delta(x-x_k)\rangle = \frac{1}{N}K_\mu(x,x) \to \tilde{\rho}_1^{\rm b}(x) = \frac{1}{\pi}\sqrt{2(\mu-V(x))}$$

Soft edges

$V(r_e)=\mu$ for $r_e<\infty$

$\Rightarrow$ Continuous $\tilde{\rho}_1^{\rm b}(x)$

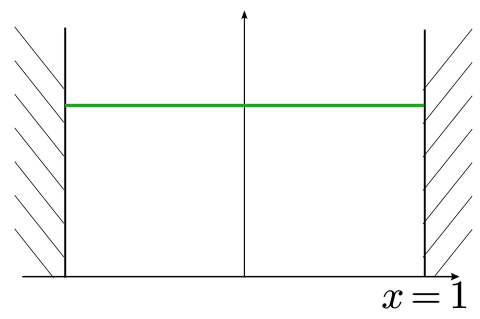


VS

Hard edges

$V(R)\to\infty$ for $R<\infty$

$\Rightarrow$ Disontinuous $\tilde{\rho}_1^{\rm b}(x)$



For specific potentials $V(x)$, mapping with RMT ($\beta=2$)

$$|\Psi_0(x_1,\cdots,x_N)|^2 \Leftrightarrow P_{\rm joint}(\lambda_1,\cdots,\lambda_N) = \frac{1}{Z_N}\prod_{i<j}|\lambda_i-\lambda_j|^2\prod_k w(\lambda_k)$$

Exact mapping

GUE AND SOFT EDGE

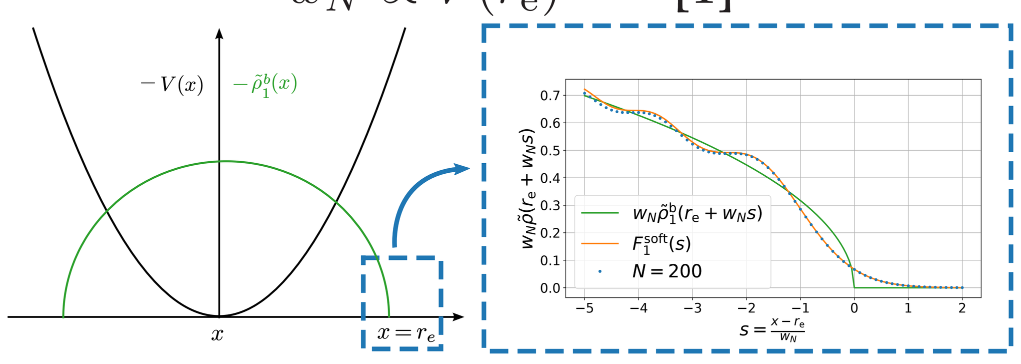
$V(x)=\frac{x^2}{2} \Leftrightarrow w(\lambda)=e^{-\lambda^2}$

$x_i \Leftrightarrow \lambda_i$

At the soft edge, typical scale $w_N \sim N^{-1/6}$

$$K_1^{\rm soft}(u,v) = \frac{\mathrm{Ai}(u)\,\mathrm{Ai}'(v) - \mathrm{Ai}'(u)\,\mathrm{Ai}(v)}{u-v}$$

The behaviour is Universal for smooth potentials $V(x)\sim x^p$ with a scale $w_N \propto V(r_e)^{-1/3}$ [1]



LUE AND HARD EDGE

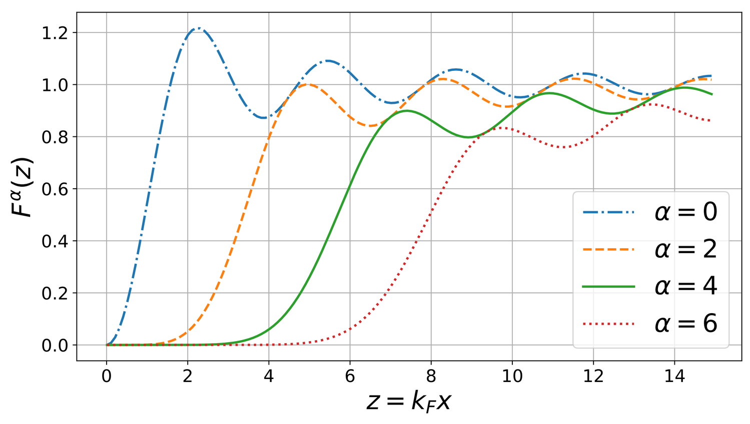
$V(x)=\frac{x^2}{2}+\frac{\alpha(\alpha+1)}{2x^2}\ ,\ x>0 \Leftrightarrow w(\lambda)=\lambda^\alpha e^{-\lambda}$

$x_i^2 \Leftrightarrow \lambda_i$

At the hard edge, $\nu=\alpha+1/2$

$$K_J^\nu(u,v) = \sqrt{uv}\,\frac{u\,J_{\nu+1}(u)\,J_\nu(v) - v\,J_{\nu+1}(v)\,J_\nu(u)}{u^2-v^2}$$

$\Leftarrow \alpha\to\infty$ $\qquad$ $\alpha\to 0 \Rightarrow$



JUE AND HARD EDGE

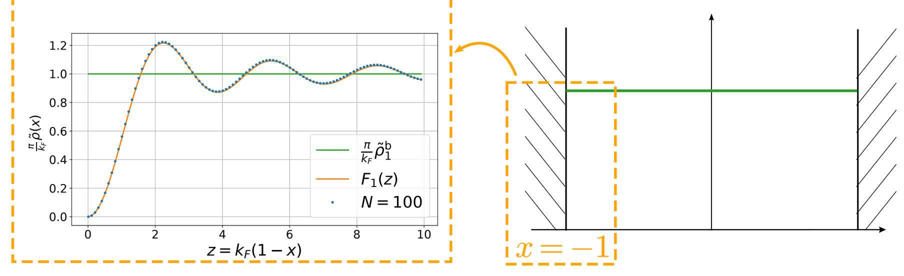
$$V(x) = \begin{cases} 0\ ,\ |x|\le 1 \\ +\infty \text{ otherwise}\end{cases} \Leftrightarrow w(\lambda)=\sqrt{\lambda(1-\lambda)}$$

$\frac{1}{2}+\frac{1}{2}\sin\left(\frac{\pi x_i}{2}\right) \Leftrightarrow \lambda_i$

At the hard edge: [2]

$$K_1^{\rm e}(u,v) = \frac{\sin(u-v)}{\pi(u-v)} - \frac{\sin(u+v)}{\pi(u+v)}$$

(see also [3] for other boundary conditions)



INVERSE POWER LAW POTENTIALS

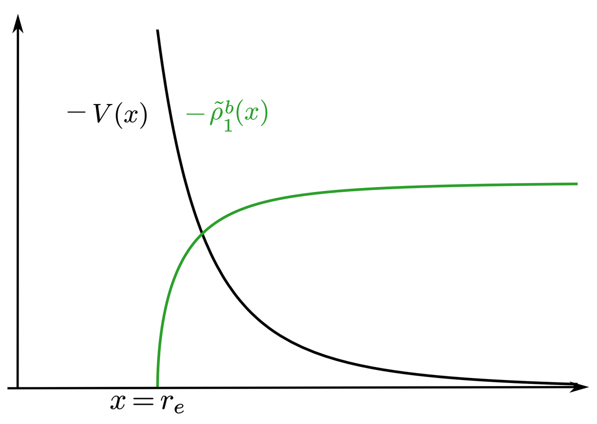
We consider the inverse power law potential $V(x)=\dfrac{\alpha(\alpha+1)}{2x^\gamma}$ with $\gamma>0$.

For $0\le\gamma<1$, the potential cannot trap N fermions.

For $\gamma\ge 1$, as the potential energy has to be finite $\langle\hat{V}\rangle = \sum_k\int_{-\infty}^{\infty}dx|\phi_k(x)|^2V(x)<\infty$, $\phi_k(0)=0$.

The behaviour at the edge depends on the value of γ: [2]

- For $1\le\gamma<2$, same behaviour as for the hard box
  $\to$ The kernel close to the hard edge in $x=0$ is $K_1^{\rm e}$
- For $\gamma=2$, critical case, same behaviour as LUE with $\nu=\alpha+1/2$
  $\to$ The kernel close to the hard edge in $x=0$ is K_J^ν
- For $\gamma>2$, a gap opens between $x=0$ and the edge of the density
  $\to$ The kernel at the edge of the density in $r_e>0$ is $K_1^{\rm soft}$



FERMIONS AND GENERAL JUE

Any JUE ensemble can be realised

$w(\lambda)=\lambda^a(1-\lambda)^b$

$$\Leftrightarrow V(x) = \frac{4a^2-1}{32\sin^2(x/2)} + \frac{4b^2-1}{32\cos^2(x/2)}\ ,\ x\in[0,\pi]$$

$\frac{1-\cos(x_i)}{2} \Leftrightarrow \lambda_i$ [2]

ROTATING FERMIONS $\to$ GINUE

In $d=2$, the complex GinUE can be realised taking the one-particle Hamiltonian [4]

$$\hat{H} = \frac{\hat{p}_x^2+\hat{p}_y^2}{2m} + \frac{1}{2}m\omega^2(\hat{x}^2+\hat{y}^2) - \Omega\hat{L}_z$$

The complex positions $z=x+iy$ of the fermions behave as GinUE eigenvalues for $0<1-\frac{\Omega}{\omega}<\frac{2}{N}$

$\to$ talk by S.N. Majumdar

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