The universality of conditional measures of the Bessel point process

By Marco Stevens – joint work with Leslie Molag

KU LEUVEN

If the location of points *outside* a compact set almost surely determine the number of points *inside* that compact set, a point process is called rigid.

After conditioning on the exterior of a compact set the induced finite point process on the compact set is called the conditional measure.

When this conditional measure converges to the original point process as one lets the compact set grow to cover the whole space, one speaks of universality.

Rigidity → **Conditional measure** → **Universality**

- Examples of rigid point processes include the sine, Airy and Bessel process [1,3]. A non-example is the Poisson process.
- The only point process for which the universality has been shown in the literature is the sine process [4].

- We study the Bessel point process (with parameter ν) , i.e. the determinantal point process governed by the Bessel kernel. Its conditional measure on $[0, R]$ is an orthogonal polynomial ensemble, with weight [2]

$$w(t) = t^{\nu} \prod_{p_n > R} (1 - \tfrac{t}{p_n})^2, \qquad 0 \leq t \leq R.$$

- The points of the Bessel process almost surely satisfy

$$p_n = \pi^2 n^2 + O\left((n \log n)^{3/2}\right), \qquad n \geq 1.$$

We study the question of universality for the Bessel point process by conducting the following:

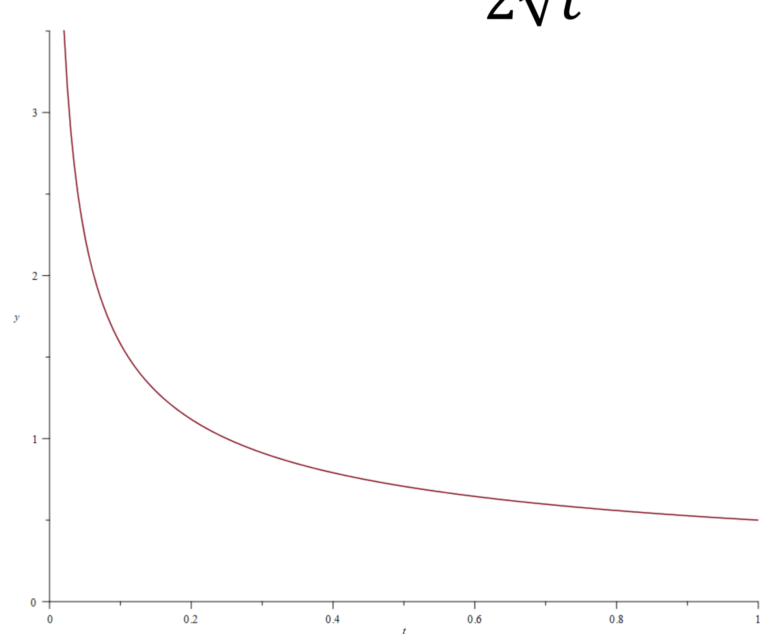
1. Rescale the weight to the interval $[0,1]$.
2. Approximate this weight by rescalings of the varying exponential weight $t^{\nu} e^{-NV(t)}$, where the external field V is $(0 \leq t \leq 1)$:

$$(1 - \sqrt{t}) \log(1 - \sqrt{t}) + (1 + \sqrt{t}) \log(1 + \sqrt{t})$$

3. Perform the Riemann-Hilbert analysis for the approximating weight to obtain the Bessel kernel.
4. Patch up the approximation to show that the original weight has the same behaviour, by using techniques from [5].

The equilibrium density for the external field V is given by $(0 < t \leq 1)$:

$$\mu(t) = \frac{1}{2\sqrt{t}}$$



In the Riemann-Hilbert problem, one needs to work around the fact that μ does not behave appropriately around $t = 1$.

References:

1. Bufetov, A. I. (2016). Rigidity of determinantal point processes with the Airy, the Bessel and the Gamma kernel. Bulletin of Mathematical Sciences, 6(1), 163-172
2. Bufetov, A. I. (2016). Conditional measures of determinantal point processes. arXiv preprint arXiv:1605.01400.
3. Ghosh, S., Peres, Y. (2017). Rigidity and tolerance in point processes: Gaussian zeros and Ginibre eigenvalues. Duke Mathematical Journal, 166(10), 1789-1858.
4. Kuijlaars, A. B., Miña-Díaz, E. (2017). Universality for conditional measures of the sine point process. arXiv preprint arXiv:1703.02349.
5. Lubinsky, D. S. (2009). A new approach to universality limits involving orthogonal polynomials. Annals of Mathematics, 915-939.