Synaptic interactions in random neural networks: critical role of eigenvectors and transients

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Continuous-time recurrent neural networks

Recurrent neural network in the limit of infinite depth can be approximated by the system of differential equations

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} W_{ij}\phi(x_j),$$

with W_{ij} a synaptic connectivity matrix and ϕ a nonlinear activation function. Sompolinsky et al. [1] considered W a random Ginibre matrix and found a transition between the stationary and chaotic dynamics, which occurs when the largest real part of an eigenvalue exceeds 1.

Rajan-Abbott model for synaptic connectivity matrix

To make this model more realistic, Rajan and Abbott considered two types of neurons: excitatory (E) and inhibitory (I) with appropriate fractions $f_{I,E}$. The synaptic strength of neurons is generated from the normal distribution $\mathcal{N}(\mu_{E,I}, \frac{\sigma^2_{E,I}}{N})$. One takes $\mu_E > 0$ and $\mu_I < 0$. The synaptix matrix can be decomposed as $W = M + G\Lambda$, where M is deterministic, G is Ginibre and Λ is diagonal with $\sigma_{I,E}$. The random part models variability in populations of neurons.

The presence of the deterministic part introduces several outliers which might induce the transition to chaos. Motivated by experiments, Rajan and Abbott introduced also the *balance condition*, which says that the sum of excitations and inhibitions incoming to neuron is balanced to 0, both on average, $\sum_j M_{ij} = 0$, and at each neuron separately, $\sum_j W_{ij} = 0$.

The balance condition tames outliers, bringing them back to the disk, but drastically increases the sensitivity of the spectrum, measured by the eigenvalue condition number

$$\kappa(\lambda_i) = ||L_i|| \cdot ||R_i||,$$

where L_i (R_i) is left (right) eigenvector to the eigenvalue λ_i.

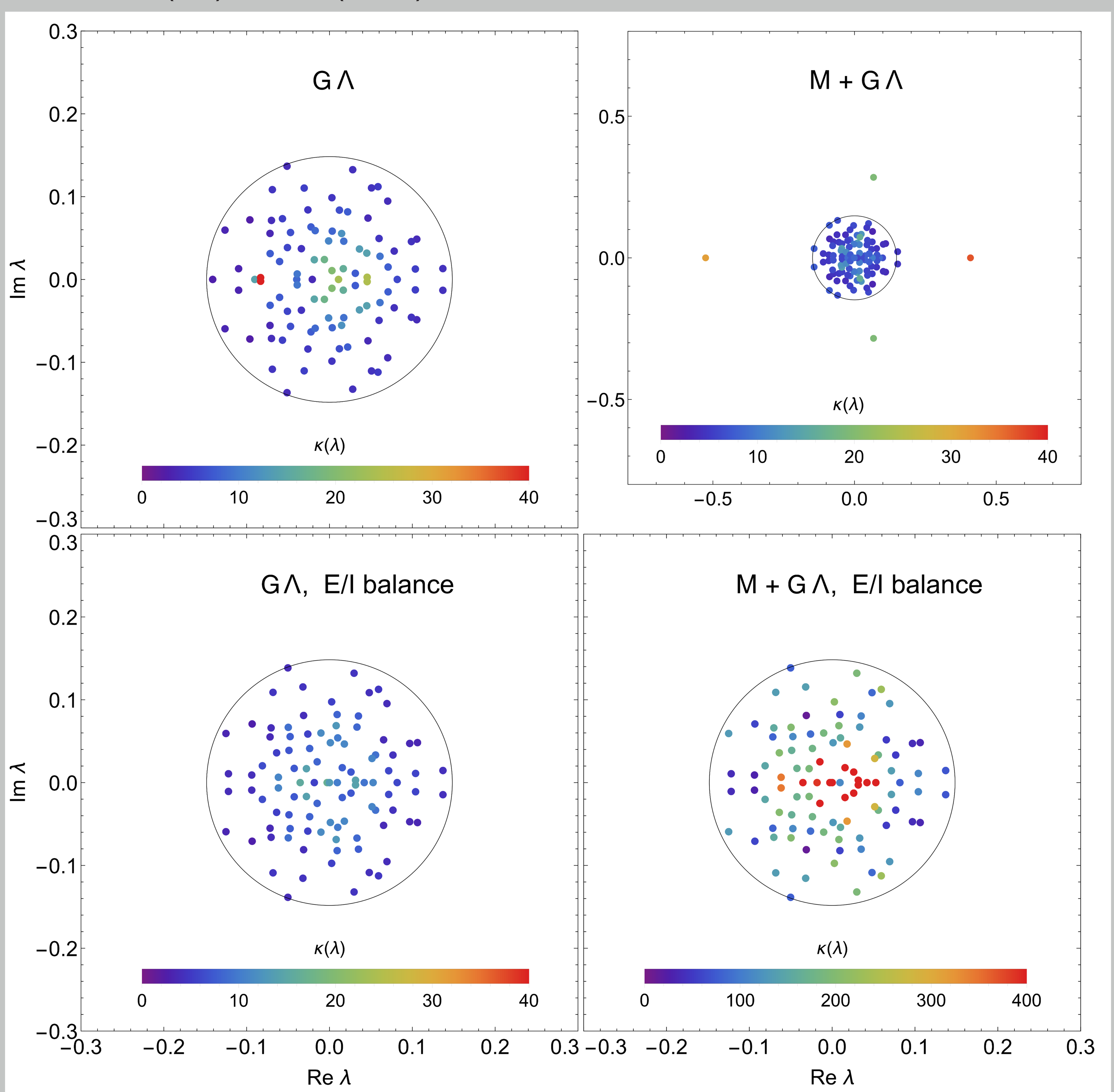

Figure 1: Eigenvalues and their condition numbers of the matrix of variances $G\Lambda$ (top left), Rajan-Abbott model of neural network (top right), matrix of variances with E/I balance imposed (bottom left) and Rajan-Abbott model with E/I balance (bottom right). The same realization of the Gaussian matrix G was taken for all plots. We used parameters $\sigma_I = 0.3$, $\sigma_E = 0.1$, $f_I = 0.15$, $f_E = 0.85$, $\mu_I = -0.85$, $\mu_E = 0.15$. The matrix is $N = 100$ in size.

References

[1] H. Sompolinsky, A. Chrisanti, H.J. Sommers, *Chaos in Random Neural Networks*, PRL **61** 259 (1988).

[2] K. Rajan, L.F. Abbott, *Eigenvalue Spectra of Random Matrices for Neural Networks* PRL **97**, 188104 (2006).

[3] E. Gudowska-Nowak, M. A. Nowak, D.R. Chialvo, J. K. Ochab, and W. Tarnowski *From synaptic interactions to collective dynamics in random neuronal networks models: critical role of eigenvectors and transient behavior* [ArXiv:1805.03592v1].

Linearized dynamics close to the fixed point x=0

The point $x = 0$ is a fixed point of the full nonlinear dynamics and the transition to chaos is associated with its instability. Assuming stability of the fixed point, we linearize equations to $\dot{x}_i = -x_i + \sum_j W_{ij}x_j$.

The relaxation towards the fixed point can be analyzed by considering the squared Euclidean distance from it, $D(t) = ||x(t)||^2$. Since equations are linear, one can write this explicitly

$$D(t) = \sum_{ij=1}^{N} \langle x_0|L_i\rangle \langle R_i|R_j\rangle \langle L_j|x_0\rangle \, e^{-2t+t(\lambda_i+\lambda_j)}.$$

There are 3 important phenomena influencing the dynamics: exponential decay of modes, which dominates late-time dynamics; projection of initial condition onto the set of non-orthogonal eigenmodes may result in the amplification of the modes, transiently overcoming the exponential decay; non-orthogonality of eigenmodes mixes them together, resulting in oscillatory behavior of $D(t)$ ('interference effects'). For normal matrices only the first effect exists.

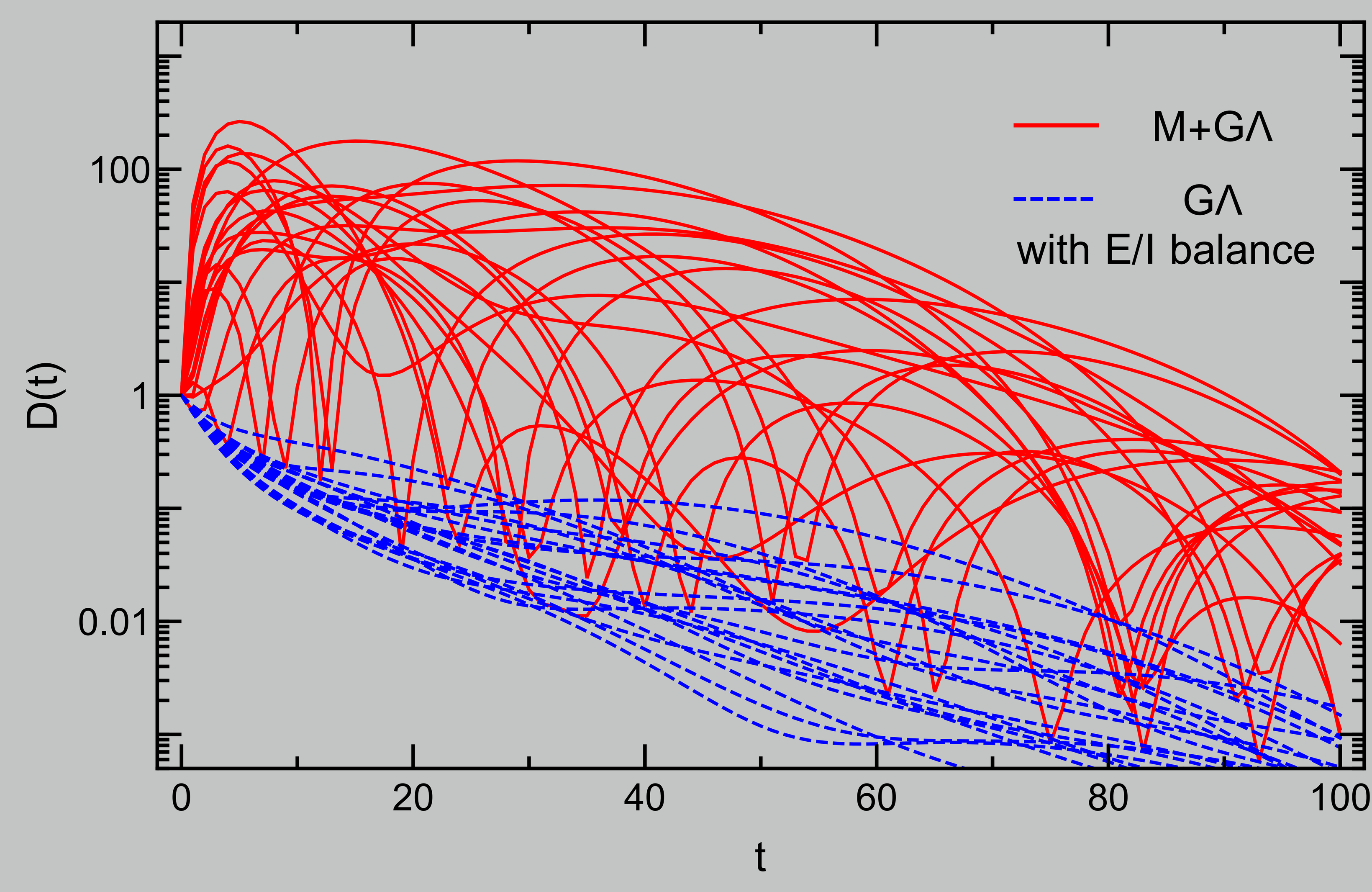

Figure 2: Squared Euclidean distance from the fixed point in the linearized dynamics. The presence of the mean synaptic strength induces strong transient behavior and wild oscillations. These effects are caused by the strong non-normality. Parameters as in Fig. 1. Each curve corresponds to a single initial condition generated randomly from the set of vectors of unit norm.

Synchronized activity

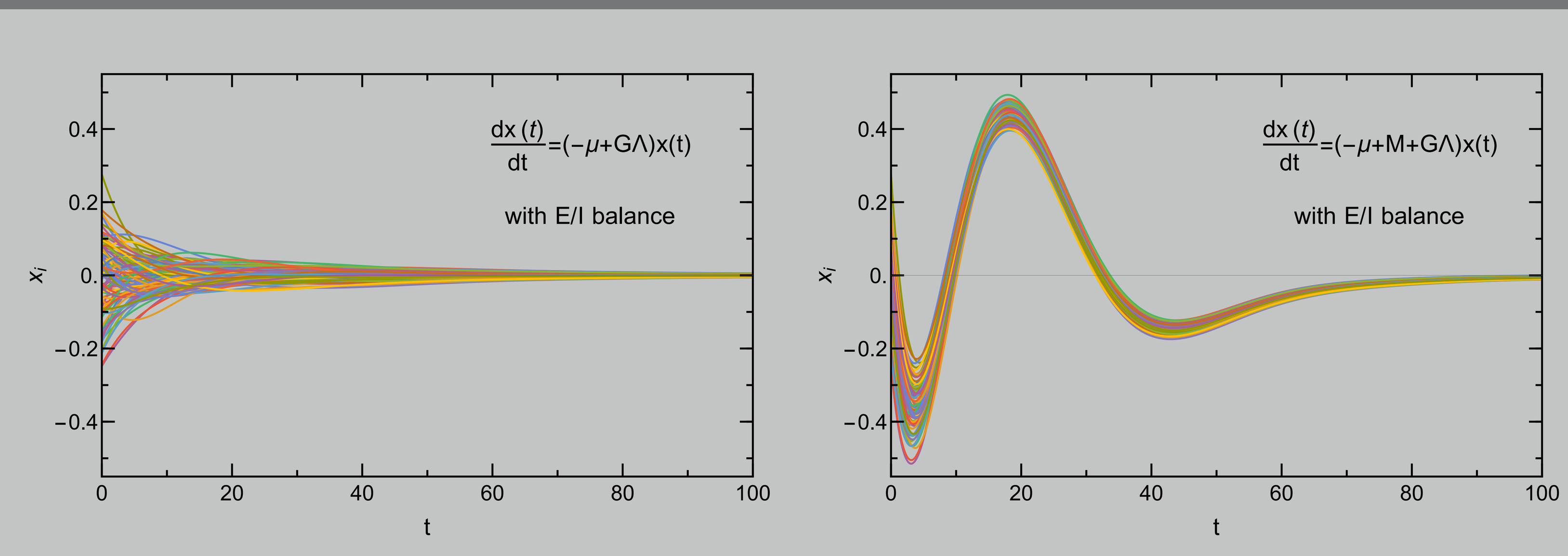

Figure 3: Activity of each neuron in the linearized dynamics. In the right panel we see the onset of collective dynamics driven by the matrix M and the balance condition. Both simulations started from exactly the same initial condition randomly chosen from the N-dimensional hypersphere. Parameters are the same as in Fig. 1.

Conclusions

- Although deterministic patterns in the connectivity matrix do not influence its spectrum, provided that the E/I balance is maintained, they significantly increase susceptibility of the spectrum to perturbations and amplify transient modes.
- Mean connectivity is responsible for the synchronization in the neural activity.

Acknowledgments

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