Superconducting RMT ensembles: recent progress and applications

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Outline

1. Introduction
   - Chiral *p*-wave superconductors
   - Edge states: detection, Majorana fermions
2. Domain wall in chiral *p*-wave superconductors
   - Charge transport properties - RMT Pt1
   - Thermal transport properties - RMT Pt2
3. Summary and outlook

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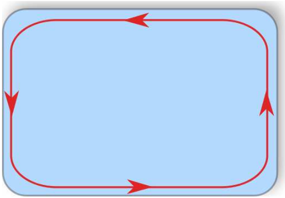
Chiral *p*-wave superconductors



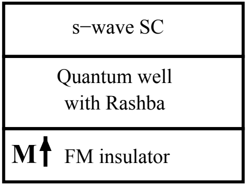
Chiral *p*-wave superconductor

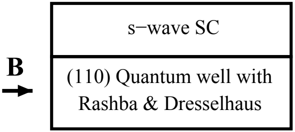
- 2D, TRS breaking
- Single chiral edge state

Examples (solid state)

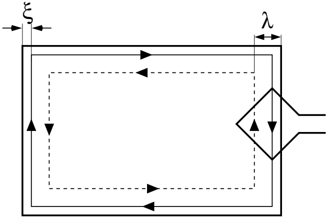
- Sr_2RuO_4
- Semiconductor realization

J. Sau *et al.* PRL 2010, J. Alicea PRB 2010





Edge states: detection difficulties



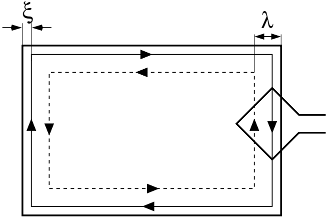
- Edge states carry equilibrium current

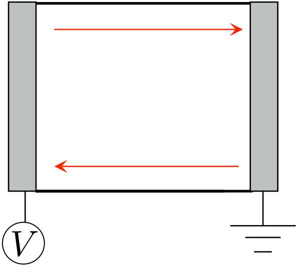
$\Rightarrow$ Measure magnetic field

Difficulty: Meissner effect

Kwon *et al.* Synth. Met. 2003, Kirtley *et al.* PRB 2007

Edge states: detection difficulties





- Edge states carry equilibrium current

$\Rightarrow$ Measure magnetic field

Difficulty: Meissner effect

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- Detect through charge transport?

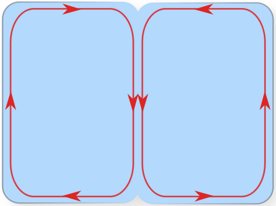
EH symmetry: $\sigma_1^{eh} H_{-k}^{*} \sigma_1^{eh} = -H_k$

$\Rightarrow$ $\boxed{\sigma_1^{eh} K |a\rangle = |a\rangle}$ Majorana mode

"neutral" excitation, won’t work

Domain wall configuration

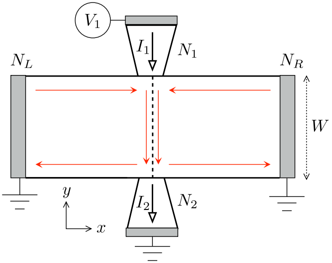
I. Serban, BB, A.R. Akhmerov, C.W.J. Beenakker PRL 2010

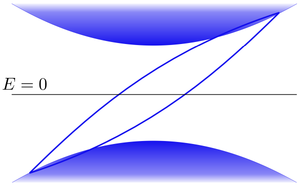


Two chiral modes in the DW

- can combine $|a_1\rangle$,$|a_2\rangle$ into non-neutral:
  $|v\rangle = \alpha|a_1\rangle + \beta|a_2\rangle$
  $\sigma_1^{eh}K\ |v\rangle = \alpha^*|a_1\rangle + \beta^*|a_2\rangle \neq |v\rangle$

Conductance setup





Conductance from Landauer-Büttiker formula

$$t = \begin{pmatrix} t_{ee} & t_{eh} \\ t_{he} & t_{hh} \end{pmatrix}$$

$$G = \frac{e^2}{h}[\mathrm{Tr}\ t_{ee}^\dagger t_{ee} - \mathrm{Tr}\ t_{he}^\dagger t_{he}]$$

$$t = T_2 t_{DW} T_1$$

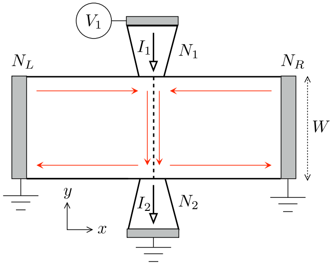
DW eigenmodes Ψ_1, Ψ_2,

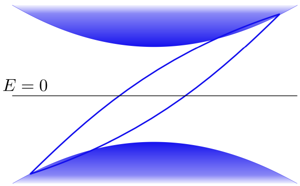
$$\Psi_2 = \sigma_1^{eh} K \Psi_1 \ \Rightarrow\ \Psi_1 = \text{"}e\text{"},\ \Psi_2 = \text{"}h\text{"}$$

$$T_j = \begin{pmatrix} T^j_{ee} & T^j_{eh} \\ T^{j*}_{eh} & T^{j*}_{ee} \end{pmatrix}$$

(rels due to EH symmetry)

Conductance setup





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$$T_j = \begin{pmatrix} T_{ee}^{j} & T_{eh}^{j} \\ T_{eh}^{j*} & T_{ee}^{j*} \end{pmatrix} \quad t_{DW} = \begin{pmatrix} e^{i\varphi} & \\ & e^{-i\varphi} \end{pmatrix}$$

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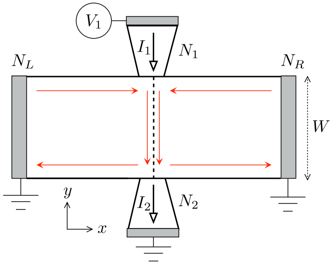

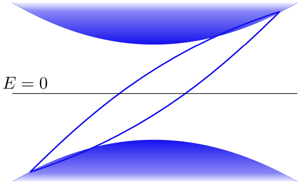

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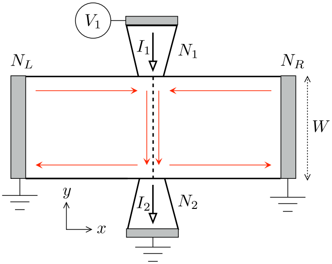
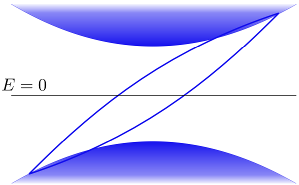
Charge transport: product rule





Conductance from Landauer-Büttiker formula

$$\boxed{G = \frac{e^2}{h}(-1)^p g_1 g_2}$$

$$g_j = \mathrm{Tr}\ T_{ee}^{j\dagger} T_{ee}^{j} - \mathrm{Tr}\ T_{he}^{j\dagger} T_{he}^{j}$$

- Vanishes upon moving contact
- Vanishes upon inverting polarity
- No definite sign

Charge transport: Effects of disorder

Some values for G?

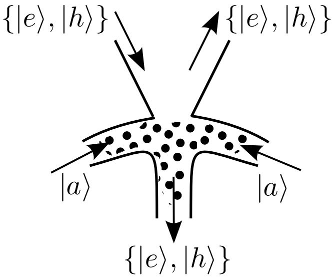
$$\boxed{G = \frac{e^2}{h}(-1)^p g_1 g_2} \qquad g_j = g(T_j),\ S_j = \begin{pmatrix} R_j & T_j' \\ T_j & R_j' \end{pmatrix}$$

Charge transport: Effects of disorder

Some values for G?

$$\boxed{G = \frac{e^2}{h}(-1)^p g_1 g_2}$$

$$g_j = g(T_j),\ S_j = \begin{pmatrix} R_j & T'_j \\ T_j & R'_j \end{pmatrix}$$



- Contact area: disordered normal metal
- Model: S_j uniformly distributed on

$$\mathcal{M}_D = \{S \in U(N) | \sigma_1^{eh} S^* \sigma_1^{eh} = S\}$$

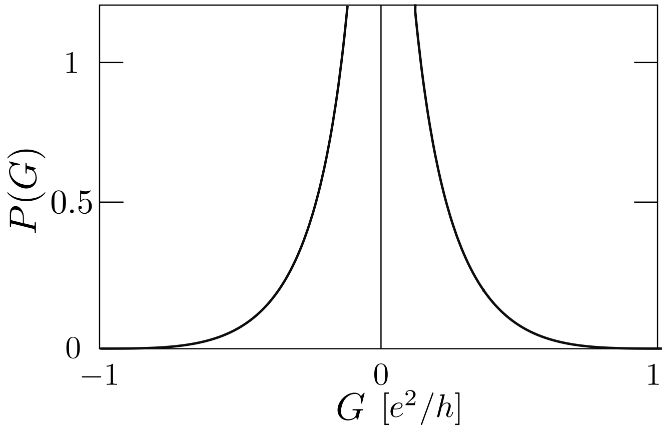
(uniform wrt invariant measure)

Assume single mode contacts $\Rightarrow N = 4$

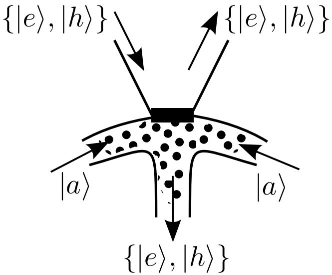
$$P(g_j) = 1 - |g_j|, \qquad |g_j| \leq 1 \qquad \text{("tent")}$$

$$P(G) = \int_{-1}^{1} dg_1 dg_2 \delta[G - g_1 g_2] P(g_1) P(g_2) = 2[2(|G|-1)-(1+|G|)\log|G|]$$

$$\langle G \rangle = 0 \qquad \sqrt{\langle G^2 \rangle} = \langle g^2 \rangle = 1/6$$



Tunnel barriers: Poisson kernel



Direct processes due to tunnel barrier: via nonrandom S_T

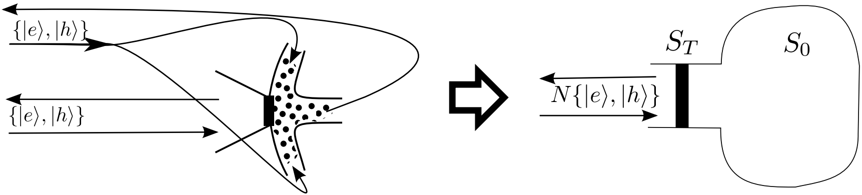
$$S \in \mathcal{M}_D, \quad S = S_T \otimes S_0$$

$S_0 \in \mathcal{M}_D$ unif distributed
$S_T \in \mathcal{M}_D$ nonrandom

$$P(S) = ?$$

Tunnel barriers: Poisson kernel

Setup as a part of a more general scheme



$P(S)$: calculate Jacobian of

$$S = r + t' S_0 (1 - r' S_0)^{-1} t \qquad S_T = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

following logic of Friedman, Mello Ann. Phys. 1985

Tunnel barriers: Poisson kernel

Main steps (Friedman, Mello Ann. Phys. 1985):

1)Inv. arclength and measure on $\mathcal{M}$: $S^\dagger dS \to \delta S$, s.t.

$$ds^2 = \mathrm{Tr}[dSdS^\dagger] = \sum_{indep.} (\delta S)^2_{kl} \Rightarrow d\mu \propto \prod_{indep.} \delta S_{kl}$$

2)Relate δS and δS_0

$$S(S_T, S_0) \Rightarrow \delta S = M(S_T, S)\delta S_0 M^\dagger(S_T, S)$$

3)Express arclength at S via δS_0

$$ds^2(S) = \sum g(S)_{ij,kl}\delta S_{0ij}\delta S_{0kl}$$

$$\Rightarrow d\mu(S) = |\det g(S)|^{1/2} \prod \delta S_{0kl} \propto |\det g(S)|^{1/2} d\mu(S_0)$$

4)Assign probability

$$P(S)d\mu(S) = d\mu(S_0) \Rightarrow P(S) \propto |\det g(S)|^{-1/2}$$

Tunnel barriers: Poisson kernel

Scheme can be adapted to include symmetries.
Symmetry classification for superconducting systems:

Altland&Zirnbauer PRB 1997

Symmetry class	D	DIII	C	CI
EH symmetry	$S=\sigma_1^{eh}S^*\sigma_1^{eh}$	$S=\sigma_1^{eh}S^*\sigma_1^{eh}$	$S=\sigma_2^{eh}S^*\tau_2$	$S=\tau_2S^*\tau_2$
TRS	$\times$	$S=\sigma_2S^T\sigma_2$	$\times$	$S=S^T$
SRS	$\times$	$\times$	$\checkmark$	$\checkmark$
t	1	2	1	2
s	-1	-1	1	1

Carried out this program for all 4 classes (BB, PRB 2009)

$$\boxed{P(S)\propto|\det(1-r^\dagger S)|^{-(N/t+s)}} \qquad S_T=\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

Tunnel barriers: Poisson kernel

Compare with normal case

(Friedman& Mello Ann. Phys 1985, Brouwer PRB 1995)

$$P_\beta(S,r) \propto |\det(1-r^\dagger S)|^{-(\beta N+2-\beta)}$$

	TRS&SRS	TRS&~~SRS~~	~~TRS~~
β	1	4	2

$$\langle S^p\rangle = \langle S\rangle^p = r^p \Rightarrow \int P_\beta(S,r)V(S) = V(r) \Rightarrow \text{"Poisson Kernel"}$$

Superconductor case:

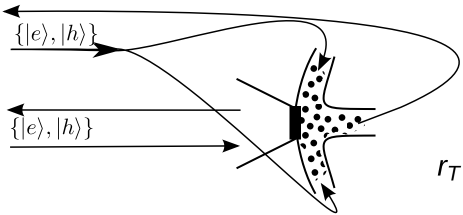
$$P_{t,s}(S,r) \propto |\det(1-r^\dagger S)|^{-(N/t+s)}$$

	Yes	No
TRS, $t =$	2	1
SRS, $s =$	1	-1

$$\langle S^p\rangle \neq \langle S\rangle^p \Rightarrow \text{Not a true PK}$$

Tunnel barriers: Poisson kernel

Apply Poisson kernel to domain wall problem.



$$N=4, t=1, s=-1$$

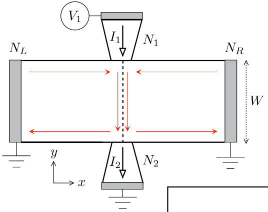
$$\Rightarrow P(S) \propto |\det(1-r_T^\dagger S)|^{-3}$$

$$r_T = r_c \oplus 0 \quad r_c = \begin{pmatrix} \sqrt{1-\Gamma} & \sqrt{\Gamma} \\ -\sqrt{\Gamma} & \sqrt{1-\Gamma} \end{pmatrix}$$

$$P(g_j,\Gamma) = \langle \delta[g_j - g(S)] \rangle = \frac{\Gamma^2}{[\Gamma+(1-\Gamma)|g_j|]^3} - \frac{\Gamma^2 |g_j|}{[\Gamma+(1-\Gamma)g_j^2]^2}$$

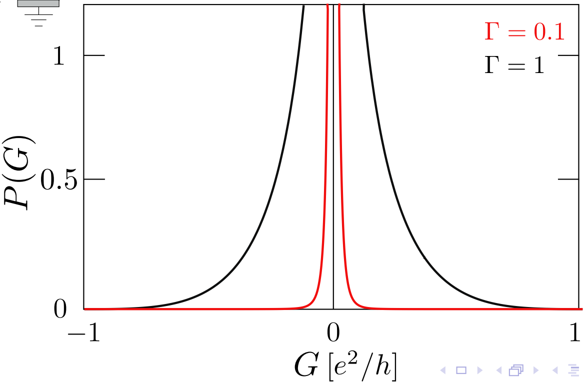
(Check: $\quad P(g_j,\Gamma\to 1) = 1-|g_j|$)

Domain wall conductance vs Γ

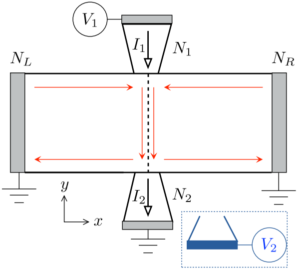


$$G = \frac{e^2}{h}(-1)^p g_1 g_2$$

$$P_\Gamma(G) = \int dg_1 dg_2 P_\Gamma(g_1) P_\Gamma(g_2)\delta[G - g_1 g_2]$$



Tunnel barriers: nonlocal resistance



$$I_1 = G_{11} V_1 \qquad (1)$$

$$0 = G_{22} V_2 + G_{21} V_1 \qquad (2)$$

$$\Rightarrow R_{21} = \frac{V_2}{I_1} = -\frac{G_{21}}{G_{11} G_{22}}$$

$$G = (-1)^p g_1 g_2 \Rightarrow \boxed{R_{21} = (-1)^p r_1 r_2}$$

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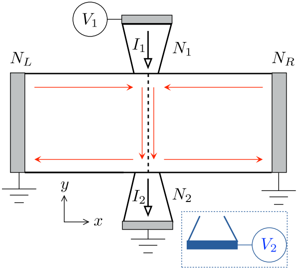

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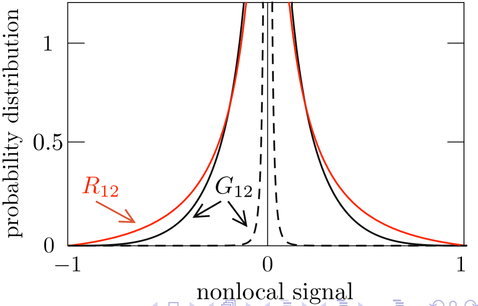
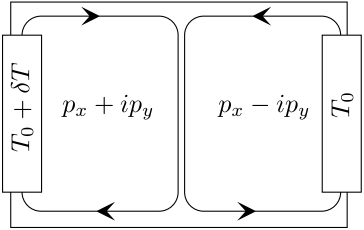
$$G = (-1)^p g_1 g_2 \Rightarrow \boxed{R_{21} = (-1)^p r_1 r_2}$$

Single-mode contact:
S_T drops from r_j

$$P(r_j) = \frac{1}{(1+|r_j|)^2} \qquad (|r_j| \leq 1)$$



Thermal transport: Transmission eigenvalues



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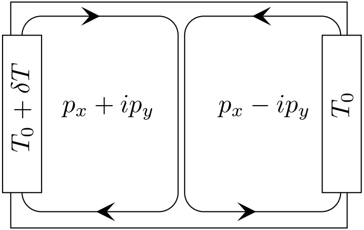

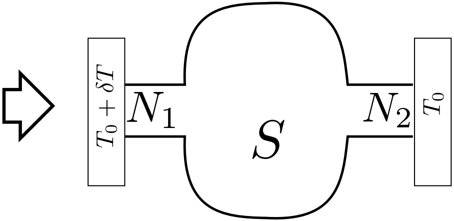

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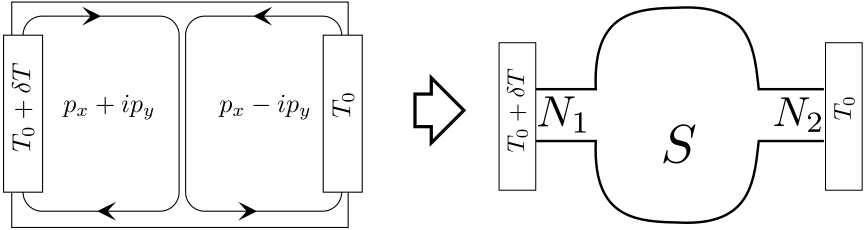

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \in \mathcal{M} \Rightarrow G = G_0 \, Tr(t^\dagger t) = G_0 \sum_n T_n$$

$G_0 = \pi^2 k_B^2 T_0 / 6h$ $\quad \{T_n\} = \mathrm{Eig}(t^\dagger t)$

$$\langle G^p \rangle = ? \quad \Rightarrow \quad P(\{T_n\}) = ?$$

Distribution of transmission eigenvalues

S unif. distr $\Rightarrow P(\{T_n\})$: main steps (Baranger&Mello PRL 1994)

1) Polar decomposition of S

$$\begin{pmatrix} r & t' \\ t & r' \end{pmatrix} = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} \begin{pmatrix} \sqrt{1-\Lambda\Lambda^T} & \frac{i\Lambda}{\sqrt{1-\Lambda^T\Lambda}} \\ i\Lambda^T & \end{pmatrix} \begin{pmatrix} V_1^\dagger & 0 \\ 0 & V_2^\dagger \end{pmatrix} \quad \Lambda_{jk} = \sqrt{T_j}\delta_{jk}$$

2) Invariant arclength and measure:

$$ds^2 = \mathrm{Tr}(dSdS^\dagger) = \sum_{kl} g_{kl}dx_k dx_l, \; \{x_l\} = \{\{T_j\},\{\theta_j\}\}$$

$$d\mu(S) = f(\{T_l\}) \prod_l dT_l \prod_j d\mu(U_j)$$

3) Unif distr. wrt $d\mu(S) \Rightarrow P(\{T_j\}) = f(\{T_l\})$

Distribution of transmission eigenvalues

J.P. Dahlhaus, BB, C.W.J. Beenakker PRB 2010

$$\boxed{\begin{aligned} P(\{T_n\}) \propto \prod_i T_i^{(\beta/2)(N_1 - N_2)} T_i^{-1+\beta/2} \\ \times (1 - T_i)^{\gamma/2} \prod_{j<k} |T_k - T_j|^{\beta} \end{aligned}}$$

Symmetry class	D	DIII	C	CI
TRS	$\times$	$\checkmark$	$\times$	$\checkmark$
SRS	$\times$	$\times$	$\checkmark$	$\checkmark$
β	1	2	4	2
γ	-1	-1	2	1
$d(T_n)$	1	2	4	4

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Compare to normal case:

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Symmetry class	A	AI	AII
TRS	$\times$	$\checkmark$	$\checkmark$
SRS	$\times$ or $\checkmark$	$\checkmark$	$\times$
β	2	1	4
$d(T_n)$	1 or 2	2	2

Distribution of transmission eigenvalues

J.P. Dahlhaus, BB, C.W.J. Beenakker PRB 2010

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Use Selberg integral (Savin&Sommers PRB 2006)

$$\langle G/G_0 \rangle = \frac{N_1 N_2}{N_\mathrm{t} + \xi} \qquad \mathrm{Var}\, G/G_0 = \frac{2N_1N_2(N_1+\xi)(N_2+\xi)}{\beta(N_\mathrm{t}-1+\xi)(N_\mathrm{t}+\xi)^2(N_\mathrm{t}+\xi+2/\beta)}$$

$$N_t = N_1 + N_2 \qquad \xi = (2-\beta+\gamma)/\beta$$

Distribution of transmission eigenvalues

J.P. Dahlhaus, BB, C.W.J. Beenakker PRB 2010

$$\boxed{P(\{T_n\}) \propto \prod_i T_i^{(\beta/2)(N_1-N_2)} T_i^{-1+\beta/2} \times (1-T_i)^{\gamma/2} \prod_{j<k} |T_k - T_j|^\beta}$$

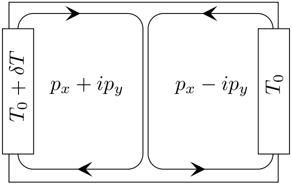
Symmetry class	D	DIII	C	CI
TRS	$\times$	$\checkmark$	$\times$	$\checkmark$
SRS	$\times$	$\times$	$\checkmark$	$\checkmark$
β	1	2	4	2
γ	-1	-1	2	1
$d(T_n)$	1	2	4	4

Large N limit $(N_1 = N_2)$

$$\delta G = \begin{cases} 0 & \text{TRS broken,} \\ -G_0/2 & \text{TRS, SRS,} \\ G_0/4 & \text{TRS } \cancel{\text{SRS}}. \end{cases} \qquad \mathrm{var}G = \frac{G_0^2}{p}$$

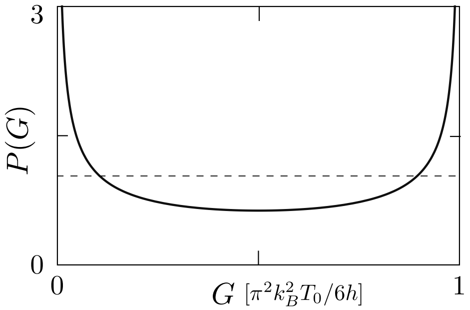
Symm	D	DIII	C	CI
p	8	4	2	1

Domain wall: thermal conductance distro



DW as minimal setup: Single T
$\Rightarrow$ have full $P(G)$:

$$P(G) = \frac{1}{\pi}\frac{1}{\sqrt{G}}\frac{1}{\sqrt{1-G}}$$



Summary

- Domain wall: channel for charge current
  - signatures of edge states in charge transport
  - outlook: effect of domain wall networks?
- Tunnel barriers in superconducting RMT
  - DW: narrowing of $P(G)$, insensitivity of $P(R)$
  - outlook: dephasing in Andreev QDs (BB PRB 2009)
- obtained $P(\{T_n\})$
  - DW: bimodal G_T distro
  - outlook: connection to integrable systems? (Osipov&Kanzieper PRL 2008)

Thank you for your attention!