Optimal Bank Regulation In the Presence of Credit and Run Risk

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June 2019

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Motivation

- Financial intermediaries perform various socially useful functions
- Both assets and liabilities are critical to delivering these services
- However, the balance sheet structure can also be a source of fragility
- We present a model featuring these interactions, study the externalities emerging from intermediation and examine regulation to mitigate their effects

Our framework

We modify the classic Diamond-Dybvig model such that banks:

- Provide liquidity and monitoring services
- Are funded by deposits and equity
- Make risky loans, hold liquidity and are subject to limited liability
- Face endogenous run risk determined by a global game
    - Akin to Goldstein and Pauzner (2005), but with a trigger based on uncertain liquidation values for loans

The economy

t = 1

- Entrepreneurs (E) borrow to invest in long-term, illiquid and risky projects
- Savers (S) invest in demandable bank deposits
- Bankers (B) raise equity and deposits to invest in risky loans and liquid safe assets

t = 2

- Each saver learns whether she is impatient or patient
- B decides whether to recall and liquidate some loans to serve early withdrawals
- Due to sequential service, decision to withdraw depends on beliefs about others' actions and loan liquidation value $\xi \in U\left(\underline{\xi}, \overline{\xi}\right)$

t = 3

- Good productivity shock (A) with probability ω and 0 otherwise
- E privately learns the value of the shock and B decides whether to monitor
- Repayment (or default on loans and deposits in the bad state)

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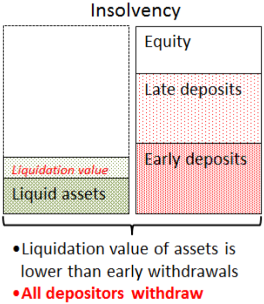

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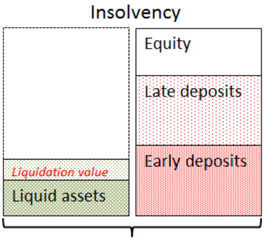
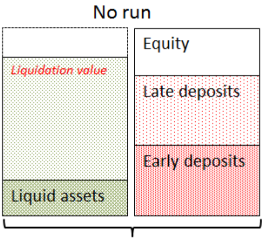
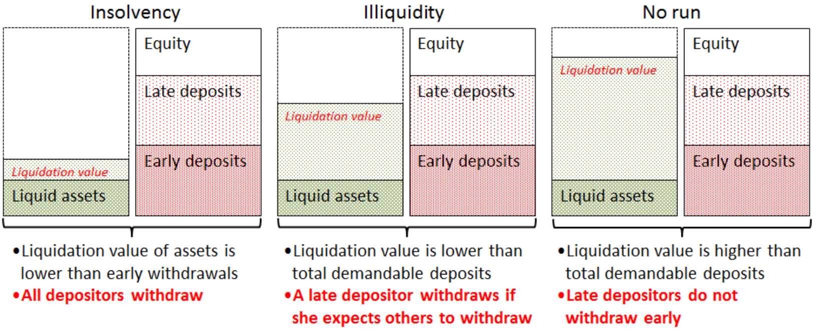


- Liquidation value of assets is lower than early withdrawals
- **All depositors withdraw**



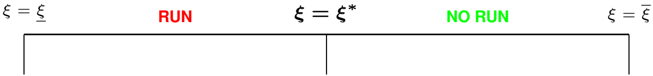
- Liquidation value is higher than total demandable deposits
- **Late depositors do not withdraw early**

Date 2 possibilities



Date 2 actions by savers

- Savers get private noisy signals $x_i = \xi + \epsilon_i$, $\epsilon_i \sim U[-\epsilon, \epsilon]$ about ξ
- Unique run threshold ξ^*, which depends on bank's balance sheet



- Thus, the probability of a run is $q = \frac{\xi^* - \underline{\xi}}{\Delta_\xi}$, where $\Delta_\xi = \overline{\xi} - \underline{\xi}$

S's Optimization problem

$$\mathbb{U}_S = U(e_S - D) + \overbrace{\int_{\underline{\xi}}^{\xi^*} \theta \cdot D(1 + r_D) \frac{d\xi}{\Delta_\xi}}^{\text{run}} + \overbrace{\int_{\xi^*}^{\overline{\xi}} \delta \cdot D(1 + r_D) \frac{d\xi}{\Delta_\xi}}^{\text{no run,impatient}}$$

$$+ \underbrace{\int_{\xi^*}^{\overline{\xi}} (1 - \delta) \cdot \omega \cdot D(1 + \bar{r}_D) \frac{d\xi}{\Delta_\xi}}_{\text{no run,patient}} + \underbrace{\int_{\xi^*}^{\overline{\xi}} V(D(1 + r_D)) \frac{d\xi}{\Delta_\xi}}_{\text{transaction services}}$$

- Assumption 1: Quasi-linear preferences for consumption and additional utility from the transactions services of deposits
- θ is the (endogenous) probability of being repaid in a run
- δ is the (exogenous) probability of being impatient

Optimization wrt D yields a **Deposit Supply** schedule, $DS(D, r_D, \bar{r}_D, \theta, \xi^*) = 0$

- Because each S is small, she takes ξ^* and θ as given

DS details

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DS details

E's Optimization problem

$$\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\overbrace{A \cdot (1-y) \cdot I}^{\text{realized output}} - \overbrace{(1-y) \cdot I \cdot (1+r_l)}^{\text{loan obligation}}] - \overbrace{c(I)}^{\text{cost}}\} \frac{d\xi}{\Delta_\xi}$$

where:

- Assumption 2: E is risk-neutral and has no endowment of her own
- Assumption 3: E has a linear production function, but incurs a convex (effort) cost
- y is the (endogenous) fraction of loans recalled and $y = 1$ in a run
- E is protected by limited liability and defaults in the bad state

Run decision

Optimization wrt I yields a **Loan Demand** schedule, $LD(r_l, I, y, \xi^*) = 0$

- Because each E is small, she takes ξ^* and y as given

LD details

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- Because each E is small, she takes ξ^* and y as given

LD details

B's Optimization problem

$$\mathbb{U}_B = U(e_B - E) + \int_{\xi^*}^{\bar{\xi}} \{\omega \cdot [\underbrace{(1-y)\cdot I}_{\text{outstanding loans}} \cdot \underbrace{(1+r_I)}_{\text{loan rate}} - \underbrace{(1-\delta)\cdot D}_{\text{patient deposits}} \cdot \underbrace{(1+\bar{r}_D)}_{\text{deposit rate}}] - \underbrace{X}_{\text{monit. cost}}\} \frac{d\xi}{\Delta_\xi}$$

- At t=1 the balance sheet identity is:

$$BS: \quad I + LIQ = D + E$$

- In a run, the probability of being repaid is:

$$\theta = \frac{LIQ + \xi \cdot I}{D \cdot (1 + r_D)}$$

- Absent a run, it liquidates $y \in (0, 1)$ of its loans to pay early withdrawals:

$$y = \frac{\delta \cdot D \cdot (1 + r_D) - LIQ}{\xi \cdot I}$$

Monitoring

- The productivity shock is privately revealed to E
- B needs to expend resources to learn it
- Given that dividends are increasing in ξ, B monitors if

$$\overbrace{\omega \left[\underbrace{(1-y(\xi^*)) \cdot I \cdot (1+r_I)}_{\text{revenue from outstanding loans}} - \underbrace{(1-\delta) \cdot D \cdot (1+\bar{r}_D)}_{\text{deposit repayments due}} \right]}^{\text{net expected benefit from monitoring}} - \overbrace{X}^{\substack{\text{monitoring}\\ \text{cost}}} \geq 0$$

- If B does not monitor, E will report the bad shock and default $\rightarrow$ implications for global game

Run threshold determination

- Global games in Diamond-Dybvig due to Goldstein-Pauzner (2005)
  - Incentives to run depend on deposit contract → important for welfare analysis
- We extend GP to allow for limited liability and uncertain liquidation value:
  - Obtain endogenously upper dominance region, but uniqueness is harder to show
- Utility differential between waiting and withdrawing for different **conjectured** level of withdrawals, λ, as a function of ξ

$$\nu(\xi,\lambda)=\begin{cases}\omega D(1+\bar{r}_D)-D(1+r_D) & \text{if} \quad \hat{\lambda}(\xi)\geq\lambda\geq\delta \qquad \textit{Partial run with monitoring}\\ -D(1+r_D) & \text{if} \quad \theta(\xi)\geq\lambda\geq\hat{\lambda}(\xi) \qquad \textit{Partial run no monitoring}\\ -(LIQ+\xi\cdot I)/\lambda & \text{if} \quad 1\geq\lambda\geq\theta(\xi) \qquad \textit{Full run}\end{cases}$$

- $\hat{\lambda}$ is the maximum level of withdrawals below which B has incentives to monitor

$\hat{\lambda}$ derivation

Run threshold determination ctd.

$$\nu(\xi,\lambda) = \begin{cases} \omega D(1+\bar{r}_D) - D(1+r_D) & \text{if} \quad \hat{\lambda}(\xi) \geq \lambda \geq \delta \qquad \textit{Partial run with monitoring} \\ -D(1+r_D) & \text{if} \quad \theta(\xi) \geq \lambda \geq \hat{\lambda}(\xi) \quad \textit{Partial run no monitoring} \\ -(LIQ + \xi \cdot I)/\lambda & \text{if} \quad 1 \geq \lambda \geq \theta(\xi) \qquad \textit{Full run} \end{cases}$$

- ”One-sided strategic complementarities”: $\nu(\xi,\lambda)$ is increasing in λ in full run
  - In a full run, the marginal gain from running is lower as more people opt to run
  - Goldstein-Pauzner deal with this issue and establish uniqueness
- Perverse state monotonicity: $\nu(\xi,\lambda)$ is *decreasing* in ξ in run region, but cut-off between regions also moves
  - In a full run, the expected return is higher for a strong bank than a weak bank
  - Not an issue in Goldstein-Pauzner because of fixed liquidation value

Existence and Uniqueness

- As $\epsilon \to 0$, ξ^* is given by $GG(\xi^*) = \int_\delta^1 \nu(\xi^*, \lambda) d\lambda = 0$

$$\underbrace{\int_\delta^{\hat{\lambda}(\xi^*)} [\omega D(1+\bar{r}_D) - D(1+r_D)]\, d\lambda}_{\substack{\text{Partial run} \\ \text{with monitoring}}} - \underbrace{\int_{\hat{\lambda}(\xi^*)}^{\theta(\xi^*)} D(1+r_D) d\lambda}_{\substack{\text{Partial run} \\ \text{no monitoring}}} - \underbrace{\int_{\theta(\xi^*)}^1 \frac{LIQ + \xi^* I}{\lambda} d\lambda}_{\text{Full run}} = 0$$

- Does a unique ξ^* exist? — (focus on noise, $\epsilon \to 0$; detailed proof for $\epsilon > 0$)

- Existence: GG is continuous and there exist thresholds $\underline{\xi} < \xi_{LD} < \xi_{UD} < \overline{\xi}$ such that $GG(\xi) < 0$ for $\xi < \xi_{LD}$ and $GG(\xi) > 0$ for $\xi > \xi_{UD}$

- Typical uniqueness proof requires that $dGG/d\xi > 0$ *everywhere*

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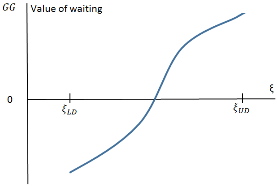
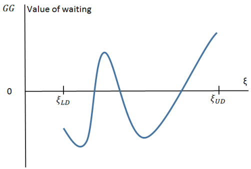


Uniqueness proof

- But, in our case $\frac{dGG}{d\xi} = \overbrace{\omega D(1+\bar{r}_D)\frac{d\hat{\lambda}}{d\xi}}^{\substack{\text{More monitoring}\\ \text{Wait on the margin}}} - \overbrace{\int_{\theta}^{1} \frac{I}{\lambda} d\lambda}^{\substack{\text{Higher recovery}\\ \text{Run on the margin}}} \overset{?}{\lesseqgtr} 0$, because $\frac{d\hat{\lambda}}{d\xi} > 0$

Bad case



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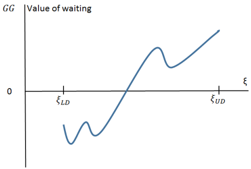

- Insight: Realize that *GG* does not need to be strictly increasing everywhere, but only at candidate solutions
- We show there are no solutions where $\{GG(\xi^*) = 0 \text{ and } dGG/d\xi|_{\xi=\xi^*} \leq 0\}$
- Hence, the run threshold is **unique**

Details

Uniqueness proof

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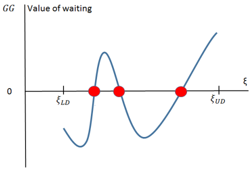

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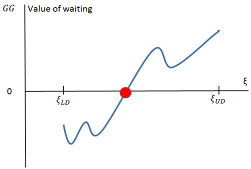

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- Hence, the run threshold is **unique**

Details

Private Equilibrium

- B chooses I, LIQ, D and E to maximize her utility while *internalizing* how these choices affect:
  - the run threshold via *GG*
  - the deposit rates that S demand via *DS*
  - the loan rates that E are willing to accept via *LD*
- Balance sheet constraint eliminates one choice variable$\rightarrow$ three (free) choices:
  - The asset mix that trades off loans and liquid assets
  - The liability mix that trades off equity and deposits
  - The overall scale of the balance sheet

Optimality conditions

Social Planner and Externalities

- Savers and Entrepreneurs are atomistic and take (ξ^*, θ, y) as given
- Consider a social planner with the following welfare function

$$\mathbb{U}_{sp} = \mathbb{U}_B + w_S \mathbb{U}_S + w_E \mathbb{U}_E$$

- If the planner respects the DS and LD constraints $\mathbb{U}_S$ and $\mathbb{U}_E$ can be replaced by

$$\mathbb{U}_S^* = \overbrace{U(e_s - D) + U'(e_S - D)D}^{\text{Surplus of deposits}} + \overbrace{\int_{\xi^*}^{\overline{\xi}} [V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)]/\Delta_\xi}^{\text{Surplus from transaction services of deposits}}$$

$$\mathbb{U}_E^* = \underbrace{\int_{\xi^*}^{\overline{\xi}} [c'(I)I - c(I)]/\Delta_\xi}_{\text{Surplus from production}}$$

- Recall S and E take ξ^* as given, but planner will explicitly account how their actions affect ξ^* and, thus, their welfare

Social Planner and Externalities ctd.

Thus, the planner maximizes $\mathbb{U}^*_{sp} = \mathbb{U}_B + w_S \mathbb{U}^*_S + w_E \mathbb{U}^*_E$ where

$$\mathbb{U}^*_S = U(e_s - D) + U'(e_S - D)D + \int_{\xi^*}^{\overline{\xi}} [V(D(1 + r_D)) - V'(D(1 + r_D))D(1 + r_D)]/\Delta_\xi$$

$$\mathbb{U}^*_E = \int_{\xi^*}^{\overline{\xi}} [c'(I)I - c(I)]/\Delta_\xi$$

Trade-offs for the Planner

- Trade-off 1: Planner trades off more deposits versus higher run risk when trying to help savers
- Trade-off 2: Planner trades off more lending versus higher run risk when trying to help entrepreneurs

Example	PE	SP for weights (w_E, w_S)		
		(0.0,0.2)	(0.1,0.1)	(0.2,0.0)
I	0.862	0.785	0.873	0.906
LIQ_1	0.052	0.221	0.060	0.000
D	0.875	0.962	0.894	0.867
E	0.038	0.044	0.039	0.038
Run prob.	0.407	0.386	0.403	0.408
Capital ratio	0.044	0.049	0.045	0.042
Liquidity ratio	0.060	0.281	0.069	0.000
$\%\Delta\mathbb{U}_E$	-	-1.66%	0.33%	1.19%
$\%\Delta\mathbb{U}_S$	-	3.63%	0.71%	-0.30%
$\%\Delta\mathbb{U}_B$	-	-0.44%	-0.05%	-0.09%

Capital ratio$= E/I$; Liquidity ratio$= LIQ/I$

- More liquid asset mix and more stable capital structure when S is favored
- More liquidity and/or capital reduce run probability
- More loans at the expense of liquidity when E is favored
- Yet, higher investment is not incompatible with more stable banking – both E and S gain
- B *loses*: already internalizes what matters to her – but total welfare is higher

Implementing the planner's solution

- The three intermediation margins differ between the private and social solutions
- One solution is to use taxes on, for example, I, LIQ and D to correct for the distorted intermediation margins
- Instead, we examine how regulation can decentralize the planner's solution
- It can be shown analytically that capital and liquidity regulations reduce the probability of runs (abstracting from GE effects)

Partial effect of regulation on run prob.

- Are these tools complements or substitutes?

Implementation example – $w_E = 0.1$, $w_S = 0.1$

	PE	CR	CR&LR	SP
I	0.862	0.861	0.858	0.873
LIQ_1	0.052	0.055	0.059	0.060
D	0.875	0.877	0.879	0.894
E	0.038	0.039	0.039	0.039
Run prob.	0.407	0.406	0.406	0.403
Cap.ratio	0.044	0.045	0.045	0.045
Liq.ratio	0.060	0.063	0.069	0.069
$\%\Delta\mathbb{U}_E$	-	-0.03%	-0.10%	0.33%
$\%\Delta\mathbb{U}_S$	-	0.04%	0.12%	0.71%
$\%\Delta\mathbb{U}_B$	-	-0.00%	-0.00%	-0.05%

$CR = E/I; \quad LR = LIQ/I$

- Tightening CR increases E and reduces run risk
- But, results in lower I
- Tightening LR too, reduces I and run risk further
- The two are not redundant
- Third tool needed to encourage intermediation – e.g. tax subsidy on D

Takeaways from regulatory tools

- Other tools that work are a liquidity coverage ratio, a net-stable funding ratio, reserve requirements, a leverage ratio
- But, at minimum the regulator needs a tool to manage capital, a tool to manage liquidity, and a tool to manage the scale of intermediation
- The distortions in the three intermediation margins are not *collinear*
- Liquidity tools can be combined with capital tools (and vice versa), but not with each other

Conclusions

- Presented a model of fragile financial intermediation where a bank offers liquidity and monitoring services
- Studied the externalities from intermediation and derived optimal regulation to address them
- Proposed a new proof for uniqueness in incomplete information bank-run models

Back-up slides

Deposit Supply

$$\mathbb{U}_S = U\left(e_S - D\right) + \int_{\underline{\xi}}^{\xi^*} \theta \cdot D(1+r_D)\frac{d\xi}{\Delta_\xi} + \int_{\xi^*}^{\overline{\xi}} \delta \cdot D(1+r_D)\frac{d\xi}{\Delta_\xi}$$
$$+ \int_{\xi^*}^{\overline{\xi}} (1-\delta)\cdot\omega\cdot D(1+\bar{r}_D)\frac{d\xi}{\Delta_\xi} + \int_{\xi^*}^{\overline{\xi}} V\left(D\right)\frac{d\xi}{\Delta_\xi}$$

- Taking θ and ξ^* as given, optimization wrt to D yields the following DS schedule

$$\underbrace{-U'(e_S - D)}_{\text{Consumption cost of depositing}} + \underbrace{(1+r_D)\int_{\underline{\xi}}^{\xi^*} \theta \frac{d\xi}{\Delta_\xi}}_{\text{Expected payoff in a run}} + \underbrace{\left[\delta(1+r_D) + (1-\delta)\omega(1+\bar{r}_D) + V'(D)\right]\int_{\xi^*}^{\overline{\xi}} \frac{d\xi}{\Delta_\xi}}_{\text{Expected payoff absent a run}} = 0$$

Back to Savers

Loan Demand

$$\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\overbrace{A \cdot (1-y) \cdot I}^{\text{realized output}} - \overbrace{(1-y) \cdot I \cdot (1+r_l)}^{\text{loan obligation}}] - \overbrace{c(I)}^{\text{cost}}\} \frac{d\xi}{\Delta_\xi}$$

- Taking y and ξ^* as given, optimization wrt to I yields the following LD schedule

$$\int_{\xi^*}^{\overline{\xi}} \{\underbrace{\omega \cdot [A - (1+r_l)] \cdot (1-y)}_{\substack{\text{Net payoff} \\ \text{from borrowing}}} - \underbrace{c'(I)}_{\substack{\text{Marginal cost} \\ \text{of effort}}}\} \frac{d\xi}{\Delta_\xi} = 0$$

Back to Entrepreneurs

The run decisions

- Patient depositors need to decide whether withdrawing at $t = 2$ or $t = 3$ is better
- To decide, the must infer:

(1) The value of a deposit at $t = 3$, which depends on

1. the number of loans they expected to be outstanding and whether they pay off
2. whether the bank will want to monitor
3. interest rate on deposits at $t = 3$

(2) The value of a deposit at $t = 2$, which depends on

1. how many loans will be recalled plus liquid assets that are available
2. how many other people will withdraw and the probability of being repaid in a run
3. interest rate on deposits at $t = 2$

Back

Derivation of $\hat{\lambda}$

- $\hat{\lambda}(\xi)$ is the level of withdrawals at which the banker is indifferent between monitoring E's projects or not when the liquidation value is ξ

$$\omega\left[(1-y(\hat{\lambda}(\xi),\xi))I(1+r_I)-(1-\hat{\lambda}(\xi))D(1+\bar{r}_D)\right]-X=0$$

$$\Rightarrow\omega\left[\frac{\xi I-\hat{\lambda}(\xi)D(1+r_D)+LIQ}{\xi}(1+r_I)-(1-\hat{\lambda}(\xi))D(1+\bar{r}_D)\right]-X=0$$

$$\Rightarrow\hat{\lambda}(\xi)=\frac{(\xi I+LIQ)(1+r_I)-\xi(D(1+\bar{r}_D+X/\omega)}{D[(1+r_D)(1+r_I)-\xi(1+\bar{r}_D)]}$$

- Because the incentives to monitor are decreasing in λ, we get that $\hat{\lambda}>\delta$

- Also, $\partial\hat{\lambda}(\xi)/\partial I>0$, $\partial\hat{\lambda}(\xi)/\partial LIQ>0$, $\partial\hat{\lambda}(\xi)/\partial D<0$, $\partial\hat{\lambda}(\xi)/\partial r_I>0$, $\partial\hat{\lambda}(\xi)/\partial r_D<0$, $\partial\hat{\lambda}(\xi)/\partial\bar{r}_D<0$

Back to Global Game

Uniqueness proof details

- At any candidate solution ξ', $GG(\xi') = 0$ yields the following necessary condition:

$$-\int_{\theta}^{1} \frac{I}{\lambda} d\lambda = \frac{1}{\xi'} \left[\int_{\theta}^{1} \frac{LIQ}{\lambda} d\lambda + \int_{\delta}^{\theta} D(1+r_D) d\lambda - \int_{\delta}^{\hat{\lambda}} \omega D(1+\bar{r}_D) d\lambda \right]$$

- Evaluating the derivative $dGG/d\xi$ at $\xi = \xi'$ and substituting in the above necessary condition yields:

$$\left.\frac{dGG}{d\xi}\right|_{\xi=\xi'} = \overbrace{\frac{1}{\xi'} \left[\int_{\theta}^{1} \frac{LIQ}{\lambda} d\lambda + \int_{\delta}^{\theta} D(1+r_D) d\lambda \right]}^{>0} + \omega D(1+\bar{r}_D) \left[\frac{d\hat{\lambda}(\xi')}{d\xi} - \frac{\hat{\lambda}-\delta}{\xi'} \right]$$

- After some algebra

$$\frac{d\hat{\lambda}(\xi')}{d\xi} - \frac{\hat{\lambda}-\delta}{\xi'} = \frac{(\hat{\lambda}-\delta)\xi' D(1+\bar{r}_D) + (\delta D(1+r_D) - LIQ)(1+r_I)}{\xi' D[(1+r_D)(1+r_I) - \xi'(1+\bar{r}_D)]} > 0$$

since $\hat{\lambda} > \delta$ to provide monitoring incentives and $\delta D(1+r_D) - LIQ > 0$ from lower dominance

Back to Global Game

Private Optimality Conditions

- Denote by ψ_{BS}, ψ_{GG}, ψ_{DS}, and ψ_{LD} the Lagrange multipliers on the balance sheet, global game, deposit supply, and loan demand constraints, respectively
- The first-order conditions of B for choices $\mathcal{C} \in \{I, LIQ, D, E, \xi^*, r_I, r_D, \bar{r}_D\}$ are:

$$\frac{d\mathbb{U}_B}{d\mathcal{C}} + \psi_{BS}\frac{dBS}{d\mathcal{C}} + \psi_{GG}\frac{dGG}{d\mathcal{C}} + \psi_{DS}\frac{dDS}{d\mathcal{C}} + \psi_{LD}\frac{dLD}{d\mathcal{C}} = 0$$

- From the foc with respect to $\bar{r}_D$ we obtain

$$\psi_{DS} = -\left(\frac{d\mathbb{U}_B}{d\bar{r}_D} + \psi_{GG}\frac{dGG}{d\bar{r}_D}\right)\frac{dDS}{d\bar{r}_D}^{-1}$$

- From the foc with respect to r_I we obtain

$$\psi_{LD} = -\left(\frac{d\mathbb{U}_B}{dr_I} + \psi_{GG}\frac{dGG}{dr_I}\right)\frac{dLD}{dr_I}^{-1}$$

Private Optimality Conditions ctd.

- From the foc with resect to ξ^*, and using ψ_{DS} and ψ_{LD}, we obtain

$$\psi_{GG} = -\frac{\frac{d\mathbb{U}_B}{d\xi^*} - \frac{d\mathbb{U}_B}{d\bar{r}_D}\frac{dDS}{d\bar{r}_D}^{-1}\frac{dDS}{d\xi^*} - \frac{d\mathbb{U}_B}{dr_l}\frac{dDS}{dr_l}^{-1}\frac{dLD}{d\xi^*}}{\frac{dGG}{d\xi^*} - \frac{dGG}{d\bar{r}_D}\frac{dDS}{d\bar{r}_D}^{-1}\frac{dDS}{d\xi^*} - \frac{dGG}{dr_l}\frac{dLD}{dr_l}^{-1}\frac{dLD}{d\xi^*}}$$

- From the foc with respect to E we obtain the shadow cost of equity

$$\psi_{BS} = -d\mathbb{U}_B/dE = U'(e_B - E)$$

- Note that the shadow cost of equity is increasing in the amount of equity raised
- Given the balance sheet constraint $E = I + LIQ - D$ and, thus, all Lagrange multiplier can be expressed as functions of I, LIQ and D
- ξ^*, r_l and $\bar{r}_D$ are also implicit functions of I, LIQ and D via constraints GG, DS and LD

Private Optimality Conditions ctd.

- Hence, there are three free choices for B
- One choice regards the asset mix which is described by combining the focs wrt *LIQ* and *I*
$$\frac{d\mathbb{U}_B}{dLIQ} - \frac{d\mathbb{U}_B}{dI} + \psi_{GG}\left(\frac{dGG}{dLIQ} - \frac{dGG}{dI}\right) + \psi_{DS}\left(\frac{dDS}{dLIQ} - \frac{dDS}{dI}\right) + \psi_{LD}\left(\frac{dLD}{dLIQ} - \frac{dLD}{dI}\right) = 0$$
- Another choice regards the liability mix which is described by the focs wrt to *E* and *D*
$$\frac{d\mathbb{U}_B}{dE} - \frac{d\mathbb{U}_B}{dD} - \psi_{GG}\frac{dGG}{dD} - \psi_{DS}\frac{dDS}{dD} - \psi_{LD}\frac{dLD}{dD} = 0$$
- The last choice regards the overall scale of the bank, which is described by the focs wrt *I* and *D*
$$\frac{d\mathbb{U}_B}{dI} + \frac{d\mathbb{U}_B}{dD} + \psi_{GG}\left(\frac{dGG}{dI} + \frac{dGG}{dD}\right) + \psi_{DS}\left(\frac{dDS}{dI} + \frac{dDS}{dD}\right) + \psi_{LD}\left(\frac{dLD}{dI} + \frac{dLD}{dD}\right) = 0$$

▸ Optimality conditions

Partial effect of regulation on run risk

- We compute the partial derivatives of run risk with respect to capital and liquidity
- Partial effects keeping the loan rate, the deposits rates and cost of equity constant
- The problem is not scale invariant so we normalize by the size of the balance sheet and partial the partial derivative with respect to:
  1. A leverage ratio: $k = E/(I + LIQ)$
  2. A liquidity ratio: $\ell = LIQ/(I + LIQ)$
- The effect on the fundamental run probability, $q_f = (\xi_{LD} - \underline{\xi})/\Delta_\xi$, is captured by the derivative of the lower dominance threshold, $\partial\xi_{LD}/\partial T$, $T \in \{k, \ell\}$, where

$$\xi_{LD} = \frac{\delta(1-k)(1+r_D) - \ell}{1-\ell}$$

- The effect of the total run probability, $q = (\xi^* - \underline{\xi})/\Delta_\xi$, is captured by the implicit derivative of the run threshold ξ^*,

$$\frac{\partial\xi^*}{\partial T} = -\frac{\partial GG/\partial T}{\partial GG/\partial\xi^*}$$

Partial effect of regulation on fundamental run probability

- Increasing capital reduces the probability of fundamental runs

$$\frac{\partial \xi_{LD}}{\partial k} = -\frac{\delta(1 + r_D)}{1 - \ell} < 0$$

- Increasing liquidity reduces the probability of fundamental runs for $\ell < \bar{\ell} \equiv 1 - \delta(1 - k)(1 + r_D)$

$$\frac{\partial \xi_{LD}}{\partial \ell} = \frac{\delta(1 - k)(1 + r_D) - (1 - \ell)}{(1 - \ell)^2} < 0 \text{ for } \ell < \bar{\ell}$$

- $\ell < \bar{\ell}$ requires $\delta(1 - k)(1 + r_D) - (1 - \ell) < 0$, which is very intuitive

- The condition says that loans in the balance sheet are higher than the expected deposit withdrawals, hence there is maturity transformation

Partial effect of regulation on total run probability

- From uniqueness proof, $\partial GG/\partial \xi^* > 0$, so suffices to sign $\partial GG/\partial T$
- The global game condition GG can be written in terms of k and ℓ as:

$$GG: \quad \int_\delta^{\hat{\lambda}} \omega(1-k)(1+\bar{r}_D)d\lambda - \int_\delta^{\theta^*} (1-k)(1+\bar{r}_D) - \int_{\theta^*}^1 \frac{\xi^*(1-\ell)+\ell}{\lambda} d\lambda = 0,$$

where $\hat{\lambda} = \frac{(\xi^*(1-\ell)+\ell)(1+r_I)-\xi^*((1-k)(1+\bar{r}_D)+X/(\omega(I+LIQ)))}{(1-k)[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)]}$

- k affects the payoff differential in a partial run as well as the range that monitoring occurs, $\hat{\lambda}-\delta$, via its effect on bank profitability
- ℓ affects the payoff differential in a full run as well as the range that monitoring occurs, $\hat{\lambda}-\delta$, via its effect on bank profitability

Partial effect of regulation on total run probability – Capital

- Trade-off from increasing capital: Monitoring more probable versus lower payoff given monitoring

$$\frac{\partial GG}{\partial k} = \underbrace{\frac{\partial \hat{\lambda}}{\partial k}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{(\hat{\lambda}-\delta)[\omega(1+\bar{r}_D)-(1+r_D)]}_{\substack{\text{Lower payoff}\\ \text{given monitoring}}} + \underbrace{(\theta^* - \hat{\lambda})(1+r_D)}_{\substack{\text{'Higher' payoff}\\ \text{absent monitoring}}}$$

- Overall, increasing capital reduces the total probability of runs

$$\frac{\partial GG}{\partial k} = \left[\frac{\xi^*(1+\bar{r}_D)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)} + \delta\right]\omega(1+\bar{r}_D) + (\theta^*-\delta)(1+r_D) > 0$$

$$\Rightarrow \frac{\partial \xi^*}{\partial k} < 0$$

Partial effect of regulation on total run probability – Liquidity

- Trade-off from increasing capital: Monitoring more probable versus higher incentives to join full run

$$\frac{\partial GG}{\partial \ell} = \underbrace{\frac{\partial \hat{\lambda}}{\partial \ell}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{\int_{\theta^*}^{1} \frac{1-\xi^*}{\lambda} d\lambda}_{\substack{\text{Higher payoff} \\ \text{in full run}}}$$

- Overall, increasing liquidity reduces the total probability of runs (but not always)

$$\frac{\partial GG}{\partial \ell} = (1-\xi^*)\left[\frac{\omega(1+\bar{r}_D)(1+r_I)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)} + \log\theta^*\right]$$

$$\Rightarrow \frac{\partial \xi^*}{\partial \ell} < 0$$

for $\delta > e^{-1}$, since $\theta^* > \delta$ and $\omega(1+\bar{r}_D) > (1+r_D)$

or $\ell > \bar{\ell} \equiv (e^{-1}(1-k)(1+r_D) - \xi^*)/(1-\xi^*)$; true for high enough ξ^*

Back to Implementation