A REVIEW OF ALGORITHMIC AND HEURISTIC BASED METHODS FOR VOLTAGE/VAR CONTROL

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Abstract

Over the last two decades voltage control has emerged as a major concern with regard to the secure operation of bulk power systems, worldwide. This is partly a consequence of the newly deregulated market environments that have been recently introduced to electricity industries. Power systems are now operating under very different conditions, particularly with regard to the distribution and location of generation. A case in point is the large-scale power transmission system operated in England and Wales by National Grid (NG), where the majority of new generation has been located in the North whilst the majority of load remains in the South. For these reasons, an increasing amount of reactive compensation equipment has been installed by NG in order to maintain the required voltage profiles throughout the transmission system. Consequently, the secure and economic control of reactive compensation equipment combined with generator reactive power outputs is an important consideration, particularly with regard to the reactive power market in England and Wales.

Keywords
Transition-optimised, voltage/VAr control.

1 INTRODUCTION

Voltage control and reactive power optimisation has been researched extensively as a static snap-shot problem, but relatively little research has addressed the time-domain aspects of the problem.

In this paper algorithmic and heuristic techniques that have been applied to the snapshot problem are reviewed in order to determine the potential of integrating such techniques to develop a transition-optimised voltage and reactive power control tool. The techniques are reviewed with specific regard to objective functions, programming techniques, control variables, decomposition, optimal power flow and problem formulation.

The ultimate objective of this research is to approach the problem of voltage control and reactive power optimisation from a schedule and dispatch viewpoint, which considers practical constraints such as the number of reactive control actions allowable within a time-domain, the time interval required between actions performed, etc.

2 OPTIMAL REACTIVE DISPATCH

Reactive power plays an important role in supporting the transfer of real power across a large-scale transmission system [30], such as the NG power system of England and Wales. In an open access system, the importance of this support is even greater as the power transfer is increased and the associated voltage issues then become a bottleneck in preventing additional power transfer [30]. In simple terms, the most important aim of reactive power dispatch is to determine the sufficient amount and correct location of reactive support in order to maintain a secure voltage profile [43].

2.1 Standard Snapshot Problem

This section reviews the standard snapshot approach to the optimisation problem of reactive dispatch. The relevance of this approach is highlighted with regard to the objective functions, mathematical programming techniques, control variables, decomposition techniques, optimal power flow and problem formulation.

Objective Functions. With regard to the problem of optimising reactive dispatch, the original objective functions were oriented economically and involved the minimisation of transmission losses, an important case in point was the formulation of Peschon et al. in 1968 [34]. Over the next decade a number of security oriented objective functions were proposed, such that either the elimination of security constraint violations was considered independently [18, 38] or the minimisation of the transmission losses and the improvement of voltage profiles were considered jointly [28, 31, 35]. In the majority of these cases the optimisation of reactive dispatch was performed as a subsequent analysis to the optimisation of active dispatch [34, 18, 28]. Active dispatch, or economic dispatch as it is also known, involves the minimisation of fuel costs [43].

More recently a number of optimisation problems have been posed, where more complex objective functions have been employed, such as the maximisa-

tion of reactive reserve and the maximisation of load voltage [6, 7]. These problems considered the optimisation of both active and reactive dispatch in a united or combined fashion and consequently required multi-objective functions [6, 7, 24]. In the united cases the objective functions can be combined in a weighted fashion [6, 7]. Alternatively, the minimisation of the adjustment of reactive power control devices has been proposed [27]. This is particularly important in an emergency situation, when the power system performance must be improved with minimal adjustments to control variables. It is also possible to weight or penalise specific control actions, such as generator terminal voltage, which the operator may not be able to control with ease [27].
Currently proposed objective functions have concerned the minimisation of control actions during the rescheduling of reactive power [12]. In all the problem formulations discussed the traditional snapshot view of the problem was adopted with regard to the objective function.

Mathematical Programming Techniques. When considering which mathematical programming technique to employ in the solution of an optimal reactive dispatch (ORD) problem, an important consideration is the computational cost involved. In 1968 a non-linear programming technique was applied to an ORD problem involving a 500-node system and required 10 minutes CPU time [34]. However, it should be noted that the linear objective function was particularised to the minimisation of transmission line losses and a limited number of control variables were considered, which considerably reduces the computational effort required. In 1989 a successive quadratic programming technique was applied to a reactive dispatch problem involving a 253-node system and required 10.5 minutes [36]. However, it should be noted that the objective function was of quadratic form and an extended number of control variables were considered, which considerably increases the computational effort required.
For these reasons, considerable research effort has been applied to improve the computational efficiency of non-linear programming techniques, when applied to either the problem of reactive dispatch or the problem of combined active and reactive dispatch [24].
As an alternative to applying non-linear programming techniques, which are inherently computationally expensive and can suffer from convergence difficulties, iterative or successive linear programming techniques can be applied to the problem of reactive dispatch [18, 28, 42, 31, 35, 27]. In 1986 a linear programming technique was applied to a reactive dispatch problem involving a 256-node system and required 2.4 seconds CPU time [31]. However, it should be noted that the problem posed only considered one case consisting of seven constraint violations (five reactive power generations and two load node voltages), therefore the performance of the algorithm was not illustrated as the number of constraints is increased. Furthermore, the transformer tap positions and switching of shunt devices were not considered as discrete control variables, which eases the computational burden of the problem.
Finally, an interior point method to determine ORD was presented in 1994 [16]. The method was applied to two networks involving 1832 and 3467 busses. The smaller network required CPU times ranging from 200-400 seconds and the larger network ranged from 400-600 seconds. The analyses were performed on a 386 25 MHz microprocessor [16].

Selected Control Variables. The standard control variables for a reactive dispatch problem are generator voltages, switchable capacitors/inductors and transformer taps [18, 28, 36, 31, 26, 35, 27]. It is important to note that in early analyses the first two control variables were considered as VAr injections [34] and that the generator voltages are continuous whereas the other two control variables are discrete. Early studies of reactive dispatch considered less control variables and in particular did not consider transformer taps [34]. More recently unified or combined approaches to both active and reactive dispatch have been developed [24, 6, 7], where additional control variables for the active dispatch are considered, such as the generator real power outputs [24].

Decomposition techniques. Realistically, reactive dispatch problems require the consideration of large-scale power networks and therefore a number of decomposition techniques can be applied. One of the earliest cases was the application of a decomposition and coordination strategy in by Mansour and Abdel-Rahman in 1984 [29]. Subsequently, in 1990, the Dantzig-Wolfe decomposition technique was applied by Deeb and Shahidehpour [10]. In both cases the problem is decomposed into several sub-problems or sub-systems, corresponding to specific areas in the power system, which are then easier and faster to solve. Additionally, the analysis can then be performed without the need for an OPF [29], which is discussed in more detail in the following sub-section. Additionally, P-Q decomposition can be efficiently applied to large-scale problems such that the problem is decomposed into a real power optimisation problem (P-problem) and a reactive power optimisation problem (Q-problem) [6, 7, 5, 24, 4, 39]. The solution of both problems can then be co-ordinated in a variety of ways [5, 4, 39].

Optimal Power Flow (OPF). The majority of reactive dispatch problems closely involve the optimisation of an AC power flow and therefore the efficiency of the overall method employed to solve the reactive dispatch problem is heavily dependent on the efficiency of the solution technique employed with regard to the AC power flow. For this reason, a review of the development of OPF methods in the context of the solution of ORD is included in this section. It should be noted that this section is closely related to the former section concerning mathematical programming techniques and therefore similar issues are discussed.
Traditionally, it was possible to develop fast methods for reactive dispatch by assuming small system

deviations from desired optimal set-point values as determined by an off-line OPF. It is important to note that the OPF could not be run on-line due to the considerable CPU time required [28, 29]. However, it should also be noted that it is possible to solve the reactive dispatch problem without the need for an OPF [29]. In such a case the solution of AC power flow is replaced by the co-ordinated solution of sub-systems involved in a decomposition approach [29]. The method is faster for larger system deviations and can run on-line without the need for an OPF [29].

Generally, the computational solution of the multivariable power flow associated with the reactive dispatch problem is solved by Newton-Raphson (NR) methods [34, 27]. The ORD problem was originally formulated and solved as a non-linear programming problem [34]. This was a direct consequence of the important developments in solving general OPF problems during the 1960s and the significant contribution of Dommel and Tinney [11]. The use of a non-linear programming approach can be attributed to non-linear constraints, such as the power flow equations [34], or to the non-linear objective functions that can also be employed in the OPF [1]. The problematic convergence properties of non-linear methods instigated the development of linear models for OPF problems [17], which were consequently employed in ORD problems [18, 27].

When considering the OPF problem generally, variables can change roles. Initially, variables can be classed as control, dependent state or, if they have violated their limits, constant [40]. Subsequently, a dual version of the linear programming approach can be employed to change the roles of variables as required [1]. Primarily, the control variables are optimised and their associated hard limits are enforced exactly by LP upper bounds. Secondarily, a significantly reduced number of the soft power flow constraints, which may be huge for a large power system, are enforced. The soft constraints can be expressed via sensitivity analysis as linear functions of the control variables [1]. Alternatively, when applying NR methods to OPF these distinctions are unnecessary, such that when variables become constant they are constrained at their limits without changing their roles. Furthermore, control and state dependent variables are processed identically and the dimensionality of the vector of variables is never changed [40].

With regard to applying LP or Newton based methods to ORD problems, an issue of historical importance is the non-separable nature of the loss minimisation objective. Traditionally, LP-based methods are naturally adaptable to OPF problems whose objectives are entirely or strongly separable [1]. With regard to the standard objectives employed in OPF only loss minimisation is outside this category. For this reason a number of researchers were originally sceptical of applying LP-based methods to such problems on large-scale power systems [1]. However, the extension of LP to non-separable problems is conceptually straightforward and a number of researchers have applied LP-based methods to ORD problems involving such objective functions [1, 28, 35].

It should also be noted that when applying Newton-based methods to OPF problems, the usual assumptions are not valid. Generally, for large-scale non-linear optimisation problems quasi-Newton methods are preferable to full Newton methods that require more expensive global matrix operations, but this assumption is not the case in OPF problems. This is a consequence of the sparse nature of the Hessian matrix of the Lagrangian function for OPF problems, which enables the use of very efficient sparse matrix operations for a full Newton method [40].

<u>Problem Formulation.</u> The ORD problem can be defined as a static optimisation in the presence of operating constraints [34]. The standard problem formulation, as originally proposed in the 1960s [34], is still widely applied. The formulation extends Lagrange multiplier methods via the Kuhn and Tucker theorem to include operating constraints when they are inequalities. The constrained problem is then transformed into an unconstrained Lagrangian function [34, 32].

An alternative formulation has been proposed when an interior point method is applied to the ORD problem. The formulation employs logarithmic barrier functions to eliminate the constraints [16].

2.2 Expert Systems, Evolutionary Programming and Genetic Algorithms

As alternatives to the conventional mathematical programming techniques, a number of novel heuristic and/or algorithmic methods have been proposed for reactive dispatch over the last 15 years. These techniques include expert system assisted approaches [26, 37, 2, 23], genetic algorithms [20] and evolutionary programming [44, 22, 15].

With regard to expert system assisted approaches, empirical rules have been employed to generate appropriate control actions to alleviate a minor voltage problem [26]. The control actions include operator control of capacitors, transformer taps and generator voltages. In this case the expert system can replace a conventional mathematical programming approach to the optimisation problem. However, when the voltage problem is so severe that empirical judgement may not be reliable, the expert system is only used to help formulate the problem for a more conventional and reliable mathematical programming technique [26]. Similarly, NG have investigated a fuzzy control system combined with fast AC load flow for control of voltage and reactive power [23]. Fuzzy control systems proceed in the same way as expert systems, but permit uncertainty to be represented in the knowledge base also. In this way more efficient use of the knowledge base is possible.

Furthermore, conventional mathematical programming techniques and heuristic (rule-based) have been hybridised to assist the system control engineer in voltage and reactive power control [37]. In the hybridisation advantage is taken of both techniques in order to find a practical number of control variables. The practical efficiency of this technique has been studied in detail and the technique has also been integrated in a power system control centre in Spain [37]. The motivation for investigating the potential of employing expert systems in reactive dispatch is the recognition that some kind of heuristics are necessary to improve the capabilities of OPF [1]. Similarly, NG have investigated the application of a knowledge based system for voltage control and monitoring on their power system [2]. In this application the OPF calculates an hourly voltage schedule and the expert system suggests remedial actions whenever voltage violation alarms are detected by the state monitoring, which occurs after every state estimation [2].

An equivalent on-line application has been developed by the same first author and installed at the national control centre of the Portuguese power system. With regard to applying genetic algorithms to reactive dispatch, multiple search directions can be employed to search for a global minimum as opposed to a local minimum [20]. The OPF associated with reactive dispatch is not necessarily a mathematically convex problem and therefore conventional techniques may converge to a local minimum instead of a unique global minimum. Furthermore, discrete control variables such as transformer tap positions and switchable capacitor banks can be handled in a more simple fashion than conventional techniques. However, it should be noted that although genetic algorithms are capable of yielding a global minimum, they have traditionally required significantly more computing time [20].

Evolutionary programming is based upon similar principles to genetic algorithms, which are basically natural selection and mutation. However, the fundamental difference is that genetic algorithms consider cross-over, while evolutionary programming does not necessarily consider cross-over. Cross-over is the combination of the genetic information of parents to produce off-spring.

With regard to both active and reactive dispatch, a conventional approach as discussed earlier is to consider two separate problems: P- and Q-, respectively. The former problem is to regulate active power outputs of generators to minimise fuel costs and the latter problem is to control reactive power flow to minimise network losses. The Q- problem is more difficult to solve due to the non-separable nature of the network losses minimisation objective and the discrete control variables involved. For this reason an evolutionary approach has been applied to the Q-problem, as opposed to conventional methods [44]. Evolutionary programming has also been applied to the problem of ORD in its own right in order to minimise network real power loss and to improve voltage profiles [22]. Additionally, evolutionary programming has been extended for problems in ORD, such that mutations in standard deviations are controlled using a strategy of dynamic limits [15].

2.3 Transition-Optimised Problem

Over the last twenty years a small but significant number of researchers have begun to consider the scheduling of reactive power compensation equipment, such as capacitors, on an hourly basis [33, 13, 3]. The intended application of one approach is to schedule the switching off of existing capacitors during periods of light load to prevent abnormally high voltages occurring during normal and post-contingency conditions [33]. The switching schedule was originally obtained by performing a series of power flow simulations, but as more capacitors were placed in service the number of power flow simulations became impractical and an OPF approach was adopted [33]. Another approach considered two objective functions associated with a two-stage optimisation scheme, where the importance of the aspects of each objective function is dependent upon the load condition of the power system. Under light to medium load conditions the reactive margins and their distributions are not critical, therefore the objective function associated with economic aspects is more important. Conversely, during load pick-up and at heavy load hours an objective function with a secure reactive management aspect is more important [13]. Another approach, which considered multi-level hierarchical control of voltage and reactive power, proposed both spatial and temporal decomposition of the problem. The spatial decomposition followed the natural multi-level hierarchy of a power system and the predominantly local nature of the voltage/VAr control problem. The temporal decomposition considers the different levels of response time for control modes attributable to different hierarchical levels [3].

However, the majority of this research was a simple extension to the techniques employed in reactive power planning [33, 13] and therefore did not consider the optimisation of control actions over a number of power system transitions on a short-term basis.

Recently, a novel technique has been developed to determine fixed transformer tap settings that are optimal for several load patterns representing winter peak, summer peak and off-peak loading patterns, for example [21]. The technique consists of two parts. The first part is concerned with the construction of a base case combining several load patterns, which retains all the features of the load patterns. The second part is concerned with a formulation of the OPF problem subject to several load patterns which includes equality constraints on selected transformer tap settings, reflecting the fact that the settings should be the same for all load patterns considered [21]. A benefit of this novel multi-case optimisation approach is the removal of the need for a trial and error strategy involving numerous load flow solutions. With regard to disadvantages, the method is limited to a small number of load patterns, three in practice, due to the large computational effort required to solve the base case. Furthermore, there is no built in discretisation of the transformer tap settings and the approach requires re-optimisation (verification) of the solution to obtain the actual discrete tap settings [21]. It is also possible to question whether the technique would be feasible for a heavily loaded system, such that it may not be possible to determine tap values that satisfy all conditions economically.

More recently, the problem has been formulated in a transition-optimised fashion that is consistent with the time-linked approach as intended for use in the research project proposed in this paper. In this case the objective function is the minimisation of control actions over a short-term operational period (1 day) [19]. In this approach, the entire problem is decomposed into master and slave levels.The master level deals with the minimisation of the depreciation cost of compensators and transformer taps while satisfying operating constraints. The slave level considers the minimisation of active power losses while satisfying system security constraints [19]. The decomposition is performed in two senses using Generalised Benders Decomposition Theory (GBDT) [14]. With regard to the slave level, the day is decomposed into sets of equality and inequality constraints associated with each of the cardinal points for the day. Six cardinal points are considered in the case study. Also, the discrete and continuous variables are decomposed to obtain the master and slave levels [19]. This research was an extension to a Newton based approach to OPF for reactive scheduling [41]. The approach was originally applied to the optimal scheduling of existing reactive power and voltage sources, but the ultimate aim was an integrated approach for long-term reactive power planning [41].

Recently, researchers have investigated the coordination between the reactive power scheduling function and the hierarchical voltage control of a large-scale power system [9]. At this point it should be noted that the architecture of control mechanisms for large-scale power systems can be fully centralised, decentralised or hierarchical. Furthermore, the control system employed by NG has recently changed from a decentralised to a fully centralised architecture [25].

Alternatively over the last 20 years, hierarchical control systems have been adopted throughout Europe. France and Italy are two notable cases in point [9]. The hierarchical system was originally proposed via the addition of a Secondary Voltage Regulation (SVR) level to the established primary Automatic Voltage Regulation (AVR) level. Basically, SVR is the control of voltage at critical pilot (or pivot) nodes in the power system by reactive power production at the electrically nearest generation plants [9]. Subsequently, the addition of a Tertiary Voltage Regulation (TVR) level was proposed with the aim of pursuing a common objective with regard to the control of voltage and reactive power in different areas of the power system [9]. The TVR coordinates the power system in a centralised mode [9]. The aim of TVR is to provide, in a real time environment, set values for the pilot nodes based upon the actual state of the power system and the optimal voltage profiles as defined by short (daily) or very short (hourly) term reactive scheduling [9]. However, it is important to note that the scheduling of reactive power is not performed in a transition-optimised sense as the reactive power scheduled for the pilot nodes is not time-linked [9]. It should also be noted that a security constrained active and reactive dispatch algorithm has been recently extended by NG to include the pseudo-scheduling of generation. This was achieved using extended generator capability charts, but does not consider reactive power and voltage control in a transition-optimised sense [8].

ACKNOWLEDGEMENTS

The authors would like to acknowledge the National Grid scholarship scheme with regard to the financial support of this research.

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