Influence of the size of a nation’s population on performances in athletics

N.P. Linthorne*

*Centre for Sports Medicine and Human Performance, School of Sport and Education, Brunel University, Uxbridge, Middlesex, UB8 3PH, United Kingdom. email: nick.linthorne@brunel.ac.uk

Abstract. Nations with a large population tend to be more successful in international sports competitions than less populous nations. The aim of this study was to examine the mathematical relations between a nation’s population and the performance by its best track and field athletes. The key assumptions in the model were that athletic performances within a nation have a Gaussian distribution, and that the mean and standard deviation of athletic performances are the same for all nations. Performances achieved by the best athletes in a nation were calculated by integrating the equation for the Gaussian distribution, and the predictions of the model were evaluated using long jump competition results obtained from athletics yearbooks. For both men and women, the performance by the best long jumper increased at the rate of about 0.3 m per factor of ten increase in population. Governments of smaller nations that invest in high performance sport therefore have a substantial performance deficit to overcome in order to achieve international success. This study suggests that a nation can increase the likelihood of international success through increasing the participation rate for the sport.

1. Introduction

The governments of many nations invest in high performance sport with the goal of achieving greater international sporting success. Sports success promotes a ‘feelgood’ factor among citizens as well as bestowing prestige upon the nation. Some governments use international sports success to drive grassroots participation in sport and physical activity, with the aim of increasing the health and quality of life of its citizens.

The two strongest indicators of international sporting success are the nation’s wealth and the nation’s population (Morton, 2002). Greater wealth is associated with more leisure time, which gives a greater fraction of the population an opportunity to participate in competitive sports. Wealthy nations are more able to provide their leading athletes with the sports facilities, coaching expertise, and sports science support services that are necessary for international success. The population of a nation influences sports success through the variation in physical characteristics within the population. The larger the population the greater the number of genetically endowed athletes with the physical characteristics (such as height, limb dimensions, body weight, cardiovascular capacity, muscle strength, and muscle fibre properties) that are suited to producing superior sports performance.

The present study investigated the relations between a nation’s population and the performances by the nation’s best athletes. The aim was to determine the magnitude of the advantage of nations with a large population relative to the less populous nations, and to see if it is realistic for small nations to aspire to outperform the larger nations.

2. Predictions of the Mathematical Model

The method of analysis used here is only applicable to sports such as athletics, where the outcome of a competition is decided by the measurement of individual performances. The independent variable in this study was the population of the nation. All other factors such as wealth, the fraction of GDP spent on sport, sports participation rates, and genetic variability, were assumed to be identical among nations. Athletic performances within a nation were assumed to have a Gaussian distribution about the mean performance, and all nations were assumed to have an identical mean performance and standard deviation. The distribution of performances within a nation is then given by (Taylor, 1997):

\[ P_{X,\sigma}(x) = \frac{N}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - X)^2}{2\sigma^2}\right], \]  

where \( x \) is the athletic performance variable, \( N \) is the population of the nation, \( X \) is the population mean performance, and \( \sigma \) is the standard deviation about the mean performance.
2.1 Best athlete in the nation

The performance by the best athlete (in standard deviations above the mean performance) was calculated by integrating equation 1 (Figure 1). As expected, the greater the population of the nation the farther the best performance is above the mean performance (Figure 2). For nations with a population of greater than about 10 000, the relation between the performance by the best athlete and the logarithm of the population is approximately linear:

\[ SD_{\text{best}} = 0.414 \log_{10} N + 2.28, \]  

where \( SD_{\text{best}} \) is the number of standard deviations above the mean performance for the performance by the best athlete, and \( N \) is the population of the nation. That is, for the larger nations, athletic performance increases at the rate of about 0.4 standard deviations per factor of ten increase in population. To achieve international success, nations with a small population must overcome this inherent disadvantage that arises from their population.

![Figure 1. Example of the extreme upper end of the Gaussian distribution of performances for two nations with different populations (\( N \)). The best athlete in a nation with a population of 2 million has a better performance than the best athlete in a nation with a population of 1 million (4.89 versus 4.75 standard deviations above the mean performance).](image1)

![Figure 2. Predicted relation between the performance by the best athlete in a nation (in standard deviations above the mean performance) and the population of the nation (\( N \)). The dashed line is the linear approximation to the relation.](image2)

2.2 National rankings

Most national athletic federations produce an annual ranking list of the top 40 athletes in the nation. The expected relation between an athlete’s performance and their ranking was calculated by integrating the equation for the Gaussian distribution (equation 1). The performances by equal-ranked athletes are better in a nation with a greater population, and at the higher rankings there is a near-linear relation between performance and the logarithm of the ranking (Figure 3).
3. Results

There are 24 athletics events in the Olympic competition programme (sprints, middle-distances, long-distances, hurdles, relays, walks, jumps, throws, and multi-events). However, this study examined just one event: the long jump. The long jump was chosen because performances in this event have arguably been the least influenced by improvements in technique and technology. The predictions of the mathematical model were compared to long jump competition statistics obtained from athletics yearbooks. The study was restricted to the industrialized nations of Europe, USA, Canada, Australia, New Zealand, and Japan. These nations are all relatively wealthy, and so have a relatively large fraction of the population with an opportunity to participate in competitive sports. The populations of the nations were obtained from the Central Intelligence Agency website (CIA, 2006).

3.1 Best athlete in the nation

One definition of the ‘best athlete in a nation’ is the athlete that produces the year’s best performance. However, this information was not readily available for all the desired nations. An alternative definition used in the present study is that the best athlete in a nation is the winner of the national championships. The Association of Track and Field Statisticians produces a yearbook which contains the winning performances at the national championships for most IAAF member nations (Matthews, 2005). A 10-year average winning performance was calculated to even out inter-year fluctuations in performance.

As predicted by the mathematical model, there was a strong linear relation between jump distance and the logarithm of the population of the nation (Figure 4). The lines of best fit to the data indicate that the mean performance is $3.7 \pm 0.3$ m for men and $2.1 \pm 0.4$ m for women. For both men and women the standard deviation about the mean performance is $0.8 \pm 0.1$ m. Here, the mean performance is determined by the intercept on the distance axis, and the standard deviation is determined by the slope of the line of best fit. In these calculations it was assumed that the population of a nation is divided equally between males and females.

A key assumption in this study was that differences in the wealth of the nation have no influence on performance. For the western industrialized nations in this study, the wealth of a nation as measured by gross domestic product (GDP) was not correlated with long jump performance (Figure 5).

Unfortunately, there are no large-scale studies of long jump performance which can be used to confirm the calculated mean performance and standard deviation from the present study. However, a study of long jump performances by 17-year old male recruits to the Swiss Army reported a mean long jump performance of $4.5$ m (Jokl and Jokl, 1968). Unfortunately, there was no report of the standard deviation about the mean.

Part of the discrepancy in the population means between the present study and the Swiss Army study arises because citizens of different ages are not all equally likely to become national champion. The ages of world-ranked long jumpers ranges from about 18 to 36 years, with an average of about 26 years (Matthews, 2006). Therefore, on age considerations alone, the national champion is selected from a subset of about 10%
of the total population of the nation. Using population values that are 10% of the total population increases the mean long jump performance in the present study to 4.0 m for men and 2.5 m for women. The mean performance will be further increased if the appropriate population is the even smaller subset of citizens that are competitive long jumpers. (See section 3.2.)

Figure 4. Effect of a nation’s population on the winning long jump performance at the national championships. Data points are 10-year averages for the western industrialized nations. Error bars indicate the standard deviation in the average performance, and the dashed lines are lines of best fit to the data.

Figure 5. The wealth of a nation is not correlated with the winning long jump performance at the national championships. Data points are for the same western industrialized nations shown in Figure 4.

The line of best fit in Figure 4 may be used to determine how a nation is performing relative to its population (Table 1). For example, we conclude that Finland and USA are overperforming, whereas Great Britain and Luxembourg are underperforming. Among the overachieving but less populous nations, the magnitude of overperformance is not sufficient to overcome the inherent disadvantage of population (about 0.3 m per factor of ten increase in population).
Table 1. Long jump performances for selected nations.

<table>
<thead>
<tr>
<th>Nation</th>
<th>Population (millions)</th>
<th>Performance (m)</th>
<th>Performance relative to population (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>men</td>
<td>women</td>
</tr>
<tr>
<td>USA</td>
<td>275</td>
<td>8.37</td>
<td>6.87</td>
</tr>
<tr>
<td>Germany</td>
<td>82</td>
<td>8.00</td>
<td>6.73</td>
</tr>
<tr>
<td>Great Britain</td>
<td>59</td>
<td>7.82</td>
<td>6.46</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>10</td>
<td>7.76</td>
<td>6.26</td>
</tr>
<tr>
<td>Finland</td>
<td>5</td>
<td>7.79</td>
<td>6.47</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>0.4</td>
<td>7.10</td>
<td>5.64</td>
</tr>
</tbody>
</table>

3.2 National rankings

National long jump rankings lists for selected nations were obtained from yearbooks and websites. The relations between performance and the logarithm of the ranking were approximately linear (Figure 6), which suggests that athletic performances within a nation have a Gaussian distribution. However, the rate of decrease in performance was similar to that expected for a population of about one thousandth of the total population, rather than that for the total population. This suggests that the appropriate population when comparing nations is the subset of citizens that are competitive athletes. For the men, the population standard deviation of 0.8 m calculated from the study of the winning performances at the national championships (section 3.1) is consistent with the value indicated by the slope of the rankings data (Figure 6). However, the ranking data for the women suggest that the standard deviation is only about 0.55 m.

Figure 6. Relation between long jump performance and national ranking for selected western industrialized nations: (a) men; (b) women. Ranking data are 4-year averages for the USA (pop. = 275 million), Great Britain (pop. = 59 million), and Switzerland (pop. = 7 million). The dashed lines are the predicted relations for selected population sizes \(N\), assuming \(X = 5.3\) m and \(\sigma = 0.8\) m for men and \(X = 4.5\) m and \(\sigma = 0.55\) m for women.

3.3 Historical increase in world standards

Long jump performances have improved substantially over the last century (Figure 7). However, very little of the improvement is due to the increase in the world’s population (from 2.6 billion in 1950, to 6.4 billion in 2005). Most of the improvement in performance is probably due to socio-economic factors that have led to increased participation in sport. That is, the historical trend in athletics performances has arisen mostly from increases in the population of competitive athletes, rather than from increases in the total population of the world.
Figure 7. Historical trends in long jump performances for men and women. Data points are the world-leading performance in each year. The dashed lines indicate the expected improvement in performance since 1950 arising from the increase in the world’s population.

4. Conclusions
Among the western industrialised nations, the performances in national rankings lists and the performances achieved by the best athlete in the nation have relations that are characteristic of a Gaussian distribution. To achieve international success in the long jump, the smaller nations have to overcome a performance deficit of about 0.3 m per factor of ten increase in population. This study suggests that when comparing athletic performances among nations the appropriate population for comparison is the subset of citizens that are competitive athletes, rather than the total population of the nation. One method for a nation to increase its chances of international athletics success is through increasing the participation rate in the sport.

Acknowledgements
Thanks to John Winter and Chris Morgan for conducting the preliminary investigations of the effects of population and wealth on athletic performance.

References